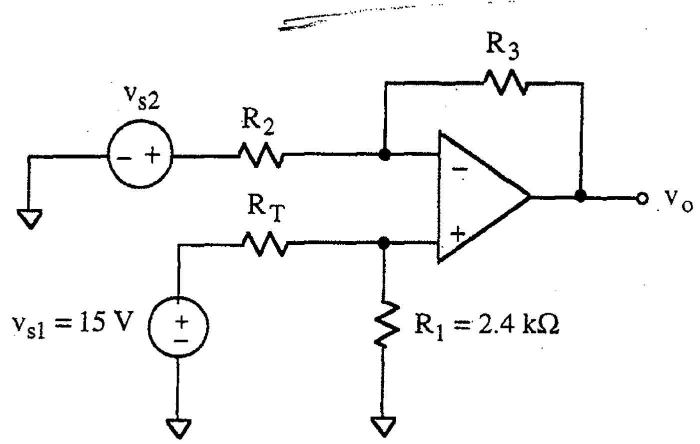
EE 1000

Final Exam soln

Spring 2001

1. (70 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{1120\left(\frac{1}{T} - \frac{1}{300}\right)}$$

where $R_o = 2490\ \Omega$ and T is temperature in °K.

Pts

10 a. ~~Calculate the numerical values of R_T (273°K) and R_T (373°K).~~

25 b. Derive a symbolic expression for v_o. The expression must contain not more than the parameters v_{s1}, v_{s2}, R_T, R_1, R_2, and R_3.
Hint: Use superposition.

25 c. Choose v_{s2}, R_2, and R_3 that will produce the following:

$v_o = 0$ V	when	$T = 273$°K
$v_o = 10$ V	when	$T = 373$°K

10 d. Using the component values you chose in (c), calculate v_o when $T = 323$°K.

sol'n 1.a)

$$R_T(273^\circ K) = 2490\, e^{1120\left(\frac{1}{273} - \frac{1}{300}\right)} = 3.6\,k\Omega \qquad \boxed{R_T(273^\circ K) = 3.6\,k\Omega}$$

$$R_T(373^\circ K) = 2490\, e^{1120\left(\frac{1}{373} - \frac{1}{300}\right)} = 1.2\,k\Omega \qquad \boxed{R_T(373^\circ K) = 1.2\,k\Omega}$$

b) Superposition:

V_{S1} on, V_{S2} off $\Rightarrow$ $V_p = V_{S1} \cdot \dfrac{R_1}{R_1 + R_T}$, $\quad V_n = V_p$

$$i_f\,(\text{thru } R_2) = \frac{0 - V_n}{R_2} = -V_{S1}\,\frac{R_1}{R_1+R_T}\,\frac{1}{R_2}$$

$$i_f\,(\text{thru } R_3) = \frac{V_n - V_{01}}{R_3} = -V_{S1}\,\frac{R_1}{R_1+R_T}\,\frac{1}{R_3}$$

$$i_f\,(\text{thru } R_2) = i_f\,(\text{thru } R_3) \Rightarrow \frac{-V_n}{R_2} = \frac{V_n - V_o}{R_3}$$

or $$V_{01} = V_n\left(1 + \frac{R_3}{R_2}\right) = V_{S1}\,\frac{R_1}{R_1+R_T}\left(1 + \frac{R_3}{R_2}\right)$$

V_{S1} off, V_{S2} on $\Rightarrow$ $V_p = 0V$, $\quad V_n = V_p$

$$i_f\,(\text{thru } R_2) = \frac{V_{S2}}{R_2} \qquad i_f\,(\text{thru } R_3) = -\frac{V_{02}}{R_3}$$

$$i_f\,(\text{thru } R_2) = i_f\,(\text{thru } R_3) \Rightarrow V_{02} = -V_{S2}\,\frac{R_3}{R_2}$$

$$V_o = V_{01} + V_{02} \qquad \boxed{V_o = V_{S1}\,\frac{R_1}{R_1+R_T}\left(1+\frac{R_3}{R_2}\right) - V_{S2}\,\frac{R_3}{R_2}}$$

c) At $273^\circ K$, $$V_{S1}\,\frac{R_1}{R_1+R_T} = 15V\,\frac{2.4k\Omega}{2.4k\Omega + 3.6k\Omega} = 15V \cdot \frac{2}{5} = 6V = V_p$$

At $373^\circ K$, $$V_{S1}\,\frac{R_1}{R_1+R_T} = 15V\,\frac{2.4k\Omega}{2.4k\Omega + 1.2k\Omega} = 15V \cdot \frac{2}{3} = 10V = V_p$$

$$V_o = \underbrace{V_p\left(1+\frac{R_3}{R_2}\right)}_{\text{proportional to } V_p} - \underbrace{V_{S2}\,\frac{R_3}{R_2}}_{\text{constant}}$$

The change in V_o vs change in V_p:

$$\Delta V_o = \Delta V_p\left(1 + \frac{R_3}{R_2}\right)$$

$$\Delta V_p = V_p(373^\circ K) - V_p(273^\circ K) = 10V - 6V = 4V$$

$$\Delta V_o = V_o(373^\circ K) - V_o(273^\circ K) = 10V - \underbrace{0V}_{\text{from prob statement}} = 10V$$

sol'n 1.c) cont. $\Delta V_o = \Delta V_P \left(1 + \frac{R_3}{R_2}\right)$ or $10V = 4V \left(1 + \frac{R_3}{R_2}\right)$

$\therefore 1 + \frac{R_3}{R_2} = 2.5$ or $\frac{R_3}{R_2} = 1.5$

Let $\boxed{R_3 = 15\,K\Omega}$, $\boxed{R_2 = 10K\Omega}$.

$V_o(273°K) = 0V \underset{\text{from prob statement}}{=} V_P\left(1 + \frac{R_3}{R_2}\right) - V_{s2}\frac{R_3}{R_2} = 6V(2.5) - V_{s2}(1.5)$

or $V_{s2} = \frac{6(2.5)V}{1.5}$ or $\boxed{V_{s2} = 10V}$

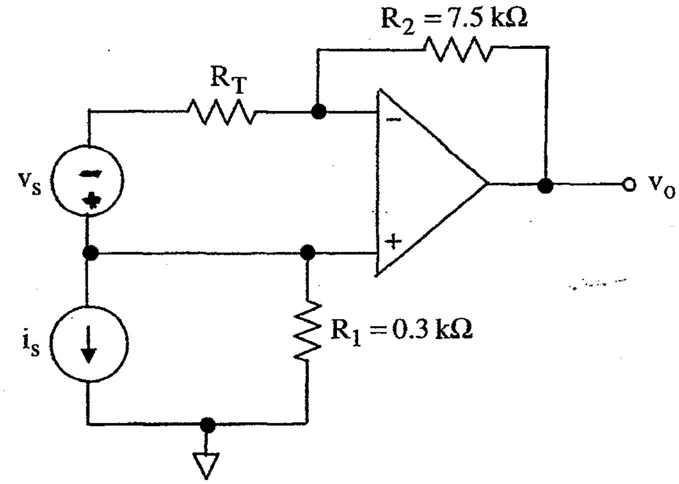
d) $R_T(323°K) = 2490\, e^{1120\left(\frac{1}{323} - \frac{1}{300}\right)} = 1.9\,K\Omega$

$V_P = 15V \cdot \frac{2.4K\Omega}{2.4K\Omega + 1.9K\Omega} = 8.37\ V$

$V_o = V_P\left(1 + \frac{R_3}{R_2}\right) - V_{s2}\frac{R_3}{R_2} = 8.37(2.5) - 10(1.5)\ V$

$\boxed{V_o(323°K) = 5.9\,V}$ Off by 18% from linear value of 5V.

1. (75 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

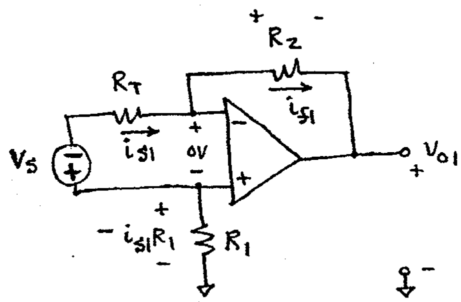
$$R_T = R_o e^{\beta\left(\frac{1}{T} - \frac{1}{300}\right)}$$

where $R_o = 2.625\ k\Omega$, $\beta = 1200°K$, and T is temperature in °K.

Pts		
30	a.	Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_s, V_s, R_1, R_2, and R_T. **Hint: Use superposition.**
10	b.	Calculate the numerical values of R_T (273°K) and R_T (373°K).
15	c.	Determine a value for v_s such that $v_o(T = 373°K) - v_o(T = 273°K) = 1V$
10	d.	Using your answer to (c), determine a value of i_s such that $v_o(T = 273°K) = 0\ V$.
10	e.	Using the component values you chose above, calculate v_o when T = 323°K. Make a rough sketch of v_o vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

Sol'n: 1. a) Use superposition

Case I: v_s on, i_s off = open



Negative feedback ⇒ 0V across + and − terminals.

From v loop around v_s, R_T, and +− terminals,

we have $i_{s1} = \frac{-v_s}{R_T}$.

From v loop around R_1, +− terminals, R_2, and v_o,

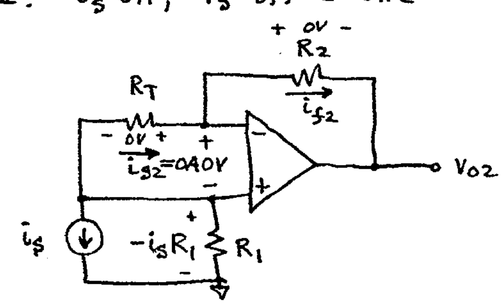
we have $-i_{s1}R_1 + 0V - i_{f1}R_2 - v_{o1} = 0V$

Now use $i_{s1} = i_{f1}$ and solve for v_{o1}:

$$- -\frac{v_s}{R_T} R_1 + 0V - -\frac{v_s}{R_T}R_2 - v_{o1} = 0V$$

$$v_{o1} = v_s \frac{R_1 + R_2}{R_T}$$

Case II: i_s on, v_s off = wire



We 0V across R_T ⇒ $i_s = 0$.

Since $i_{s2} = i_{f2} = 0A$, we have $v_{o2} = v_- = v_+$

$v_+ = -i_s R_1$ ⇒ $v_{o2} = -i_s R_1$

Sum $$\boxed{v_o = v_{o1} + v_{o2} = v_s \frac{R_1 + R_2}{R_T} - i_s R_1}$$

soln: 1. b) $R_T(273°K) = 2.625k\Omega\, e^{1200°K\left(\frac{1}{273°K} - \frac{1}{300°K}\right)}$

$$\boxed{R_T(273°K) \doteq 3.9k\Omega}$$

$R_T(373°K) = 2.625k\Omega\, e^{1200°K\left(\frac{1}{373°K} - \frac{1}{300°K}\right)}$

$$\boxed{R_T(373°K) \doteq 1.2k\Omega}$$

c) $1V = V_o(373°K) - V_o(273°K) = V_S\frac{(R_1+R_2)}{R_T(373°K)} - \cancel{i_S R_1}$

$$-\left(V_S\frac{R_1+R_2}{R_T(273°K)} - \cancel{i_S R_1}\right)$$

$$= V_S(R_1+R_2)\left(\frac{1}{R_T(373°K)} - \frac{1}{R_T(273°K)}\right)$$

$$= V_S(\underbrace{0.3k\Omega + 7.5k\Omega}_{7.8k\Omega})\left(\frac{1}{1.2k\Omega} - \frac{1}{3.9k\Omega}\right)$$

$$V_S = \frac{1.2k\Omega \,\|\, (-3.9k\Omega)}{7.8k\Omega}V = \frac{\cancel{0.3k\Omega}}{\cancel{0.3k\Omega}}\cdot\frac{4\,\|\,-13}{26}V = \frac{-4\cdot\cancel{13}^{\,2}}{(4-13)\,\cancel{26}}V$$

$$\boxed{V_S = \frac{2}{9}V}$$

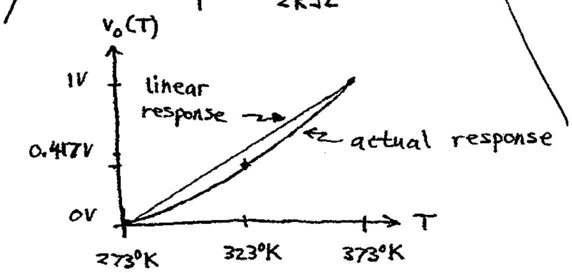
d) $0V = V_o(273°K) = V_S\frac{R_1+R_2}{R_T(273°K)} - i_S R_1 = \frac{2}{9}V\,\frac{\cancel{7.8}^{\,2}k\Omega}{\cancel{3.9}k\Omega} - i_S\, 0.3k\Omega$

$i_S\, 0.3k\Omega = \frac{4}{9}V \Rightarrow i_S = \frac{4}{9}\frac{1V}{0.3k\Omega} = \frac{4}{2.7}mA$

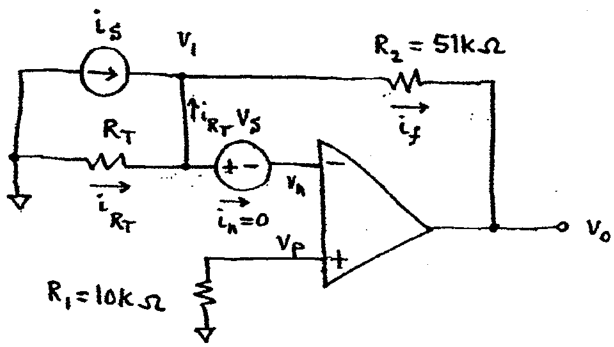
$$\boxed{i_S \doteq 1.48\,mA \approx 1.5\,mA}$$

e) $R_T(323°K) = 2.625k\Omega\, e^{1200°K\left(\frac{1}{323°K} - \frac{1}{300°K}\right)} = 1.97k\Omega \approx 2k\Omega$

$V_o(323°K) \approx \frac{2}{9}V\,\frac{7.8k\Omega}{2k\Omega} - 1.5\,mA\; 0.3k\Omega = 0.417V$



sol'n: 1.a)



- Find v_p: $v_p = 0V$ since no current in R_1 so no v-drop for R_1

$v_n = v_p = 0V$

- Find i_f on left side (total current into v_1 node from i_s and R_T)

$v_1 = v_n + v_s = v_s$, and no current flows thru v_s since no current flows into op-amp.

$\therefore$ i_f on left side $= i_s + i_{R_T} = i_s + \frac{0 - v_1}{R_T} = i_s + \frac{-v_s}{R_T}$

- Find i_f on right side

i_f on right side $= \frac{v_1 - v_o}{R_2} = \frac{v_s - v_o}{R_2}$

- Set i_f on left side $= i_f$ on right side

$$i_s - \frac{v_s}{R_T} = \frac{v_s - v_o}{R_2} \quad \text{or} \quad \boxed{v_o = v_s\left(1 + \frac{R_2}{R_T}\right) - i_s R_2}$$

verify: Superposition $v_s = 0 \Rightarrow i_{R_T} = 0 \Rightarrow v_o = 0V - i_s R_2$ ✓

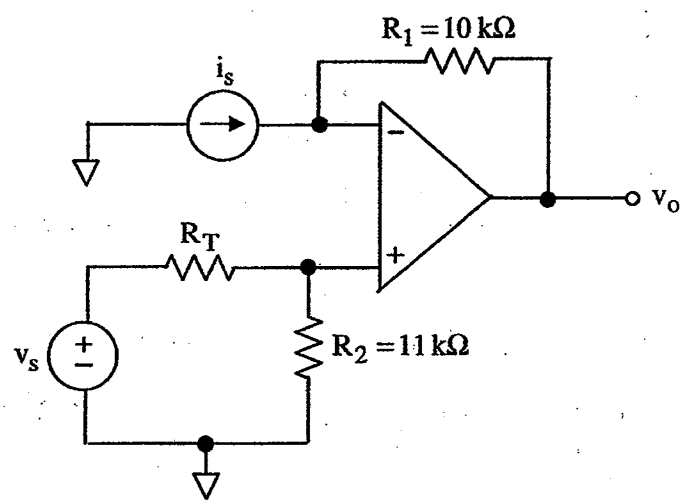
$i_s = 0 \quad i_{R_T} = \frac{-v_s}{R_T} = i_f = \frac{v_s - v_o}{R_2}$

Same as having $v_p = v_s$ $v_o = v_s\left(1 + \frac{R_2}{R_T}\right)$ ✓

UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 HOMEWORK #9 Spring 2005

1.



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{\beta\left(\frac{1}{T} - \frac{1}{300}\right)}$$

where $R_o = 12.25\,k\Omega$, $\beta = 170°K$, and T is temperature in °K. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_s, V_s, R_1, R_2, and R_T. **Hint: Use superposition.**

2. a. ~~Calculate the numerical values of R_T (273°K) and R_T (373~~°K).

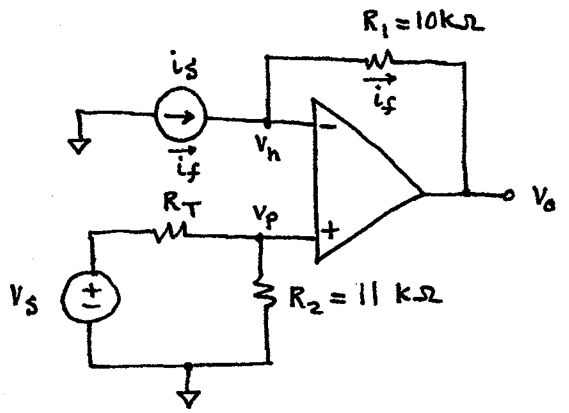
b. ~~Determine a value for v_s such that $v_o(T = 373°K) - v_o(T = 273°K) = 1V$~~

c. ~~Using your answer~~ to (c), ~~determine a value of i_s such that~~

v_o ~~(T = 273°K) = 0~~ V.

d. ~~Using the component values you~~ chose above, calculate v_o when T = 323°K. Make a rough sketch of ~~v_o vs. T on the basis of the~~ values when T = 273~~°K, 323°K, and 373°K. On the same axes, sketch the ide~~al linear response.

sol'n: 1.



Find v_p:

$$v_p = v_s \frac{R_2}{R_2 + R_T} \qquad \text{V-divider}$$

$v_n = v_p$

Find i_f on left: $i_f = i_s$

Find i_f on right: $i_f = \dfrac{v_n - v_o}{R_1}$

Set i_f's equal and use $v_n = v_p$

$$i_s = \frac{v_n - v_o}{R_1} \quad \text{or} \quad v_o = v_n - i_s R_1$$

$$\boxed{v_o = v_s \frac{R_2}{R_2 + R_T} - i_s R_1}$$

2. a) $R_T(273°K) = 12.25 k\Omega \cdot e^{170°K\left(\frac{1}{273°K} - \frac{1}{300°K}\right)} = 13 k\Omega$

$R_T(373°K) = 12.25 k\Omega \cdot e^{170°K\left(\frac{1}{373°K} - \frac{1}{300°K}\right)} = 11 k\Omega$

$$\boxed{\begin{aligned} R_T(273°K) &= 13k\Omega \\ R_T(373°K) &= 11k\Omega \end{aligned}}$$

b) $1V = v_o(373°K) - v_o(273°K) = v_s \dfrac{R_2}{R_2 + R_T(373°K)} - \cancel{i_s R_1}$

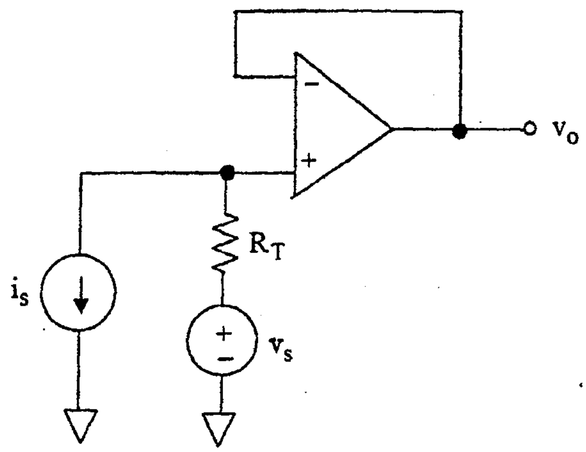
$$- \left(v_s \frac{R_2}{R_2 + R_T(273°K)} - \cancel{i_s R_1}\right)$$

$$1V = v_s \cdot 11k\Omega \cdot \left(\frac{1}{11k\Omega + 11k\Omega} - \frac{1}{11k\Omega + 13k\Omega}\right)$$

$$\boxed{v_s = \frac{1V}{11k} \frac{22k\Omega \cdot 24k\Omega}{24k\Omega - 22k\Omega} = 24\ V}$$

ECE 1000 Final Exam sol'n Sp 03

1. (75 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

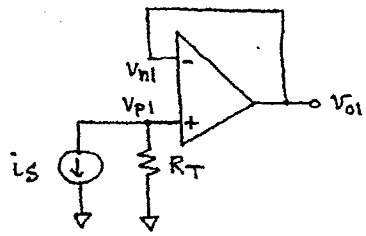
$$R_T = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{300}\right)}$$

where $R_o = 2.8\ k\Omega$, $\beta = 1300^\circ K$, and T is temperature in °K.

Pts		
30	a.	Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_s, V_s, and R_T. **Hint: Use superposition.**
10	b.	Calculate the numerical values of R_T (273°K) and R_T (373°K).
15	c.	Determine a value for i_s such that $v_o(T = 373^\circ K) - v_o(T = 273^\circ K) = 1V$
10	d.	Using your answer to (c), find the numerical value of v_s such that $v_o\ (T = 273°K) = 0\ V$.
10	e.	Using the component values you chose above, calculate v_o when T = 323°K. Make a rough sketch of v_o vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

10

sol'n: 1.a) Superposition   case I: i_s on, v_s off = wire

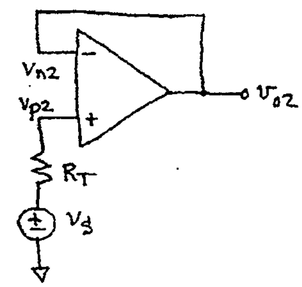


$v_{p1} = -i_s R_T$

$v_{n1} = v_{p1} = -i_s R_T$

$v_{o1} = v_{n1} = -i_s R_T$

case II: i_s off = open, v_s on



$v_{p2} = v_s$ (no current in R_T so no V-drop)

$v_{n2} = v_{p2} = v_s$

$v_{o2} = v_{n2} = v_s$

$$v_o = v_{o1} + v_{o2}$$

$$\boxed{v_o = -i_s R_T + v_s}$$

b) $R_T(273°K) = 2.8\,k\Omega\, e^{1300°K\left(\frac{1}{273°K} - \frac{1}{300°K}\right)}$

$\boxed{R_T(273°K) = 4.3\,k\Omega}$

$R_T(373°K) = 2.8\,k\Omega\, e^{1300°K\left(\frac{1}{373°K} - \frac{1}{300°K}\right)}$

$\boxed{R_T(373°K) = 1.2\,k\Omega}$

c) Change in v_o: $\Delta v_o = -i_s \, \Delta R_T$ $\Delta \equiv$ "change"
(for $\Delta T = 100°K$) $= 1V$ (desired)   (v_s constant does not change with T)

$\Delta R_T = 1.2k\Omega - 4.3k\Omega$

$\Delta R_T = -3.1\,k\Omega$

$\therefore \; i_s = \dfrac{-\Delta v_o}{\Delta R_T} = \dfrac{-1V}{-3.1k\Omega} = 0.323\text{ mA}$

$\boxed{i_s = 0.323\,mA}$

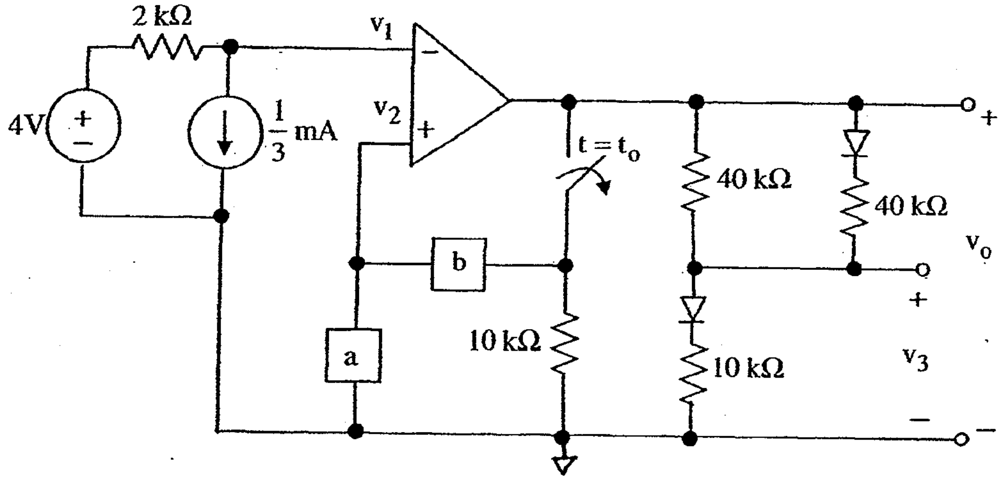
d) $v_o(273°K) = 0V = \underbrace{-0.323mA}_{-i_s} \cdot \underbrace{4.3k\Omega}_{R_T} + v_s$

$\therefore \; v_s = 0.323\,mA \cdot 4.3k\Omega$

$\boxed{v_s = 1.39\,V \text{ or } 1.4V}$

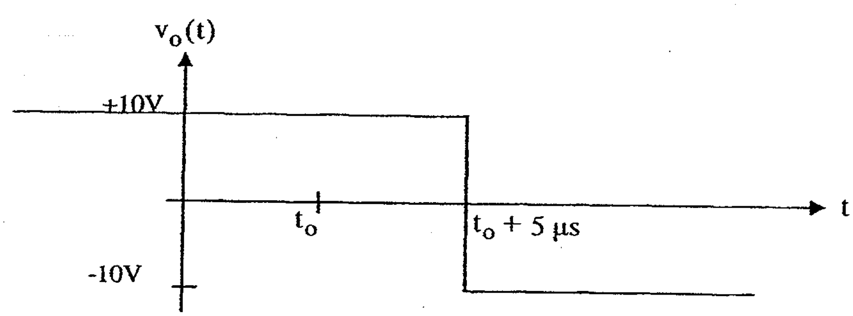
HW #10 cont. Su 2005

2.



Rail voltages = ±10 V

After being closed for a long time, the switch closes at $t = t_o$.



a. Choose either an R or L to go in box a and either an R or L to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.

b. Sketch $v_1(t)$, showing numerical values appropriately.

c. Sketch $v_2(t)$, showing numerical values appropriately.

d. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 5\ \mu s$, and for $t_o + 5\ \mu s < t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

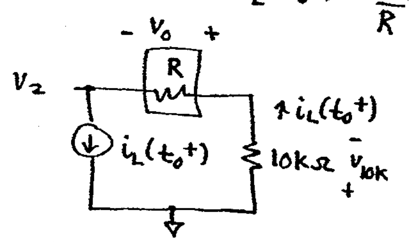
Explain your work carefully.

sol'n: 2. a) Consider possibilities.

$a = R$ and $b = R$ — Doesn't work. v_2 would change at t_0 but never again. Delay of $5\,\mu s$ not possible.

$a = L$ and $b = L$ — Doesn't work. Before time t_0, the L's look like wires and would short v_0 to ref. But $v_0 = \pm v_{Rail} = \pm 10V$. Thus, we would have an invalid circuit.

$a = L$ and $b = R$ — Before t_0, the L looks like a wire, and $v_2 = 0V$. $i_L(t_0^-) = \frac{v_0}{R} = i_L(t_0^+)$.



Circuit at t_0^+.

At t_0^+, the current in R will be $i_L(t_0^+) = i_L(t_0^-) = \frac{v_0}{R}$ or the same as at t_0^-. Thus, the v-drop for R at t_0^+ will be v_0. Current $i_L(t_0^+)$ flowing in the 10kΩ will cause a voltage drop in series with the drop for R, resulting in a very negative voltage at v_2. This would cause v_0 to go low at $t = t_0^+$ rather than after a delay of $5\,\mu s$.

∴ This case doesn't work.

HW #10 Cont.

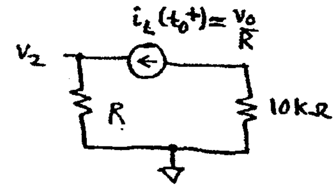
ECE 1000

Su 05

Sol'n: 2.a) cont. $\boxed{a = R \text{ and } b = L}$

$i_L(t_0^-) = \frac{V_0}{R}$ as in prev case.

At t_0^+ with $i_L(t_0^+) = i_L(t_0^-) = \frac{V_0}{R}$:



we have $V_2(t_0^+) = \underbrace{\frac{V_0}{R}}_{i_L(t_0^+)} \cdot R = V_0$.

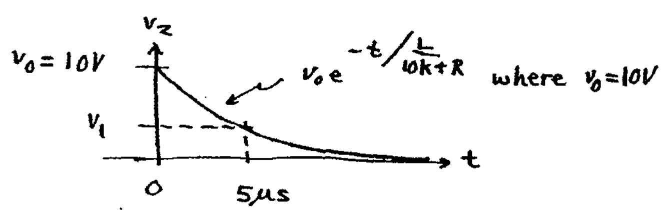
So V_2 doesn't change immediately.
This will work!
For $t \to \infty$ we will have $V_2 = 0V$
since there is no power source.

Using the general formula for
$V_2(t)$, we have:

$$V_2(t) = \underset{0V}{\underset{\parallel}{V_2(t\to\infty)}} + [\underset{V_0}{\underset{\parallel}{V_2(t_0^+)}} - \underset{0V}{\underset{\parallel}{V_2(t\to\infty)}}]\, e^{-t/\frac{L}{R_{Th}}}$$

where $R_{Th} = 10k\Omega + R$

$$V_2(t) = V_0\, e^{-t/\frac{L}{10k+R}}$$



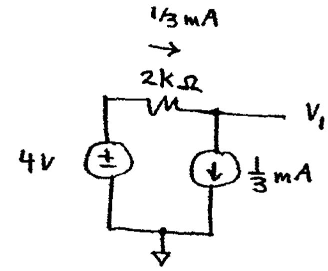
The output, V_0, will drop at
$t_0 + 5\mu s$ if $V_2(t_0 + 5\mu s) = V_1$.
[Assume $t_0 = 0$ for convenience.]

Now find V_1.

HW #10 Cont.

ECE 1000 Su 05

sol'n: 2.a) cont.



$$V_1 = 4V - \frac{1}{3}mA \cdot 2k\Omega$$

$$V_1 = 4V - \frac{2}{3}V = \frac{10}{3}V$$

$\therefore$ we want $V_2(5\mu s) = V_1 = \frac{10}{3}V$

or $$\cancel{10}V\, e^{-5\mu s / \frac{L}{10k\Omega + R}} = \frac{\cancel{10}}{3}V$$

$$-5\mu s \Big/ \frac{L}{10k\Omega + R} = \ln\frac{1}{3} \doteq -1.1$$

or $$5\mu s = 1.1 \frac{L}{10k\Omega + R}$$

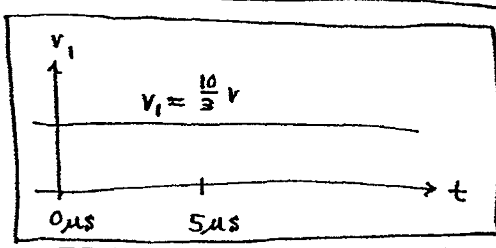
If we use R = 12k we get a convenient value for L. Many solutions for R and L will work, however.

$R = 12k\Omega$ gives $$L = 5\mu s \cdot \frac{10k\Omega + 12k\Omega}{1.1}$$

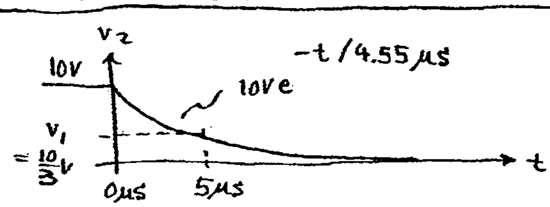
$$L = 5\mu s \cdot 20k\Omega = 100\,mH$$

$\boxed{R = 12k\Omega \quad L = 100\,mH}$ One solution among many.

b)

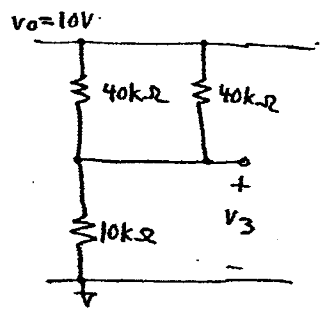


c)



Note: $$\frac{L}{R_{Th}} = \frac{100\,mH}{22k\Omega} = 4.55\,\mu s$$

sol'n: 2.d) For $v_o = +10V$, both diodes are forward biased and look like wires.

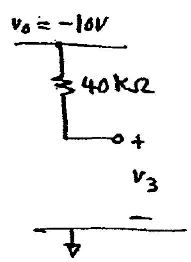


$$v_3 = 10V \cdot \frac{10k\Omega}{10k\Omega + 40k\Omega \| 40k\Omega}$$

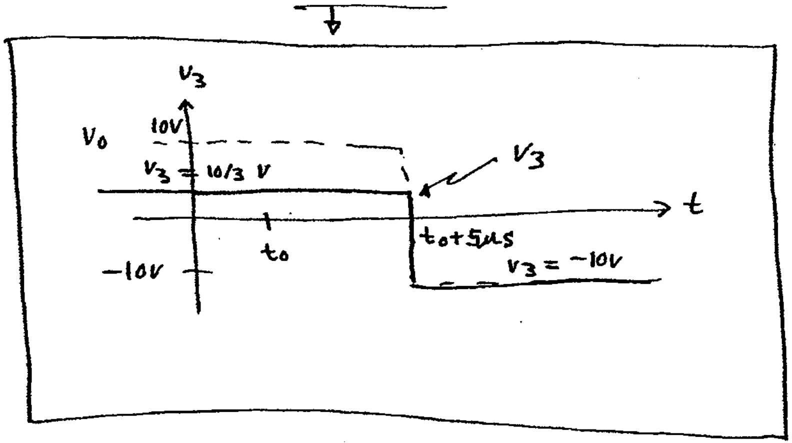
$$'' = 10V \cdot \frac{10k\Omega}{10k\Omega + 20k\Omega}$$

$$v_3 = \frac{10}{3}\ V$$

For $v_o = -10V$, both diodes are reverse biased and look like open circuits.



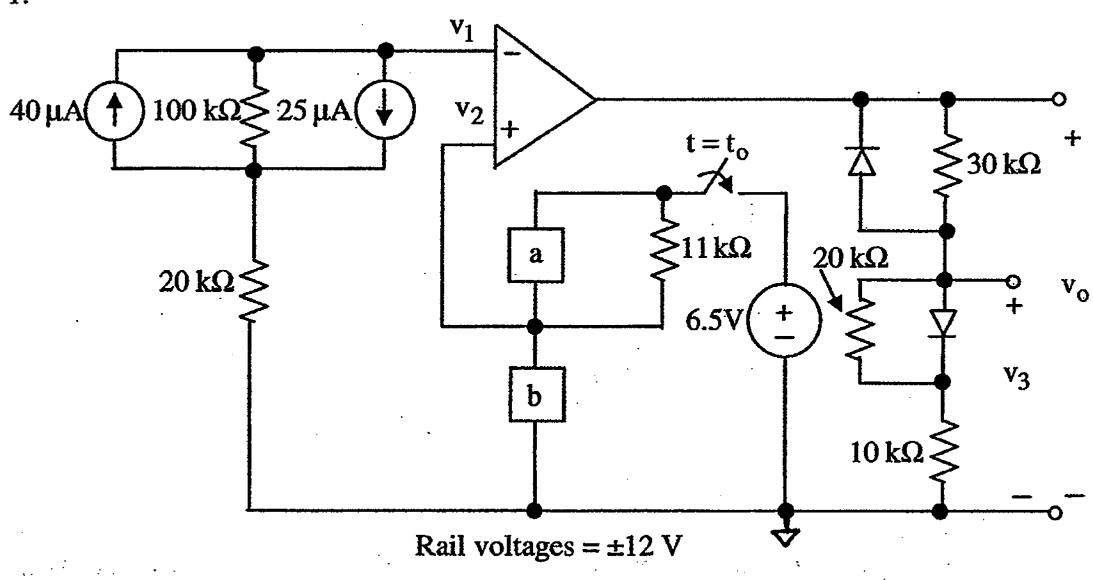
$v_3 = -10V$



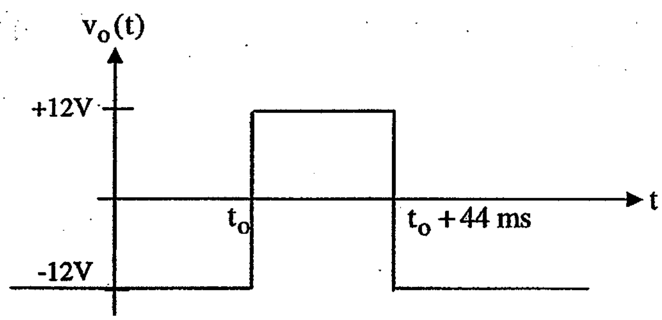
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 HOMEWORK #10 Spring 2005

1.



After being open for a long time, the switch closes at $t = t_o$.



a. Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value. **Hint:** Use $v_2(t \to \infty) = 1V$

b. Sketch $v_1(t)$, showing numerical values appropriately.

2. a. Sketch $v_2(t)$, showing numerical values appropriately.

b. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 44$ ms, and for $t_o + 44$ ms $< t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

sol'n: 1.a) Consider possibilities.

$a = R$ and $b = R$: v_2 would change at time $t = t_0$ but never change again. Thus, v_0 could not go high and then low again.

$a = C_1$ and $b = C_2$: Before $t = t_0$, there would be no path for C_2 in b to discharge. Thus, we would need to know $v_{C2}(t = t_0^-)$.

We would also have $v_{C1}(t = t_0^-) = 0V$ since C_1 will discharge thru the R in parallel with it.

When the switch closes, we have an invalid circuit if $v_{C2}(t_0^+) + v_{C1}(t_0^+) \neq 6.5V$. This means we must have $v_{C2}(t_0^+) = v_C(t_0^-) = 6.5V$ since $v_{C1}(t_0^+) = v_C(t_0^-) = 0V$.

But if $v_{C2}(t_0^+) = 6.5$, nothing changes when the switch closes. Thus, we could not get a waveform that goes high and then low again.

$a = R$ and $b = C$: As in the case of $a = C_1$ and $b = C_2$, we would have some value of $v_C(t_0^-) = v_C(t_0^+)$. Since $v_2 = v_C(t_0^+)$ at $t = t_0^+$, v_0 would not change at t_0. Thus, this will not work.

$\boxed{a = C \text{ and } b = R}$: The C in a will discharge thru the R in parallel with it. $\therefore v_C(t_0^-) = 0V$. Also, no current flows in R for $t < t_0$ because there is no closed circuit path in which current could flow. Thus, $v_2(t_0^-) = i \cdot R = 0V$.

sol'n: 1.a) cont.

At $t = t_0^+$, we have $v_2(t_0^+) = 6.5V - v_C(t_0^+)$ or $v_2(t_0^+) = 6.5V$ since $v_C(t_0^+) = v_C(t_0^-) = 0V$.

Thus, v_2 jumps from 0V to 6.5V at t_0.

If $v_2(t_0^+) > v_1$, then v_o would go high, (assuming $v_1 > 0V$ and $v_1 < 6.5V$).

After t_0, C will charge and v_2 will start to drop. If v_2 eventually drops below v_1, then v_o will go low again.

Thus, this will work.

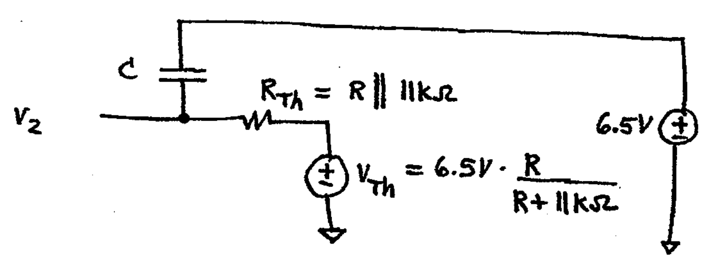
Now find values of R and C:

v_1: Since no current flows into the − input of the op amp, no current flows thru the 20kΩ resistor (lower left).

Combining the parallel current sources, we have $40\mu A - 25\mu A = 15\mu A$. This current flows thru the 100kΩ to produce voltage $15\mu A \cdot 100k\Omega = 1.5V$.

$\therefore v_1 = 2.5V$ (constant)

v_2: For $t > t_0$, we use a Thevenin equivalent circuit for the 6.5V source, the 11kΩ, and R in b.



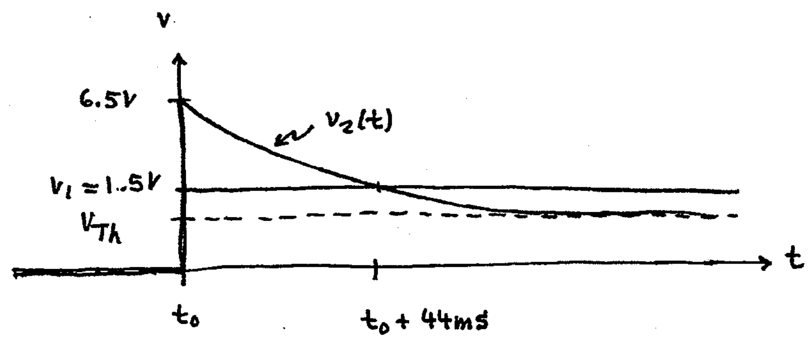
soln: 1.a) cont. We use the general RC soln for $v_2(t)$:

$$v_2(t) = v_2(t\to\infty) + [v_2(t_0^+) - v_2(t\to\infty)]\, e^{-t/R_{Th}C}$$

To find $v_2(t\to\infty)$, we observed that, when C is charged, no current flows in R_{Th}.

Thus $v_2(t\to\infty) = V_{Th} = 6.5V \dfrac{R}{R+11k\Omega} < 6.5V.$

From earlier, $v_2(t_0^+) = 6.5V.$



Any $V_{Th} < v_1$ will suffice. For convenience,

let $V_{Th} = 1V = 6.5V \cdot \dfrac{R}{R+11k\Omega} \Rightarrow \boxed{R = 2k\Omega}$

Assume $t_0 = 0$. For v_0 to switch at $t = 44ms$,

we have $v_2(44ms) = v_1 = 1.5V$

or $1V + [6.5V - 1V]\, e^{-44ms/R_{Th}C} = 0.5V$

or $5.5V\, e^{-44ms/R_{Th}C} = 0.5V$ where

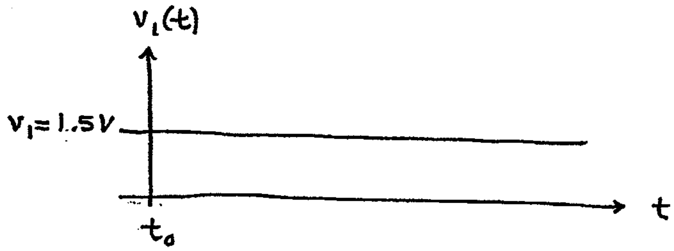
$R_{Th} = R \| 11k\Omega = 2k \| 11k\Omega = \dfrac{22}{13} k\Omega.$

Take ln of both sides of exponential RC eqn:

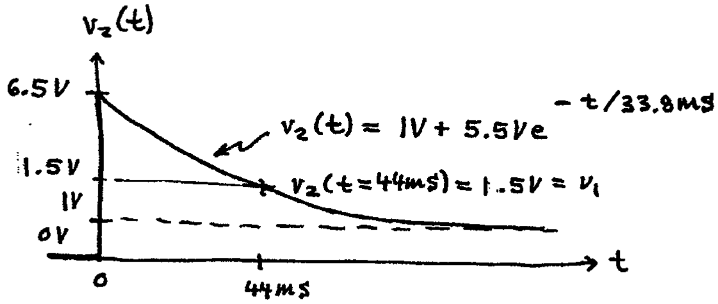
$$-\frac{\overset{2}{\cancel{44}}ms}{\frac{\cancel{22}}{13}k\Omega \cdot C} = \ln \frac{0.5V}{5.5V} \doteq -2.39$$

$$C = \frac{2}{\frac{2.39}{13}} \mu F = 10.9\mu F \qquad \boxed{C = 11\mu F}$$

Sol'n: 1.b) $v_1(t) = 1.5V$ constant



2.a)

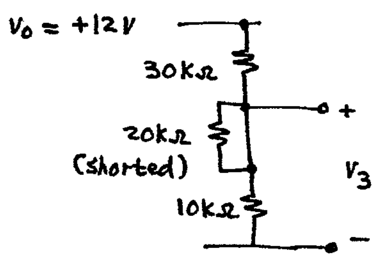


Note: $R_{Th} C = \frac{22}{13} k\Omega \cdot 20\mu F \doteq 33.8ms$

Note: Assume $t_0 = 0$.

b) When $v_0 = +12V$, top diode is reverse biased = open.

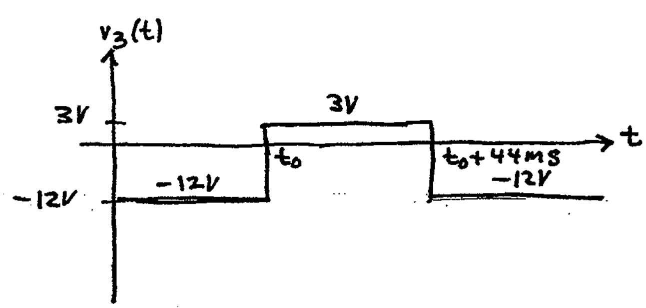
The bottom diode is forward biased = wire.



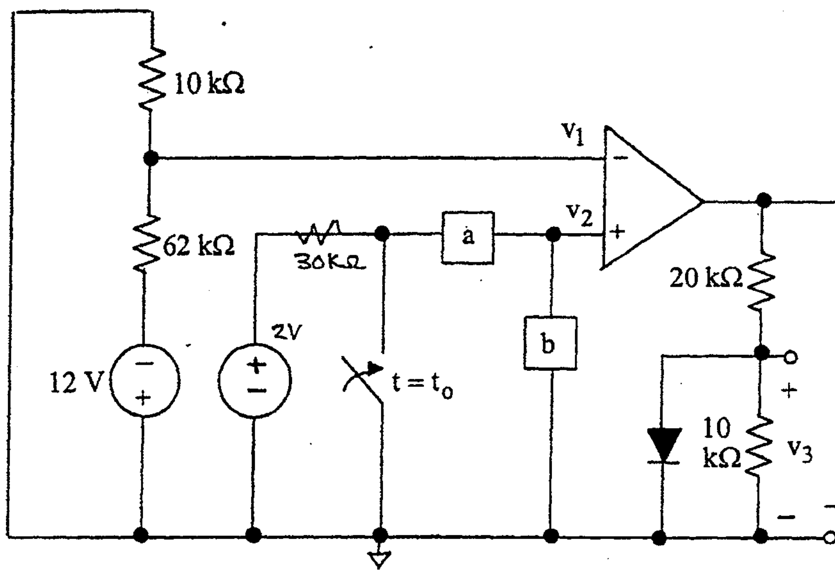
$$v_3 = v_0 \cdot \frac{10k\Omega}{10k\Omega + 30k\Omega} = 12V \cdot \frac{1}{4} = 3V$$

when $v_0 = -12V$, top diode is forward biased = wire.

Thus, v_0 is shorted to $-12V$.

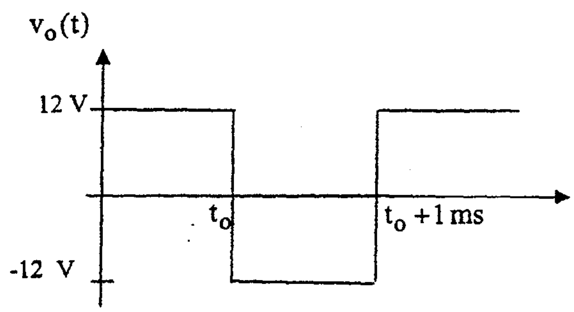


2. (65 points)



Rail voltages = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



Pts

35 a. Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.

5 b. Sketch $v_1(t)$, showing numerical values appropriately.

15 c. Sketch $v_2(t)$, showing numerical values appropriately.

10 d. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 1$ ms, and for $t < t_o + 1$ ms. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

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sol'n: 2. a) At $t = t_o^-$: $V_1 = -12V \dfrac{10k\Omega}{10k\Omega + 62k\Omega}$ V-divider

$V_1 = -\frac{5}{3}V$ Actually, $V_1 = -\frac{5}{3}V$ for all t

$V_o = +12V \Rightarrow V_2 > V_1 = -\frac{5}{3}V$

Consider $a = R$, $b = R$: V_o would never switch because V_2 would be 0V after switch closed, and $V_2 > V_1$ for all time

Will not work.

Consider $a = C$, $b = C$: C's will charge to $V_{tot} = 2V$.

Part of 2V across a, part across b.

When switch closes, the C's instantly charge to $V_{tot} = 2V$.

Then their voltages remain fixed.

$\therefore$ V_o would not switch after 1 ms.

Will not work.

Consider $a = R$, $b = C$: C will charge to 2V. $V_2 = 2V$

When switch closes, V_2 will start at V_2 and charge toward 0V.

$V_2 > V_1$ for all time.

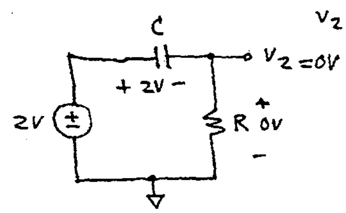
V_o never switches.

Will not work.

Consider $\boxed{a = C,\ b = R}$: C will charge to 2V at t_o^-

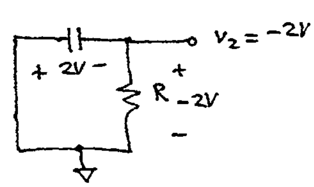
$V_2 > V_1$ so $V_o = +12V$ at t_o^- ✓

t_o^-:



When switch closes, V_C will stay at 2V for $t = t_o^+$:

t_o^+:



Thus, V_2 drops to −2V.

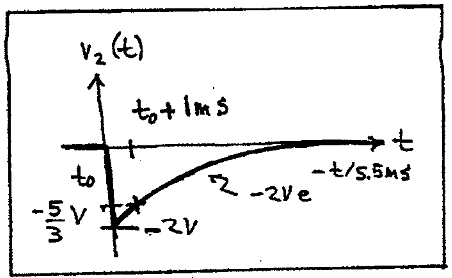
Then $V_2 < V_1$ and V_o drops to −12V. ✓

$t > t_o$: C charges to 0V and V_2 climbs toward 0V.

We want $V_2 = V_1 = -\frac{5}{3}V$ at $t_o + 1ms$. ✓

Works

sol'n: 2.a) (cont.)



sol'n: 2.c)

Let $t_o = 0$

$$v_2(t) = -2V\, e^{-t/RC}$$

$$v_2(1ms) = -2V e^{-1ms/RC} = -\frac{5}{3}V$$

$$e^{-1ms/RC} = \frac{5}{6}$$

$$-1ms/RC = \ln \frac{5}{6}$$

$$RC = \frac{-1ms}{\ln \frac{5}{6}} = 5.48\,ms$$

$$RC \approx 5.5\ ms$$

Let $\boxed{C = 1\mu F, \quad R = 5.5k\Omega \text{ or } 5.6k\Omega \text{ (standard value)}}$

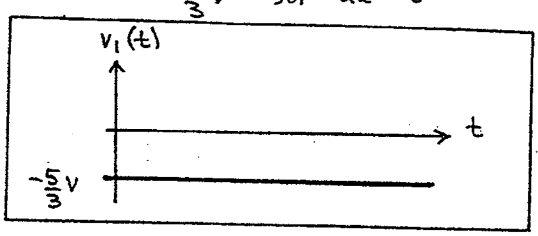
Any $RC = 5.5\ ms$ acceptable if $1pF < C < 1F$ and $1\Omega < R < 1G\Omega$.

Note: If R is $\frac{1}{8}W$ then we would really want $\max i_R^2 R < \frac{1}{8}W$.

$$\max i_R^2 = \left(\frac{2V}{R}\right)^2 \Rightarrow \max i_R^2 R = \frac{4V^2}{R} < \frac{1}{8}W.$$

$$\therefore\ R > \frac{4V^2}{1/8\ W} = 32\Omega \text{ required.}$$

2.b) $v_1 = -\frac{5}{3}V$ for all t

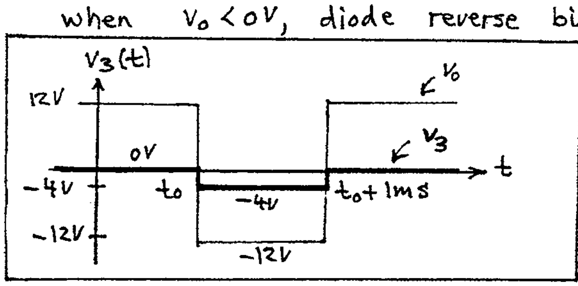


2.c) See plot of $v_2(t)$ in sol'n to 2.a), above.

2.d) When $v_o > 0V$, diode forward biased = wire. $\therefore V_3 = 0V$

when $V_o < 0V$, diode reverse biased = open. $\therefore V_3 = V_o \dfrac{10k\Omega}{10k\Omega + 20k\Omega}$

$$V_3 = -12V \cdot \frac{1}{3} = -4V$$



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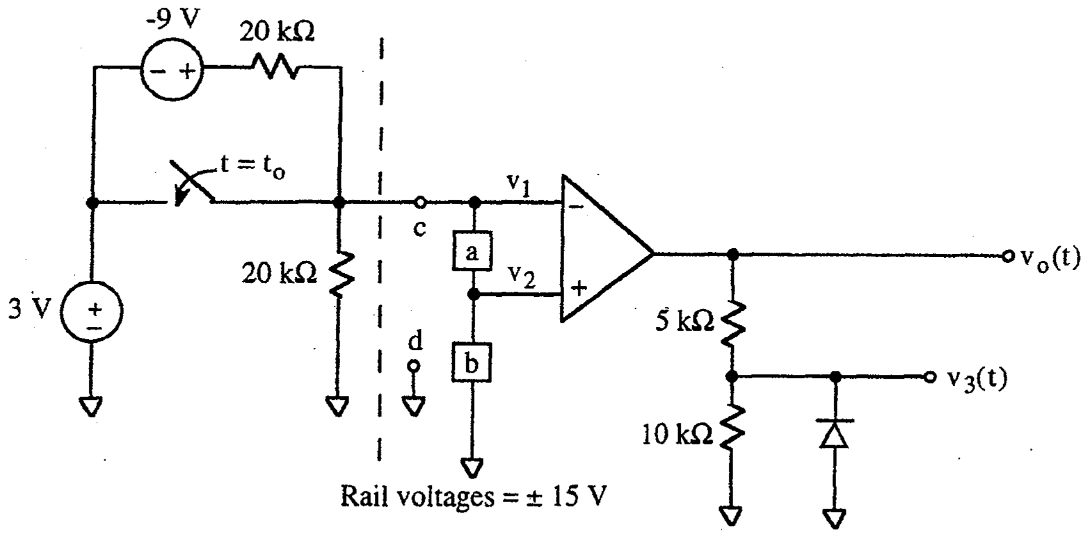
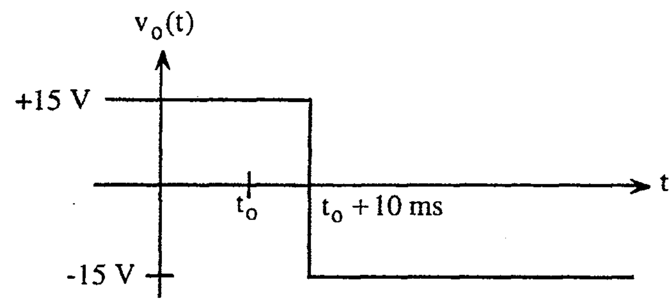
2. (70 points)



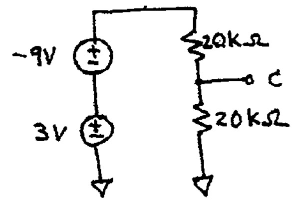
After being open for a very long time, the switch closes at $t = t_o$.



Pts		
10	a.	Find the Thevenin equivalent of the circuit left of the dashed line with respect to terminals c, d for $\mathbf{t < t_o}$.
10	b.	Find the Thevenin equivalent of the circuit left of the dashed line with respect to terminals c, d for $\mathbf{t > t_o}$.
20	c.	Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.
20	d.	Using the elements found in (b), sketch $v_2(t)$. Show numerical values appropriately.
10	e.	Sketch $v_3(t)$. Show numerical values for $t < t_o$, $t_o < t < t_o + 10$ ms, and $t > t_o + 10$ ms. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

sol'n 2. a) $t < t_0$ switch is open

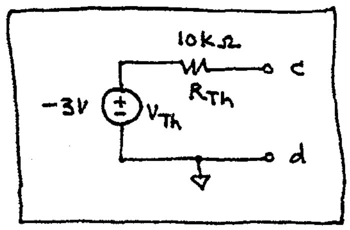


$V_{Th} = (3V - 9V) \dfrac{20k\Omega}{20k\Omega + 20k\Omega} = -6V \cdot \dfrac{1}{2} = -3V$

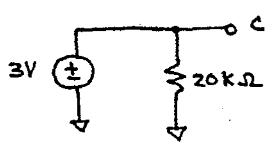
use V-divider

R_{Th} = R looking into c with V sources shorted

$= 20k\Omega \parallel 20k\Omega = 10k\Omega$



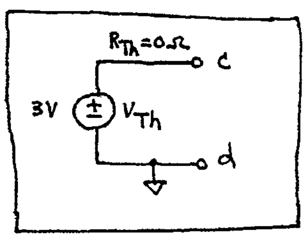
b) $t > t_0$ switch is closed (-9V source and 20kΩ bypassed)



$V_{Th} = 3V$ since c connected to 3V source

R_{Th} = R looking into c with 3V source shorted

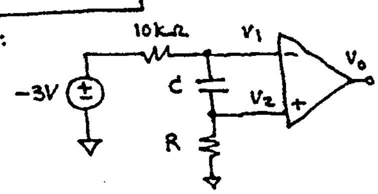
$= 0\Omega$ (20kΩ bypassed by short)



c) R in a and C in b would immediate switching: <u>wrong</u>

$\therefore$ $\boxed{\text{C in a and R in b}}$ must be correct.

$t < t_0$ circuit:
$(t = t_0^-)$



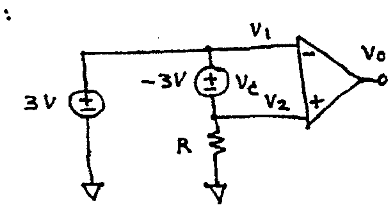
C is open circuit
no V drop across R's

$\therefore$ $V_2 = 0V$

$V_1 = -3V$

$V_1 < V_2 \Rightarrow V_o = +15V$ (rail)

$t = t_0^+$ circuit:



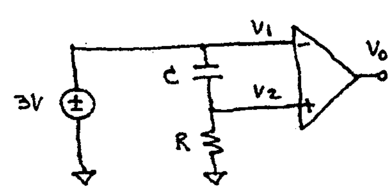
C acts like -3V source

$\therefore$ $V_1 = 3V$

$V_2 = 3V - -3V = 6V$

$V_1 < V_2 \Rightarrow V_o = +15V$ (rail)

$t \to \infty$ circuit:



C acts like open circuit
no V drop across R

$\therefore$ $V_1 = 3V$

$V_2 = 0V$

$V_1 > V_2 \Rightarrow V_o = -15V$ (rail) soln 2.c) cont. Let $t_0 = 0$. Want $V_1 = V_2$ at $\Delta t \equiv 10ms$ for V_0 transition.

$V_1 = 3V$ for $t > t_0 = 0$

$V_2 = 3V - V_C$ for $t > t_0$.

$$V_2(t>0) = V_2(t\to\infty) + [V_2(0^+) - V_2(t\to\infty)]\, e^{-t/RC}$$

$$'' = 0V + [6V - 0V]\, e^{-t/RC} = 6V\, e^{-t/RC}$$

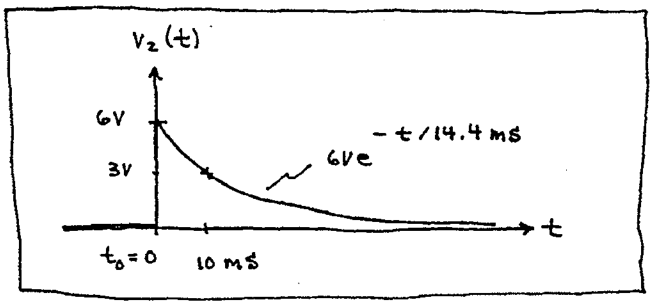
We want $V_2(\Delta t) = 3V$. $\therefore$ $6V\, e^{-\Delta t/RC} = 3V$

$$e^{-\Delta t/RC} = \frac{3V}{6V} = \frac{1}{2} \quad \text{or} \quad -\frac{\Delta t}{RC} = \ln\frac{1}{2}$$

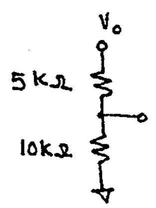
$$RC = \frac{-\Delta t}{\ln\frac{1}{2}} = \frac{\Delta t}{\ln 2} = \frac{10ms}{\ln 2} = 14.4\ ms$$

Use $\boxed{C = 1\mu F, \quad R = 14.4\ k\Omega}$ (15kΩ is closest standard value)

d) $V_2(t<0) = 0V$ $\qquad V_2(t>0) = 6V e^{-t/14.4ms}$ $\qquad V_2(10ms) = 3V$

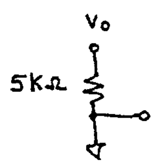


e) When $V_0 = +15V$, the diode is reverse biased ⇒ open circuit (i.e. disappears)

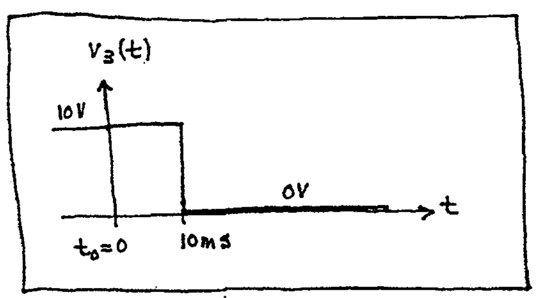


$$V_3 = V_0 \cdot \frac{10k\Omega}{10k\Omega + 5k\Omega} = 15V \cdot \frac{2}{3} = 10V$$

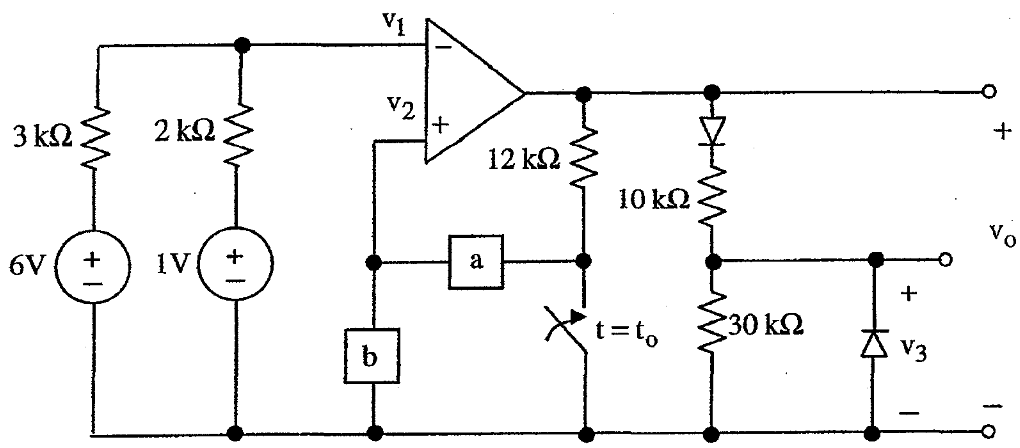
When $V_0 = -15V$, the diode is forward biased ⇒ short



$V_3 = 0V$

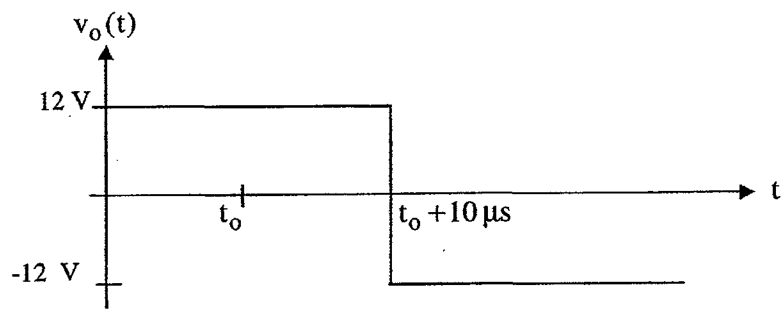


2. (65 points)



Rail voltages = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



Pts

35 a. Choose either an R or L to go in box a and either an R or L to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.

5 b. Sketch $v_1(t)$, showing numerical values appropriately.

15 c. Sketch $v_2(t)$, showing numerical values appropriately.

10 d. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 10\ \mu s$, and for $t_o + 10\ \mu s < t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

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soln: 2.a) Ignore diode and resistor network on output (since it doesn't affect v_o).

For $t < t_o$, $v_o = +12V \Rightarrow v_2 > v_1$.

Calculate v_1 using node-v method: $\frac{v_1 - 6V}{3k\Omega} + \frac{v_1 - 1V}{2k\Omega} = 0A$

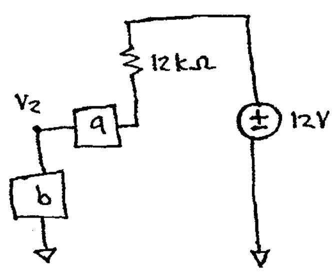
$$v_1\left(\frac{1}{3k\Omega} + \frac{1}{2k\Omega}\right) = \frac{6V}{3k\Omega} + \frac{1V}{2k\Omega}$$

mult both sides by $6k\Omega$

$$v_1\ (2+3) = 6V \cdot 2 + 1V \cdot 3 = 15V, \qquad v_1 = \frac{15V}{5} = 3V$$

So we need $\underline{\underline{v_2 > 3V.}}$

Circuit model: $v_o = 12V$, switch open



If we have an L, it will be equivalent to a wire.

Consider possibilities:

case I: $a = L$ and $b = L$ $L = \text{wire}$

$v_2 = 0V$ from v divider

Doesn't work.

case II: $a = R$ and $b = L = \text{wire}$

$v_2 = 0V$ from v divider

Doesn't work.

case III: $a = R_1$ and $b = R_2$

We can choose R_1 and R_2 to achieve $v_2 > 3V$, but we cannot get a delay in v_o dropping from $+12V$ to $-12V$.

case IV: $a = L$ and $b = R$

Since $L = \text{wire}$ and we can pick R, we can achieve $v_2 > 3V$. When switch moves, the L continues to carry same current initially. Thus, $v_2 > v_1$ is sustained for delay. Should work.

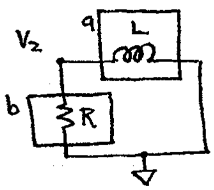
sol'n: 2.a) cont.

When switch closes, we have RL circuit that determines v_2. Time constant $\tau = L/R$. Output v_o drops when v_2 drops below 3V.

As $t \to \infty$, the L in 'a' acts like a wire and the switch is closed $\Rightarrow$ $v_2(t\to\infty) = 0V$

Without additional constraints, we may choose any v_2 between 3V and 12V. One choice is $v_2(0^-) = 6V$. Using v-divider of 12kΩ and 'b', $\boxed{b = 12k\Omega}$.

We want $v_2(t = 10\mu s) = 3V$ so v_o drops at time $t_o + 10\mu s$. (Assume $t_o = 0s$)

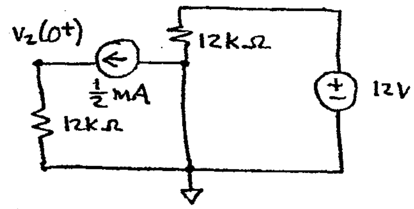


$$v_2(t>0) = v_2(t\to\infty) + [v_2(0^+) - v_2(t\to\infty)]e^{-t/\tau}$$
$$" \quad = 0V + [v_2(0^+) - 0V]e^{-t/\tau}$$
$$" \quad = v_2(0^+)e^{-t/\tau},$$ Now find $v_2(0^+)$.

Consider $t = 0^-$: L = wire $\quad R = 12k\Omega$

$$i_L(0^-) = \frac{12V}{12k\Omega + 12k\Omega} = \frac{1}{2}\,mA$$

$t = 0^+$: L = i src where $i_L(0^+) = i_L(0^-) = \frac{1}{2}mA$



$$v_2(0^+) = \tfrac{1}{2}mA \cdot 12k\Omega$$
$$v_2(0^+) = 6V$$

$$v_2(t>0) = 6V e^{-t/\tau}$$

We want $v_2(10\mu s) = 3V = 6V e^{-10\mu s/\tau}$

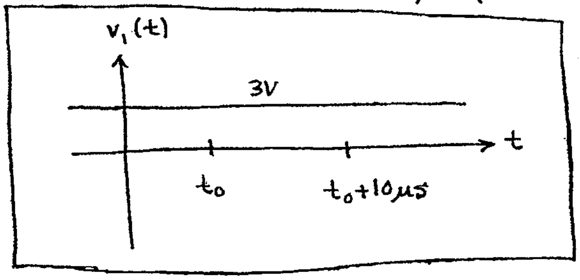
$$\frac{3V}{6V} = \frac{1}{2} = e^{-10\mu s/\tau}, \qquad \ln\tfrac{1}{2} = -10\mu s/\tau$$

$$\tau = \frac{-10\mu s}{\ln\frac{1}{2}} = \frac{10\mu s}{\ln 2} = 14.4\,\mu s = \frac{L}{R} = \frac{L}{12k\Omega}$$

$$L = 14.4\mu s \cdot 12k\Omega = 173\ mH$$

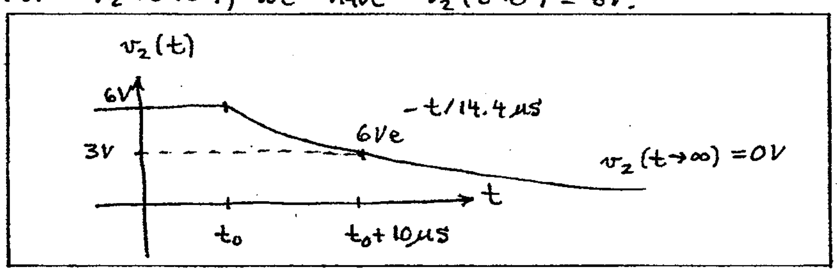
Summary: $\boxed{\begin{array}{l} a = L = 173\ mH \\ b = R = 12k\Omega \end{array}}$

soln: 2.b) As shown in soln for (a), $v_1(t) = 3V$. It never changes.

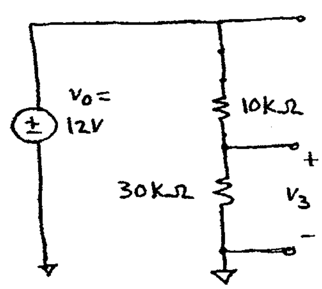


c) From soln to (a), we have $v_2(t>0) = 6V e^{-t/14.4\mu s}$.

For $v_2(t<0)$, we have $v_2(t<0) = 6V$.

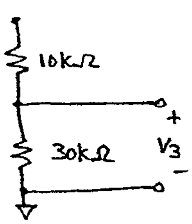


d) When $v_o > 0V$, top diode = wire, bottom diode = open.

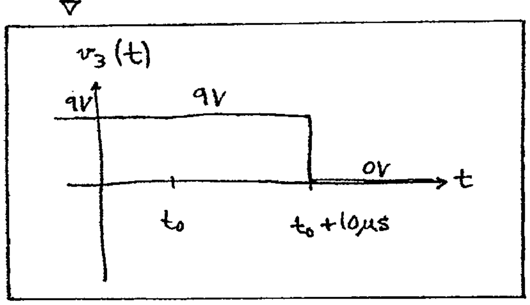


$$V_3 = 12V \cdot \frac{30k\Omega}{30k\Omega + 10k\Omega} = 9V$$

When $V_o < 0V$, top diode = open, bottom diode doesn't matter since no current

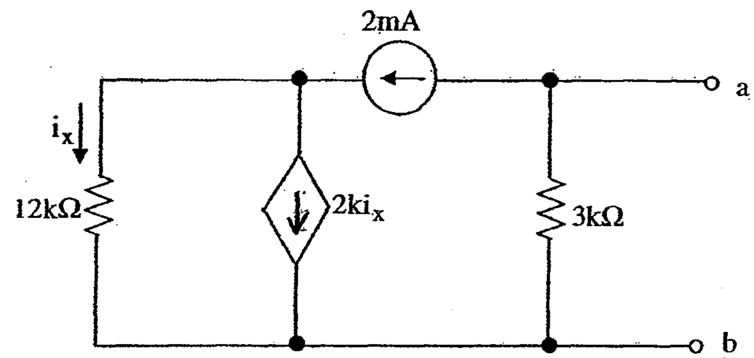


$V_3 = 0V$ since no current flowing



HW #10 Cont.

3.



a. Find the Thevenin equivalent of the above circuit relative to terminals a and b.

b. If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.

c. Calculate the value of that maximum power absorbed by R_L.

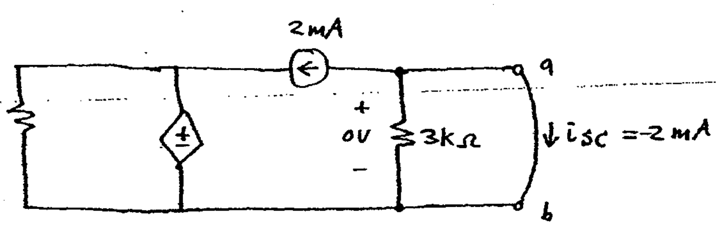
HW#10 Cont.

ECE 1000 Su 05

sol'n: 3. a) $V_{Th} = V_{a,b \text{ open circ}} = -2\,mA \cdot 3k\Omega = -6V$

The current source is between a, b and the 12kΩ and dependent source. The current source thus isolates behavior at a, b from the 12kΩ and dependent source.

Use i_{sc} to find R_{Th}:



We short a, b so no v-drop across 3kΩ.

So $i_{3k\Omega} = 0$ and $i_{sc} = -2\,mA$.

$$R_{Th} = \frac{V_{Th}}{i_{s}} = \frac{-6V}{-2mA} = 3k\Omega$$

$$\boxed{V_{Th} = -6V \qquad R_{Th} = 3k\Omega}$$

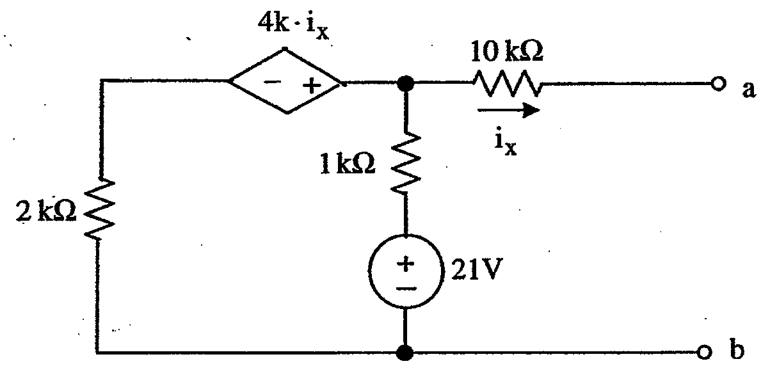
Comments: 1) We could also just turn off the 2 mA source and look into a, b to see $R_{Th} = 3k\Omega$.

2) We really just started with a Norton equivalent that we converted to Thevenin equivalent.

b) max pwr when $R_L = R_{Th} = \boxed{3k\Omega = R_L}$

c) max pwr $P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(-6V)^2}{4 \cdot 3k\Omega} = \boxed{3\,mW = P_{max}}$

3.

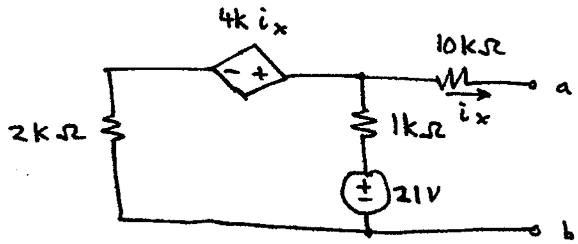


a. Find the Thevenin equivalent of the above circuit relative to terminals a and b.

b. If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.

c. Calculate the value of that maximum power absorbed by R_L.

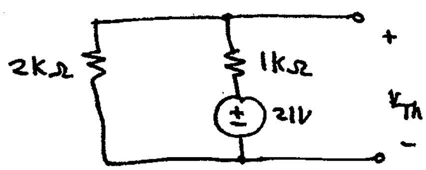
soln: 3. a)



$V_{Th} = V_{ab}$ open circuit

$i_x = 0$ for open circuit a, b. $\therefore 4k \cdot i_x = 0V$

No $i_x \Rightarrow$ no v drop across 10kΩ $\Rightarrow$ ignore 10kΩ

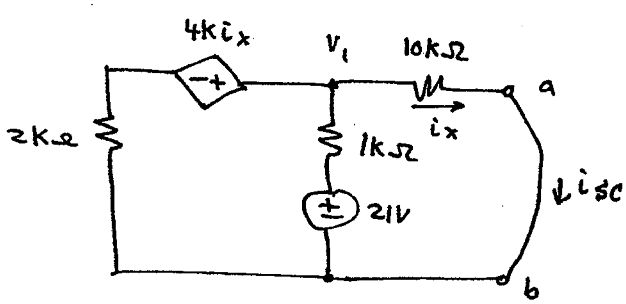


This is V-divider.

$$V_{Th} = 21V \cdot \frac{2k\Omega}{2k\Omega + 1k\Omega} = 14V$$

$$\boxed{V_{Th} = 14V}$$

To find R_{Th} we can use the i_{sc} method:



$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$

Need to find i_{sc}. $\therefore$ Find v_1

Find v_1 by node-voltage method:

$$i_x = \frac{v_1}{10k\Omega}$$ so we can eliminate i_x

$$\frac{v_1 - 4k\frac{v_1}{10k\Omega}}{2k\Omega} + \frac{v_1 - 21V}{1k\Omega} + \frac{v_1}{10k\Omega} = 0A$$

mult everything by 10kΩ

$$v_1 \cdot 5 - v_1 \cdot 2 + v_1 \cdot 10 + v_1 \cdot 1 = 210\ V$$

$v_1(5 - 2 + 10 + 1) = 210V$ $\quad v_1 \cdot 14 = 210V$ $\quad v_1 = 15V$

$i_{sc} = i_x = \frac{v_1}{10k\Omega} = 1.5\ mA$ $\qquad R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{14V}{1.5mA}$

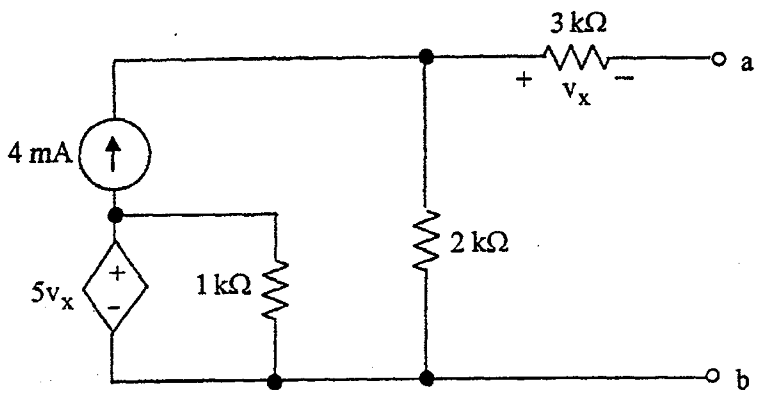
$$\boxed{R_{Th} = 9.33\ k\Omega}$$

sol'n: 3.b) max pwr when $\boxed{R_L = R_{Th} = 9.33\ k\Omega}$

c) max pwr $p_{Max} = \dfrac{v_{Th}^2}{4 R_{Th}} = \dfrac{(14V)^2}{4 \cdot \frac{14}{1.5}\ k\Omega} = \dfrac{14\ (1.5)}{4}\ mW$

$$\boxed{P_{max} = \frac{21}{4}\ mW = 5.25\ mW}$$

3. (30 points)



Pts		
20	a.	Find the Thevenin equivalent of the above circuit relative to terminals a and b.
5	b.	If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.
5	c.	Calculate the value of that maximum power absorbed by R_L.

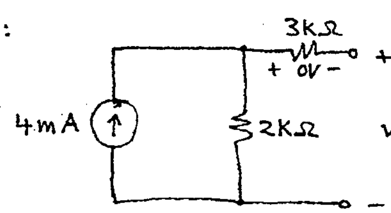
sol'n: a) $v_{Th} = v_{a,b}$ open circuit

open circuit $\Rightarrow$ $v_x = 0$ since no current in 3kΩ.

∴ $5v_x$ src $= 0V$ = wire

1kΩ across v_x shorted so can be ignored.

So we have:



$V_{Th} = 4mA \cdot 2k\Omega = 8V$

$\boxed{V_{Th} = 8V}$

$R_{Th} = \frac{V_{Th}}{i_{sc}}$ If we short a,b we have current divider. $i_{sc} = 4mA \cdot \frac{2k\Omega}{2k\Omega + 3k\Omega} = \frac{8}{5}mA$

$R_{Th} = \frac{8V}{8/5\ mA} = 5k\Omega$ $\boxed{R_{Th} = 5k\Omega}$

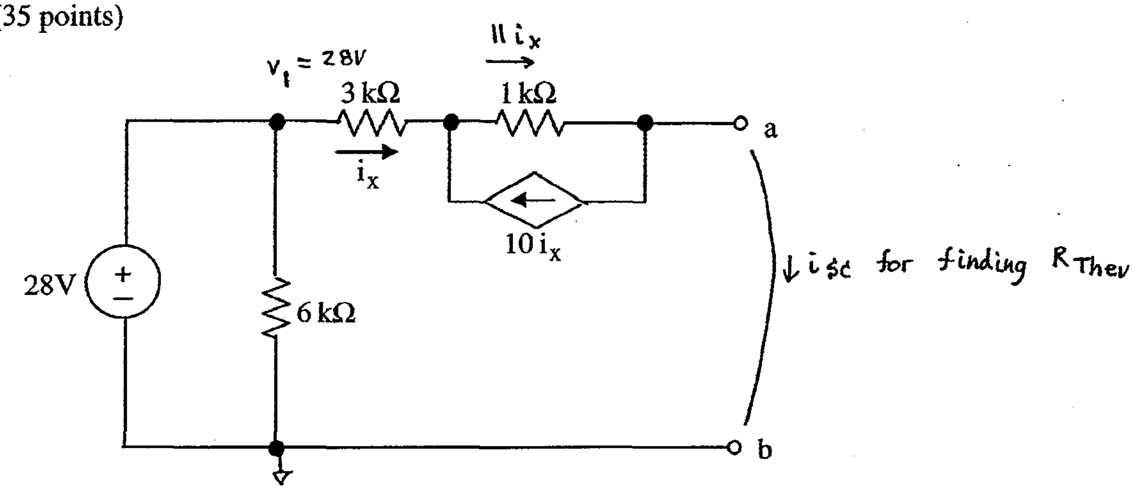
Note: Easier sol'n is to say $5v_x$ and 1kΩ don't matter because they are in series with current source. Then $R_{Th} = 2k\Omega + 3k\Omega$ seen from a,b with 4mA off.

sol'n: 3.b) max pwr when $\boxed{R_L = R_{Th} = 5\,k\Omega}$

3.c) max pwr $= \dfrac{V_{Th}^2}{4R_{Th}} = \dfrac{(8V)^2}{4 \cdot 5k\Omega} = \dfrac{64}{20}$ mW $= 3.2$ mW

$\boxed{\text{max pwr} = 3.2\ mW}$

3. (35 points)



Pts

25 a. Find the Thevenin equivalent of the above circuit relative to terminals a and b.

5 b. If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.

5 c. Calculate the value of that maximum power absorbed by R_L.

sol'n: a) The 6kΩ resistor is across the 28V source, so it may be ignored.

For V_{Thev} we use $V_{a,b}$ with no load. Since no current flows out of the 'a' terminal, $i_x = 0$.

$\therefore$ $10 i_x = 0A$ and v drop across 3kΩ and 1kΩ is zero.

$\therefore$ $V_{a,b} = 28V$ from v src $\therefore$ $\boxed{V_{Thev} = 28V}$

Now find i_{sc} flowing in wire connected from a to b.

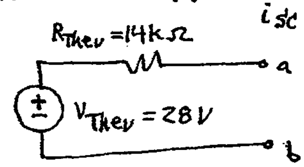
From current sum at node on left end of 1kΩ, we have current $11 i_x$ flowing in 1kΩ resistor.

Using v drops for 3kΩ and 1kΩ, we must have

$i_x \cdot 3k\Omega + 11 i_x \cdot 1k\Omega = 28V$ or $14k\Omega \cdot i_x = 28V$

or $i_x = 2\ mA$. Since $i_{sc} = i_x$ and $R_{Th} = \dfrac{V_{Th}}{i_{sc}}$,

$$\boxed{R_{Th} = \frac{28V}{2mA} = 14k\Omega}$$



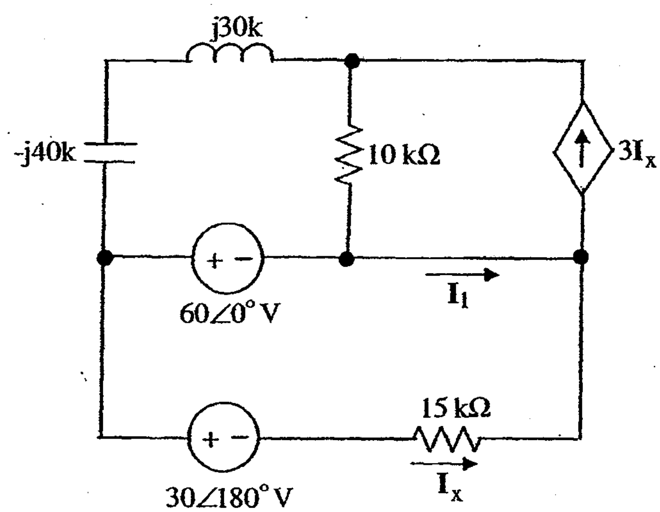
sol'n: 3. b) Max pwr when $\boxed{R_L = R_{Thev} = 14k\Omega}$

3. c) Max pwr $= \dfrac{V_{Thev}^2}{4\,R_{Thev}} = \dfrac{(28V)^2}{4 \cdot 14k\Omega} = 7(2)W/k$

$\boxed{\text{Max pwr} = 14mW}$

HW #10 Cont.

4.



a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

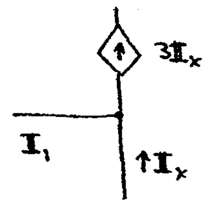
b. Given $\omega = 53.13$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

HW #10 Cont.

Su 05

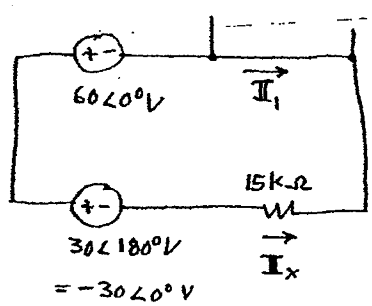
ECE 1000

sol'n: 4. a) Sum of currents for node on right side:



We see that $\mathbf{I}_1 = 2\mathbf{I}_x$ from sum of currents out of node $= 0$.

From the bottom half of the circuit, we can compute $\mathbf{I}_x$ directly:



From V-loop we have

$$\mathbf{I}_x = \frac{60\angle 0^\circ V - -30\angle 0^\circ V}{15k\Omega}$$

$$\mathbf{I}_x = \frac{90\angle 0^\circ V}{15k\Omega}$$

$$\mathbf{I}_x = 6\text{ mA}\angle 0^\circ$$

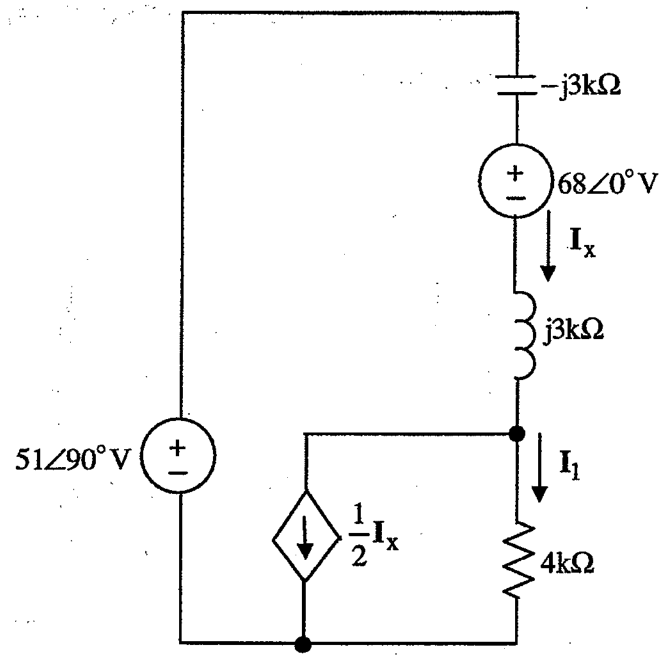
So $\mathbf{I}_1 = 2\mathbf{I}_x = 12\text{ mA}\angle 0^\circ$

$$\boxed{\mathbf{I}_1 = 12\angle 0^\circ \text{ mA}}$$

b) $$\boxed{i_1(t) = 12\cos(53.13t)\text{ mA}}$$

or πt

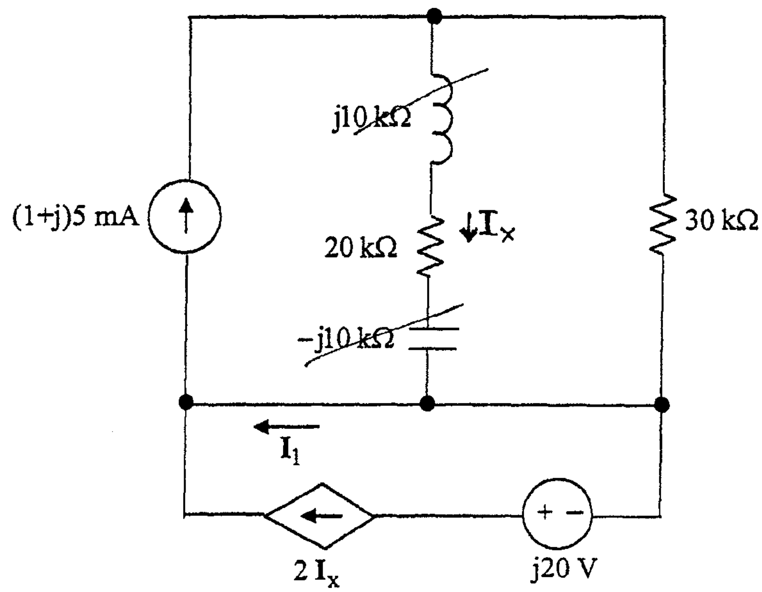
3.



a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

b. Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

4. (25 points)



Pts

20 pts a. A frequency-domain circuit is shown above. Write the value of $\mathbf{I}_1$ in polar form.

5 pts b. Given $\omega = 100$ k rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

sol'n: 4. a) $j10\,k\Omega - j10\,k\Omega = 0\,\Omega$ so L and C cancel.

Current divider for 20kΩ and 30kΩ.

$$\therefore\ \mathbb{I}_X = (1+j)\,5\,mA \cdot \frac{30\,k\Omega}{20\,k\Omega + 30\,k\Omega} = (1+j)\,3\,mA$$

Find $\mathbb{I}_1$ from sum of currents at node on left side:

$$(1+j)\,5\,mA - 2\underbrace{(1+j)3\,mA}_{\mathbb{I}_X} - \mathbb{I}_1 = 0$$

$$\mathbb{I}_1 = (1+j)\,5\,mA - 2(1+j)\,3\,mA = (1+j)(5-6)\,mA$$

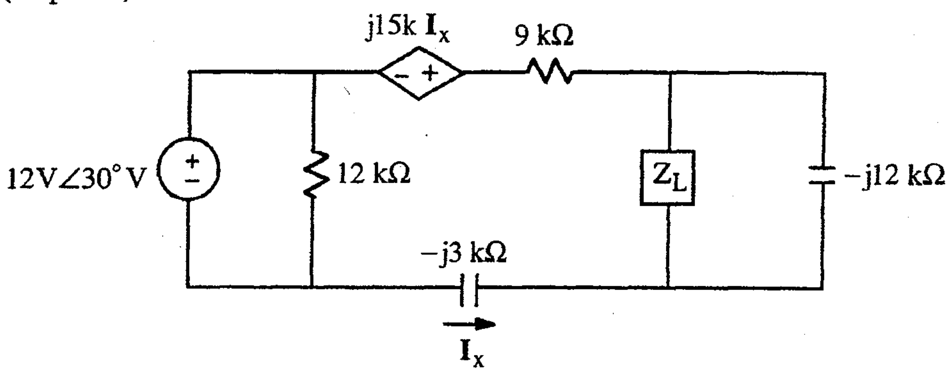
$$\mathbb{I}_1 = (1+j)(-1)\,mA = -(1+j1)\,mA$$

$$\boxed{\mathbb{I}_1 = -\sqrt{2}\ \angle 45^\circ\ mA} \boxed{= \sqrt{2}\ \angle -135^\circ\ mA \text{ or } 225^\circ}$$

b) $\boxed{i_1(t) = \sqrt{2}\cdot \cos(100\,k\,t - 135^\circ)\ mA}$

45

3. (30 points)



<u>Pts</u>

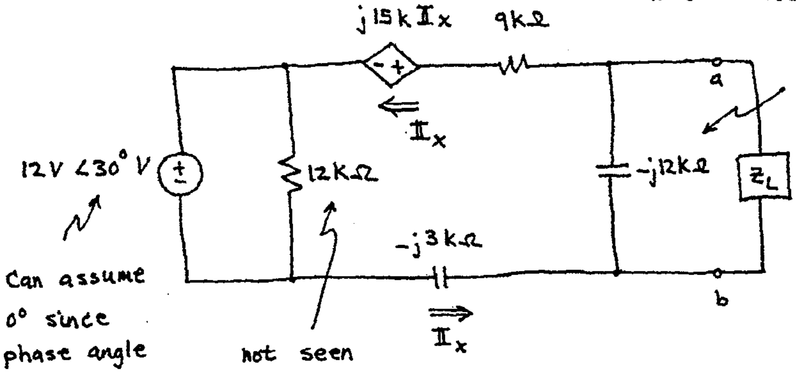
15 a. Choose the value of Z_L that will absorb maximum average power.

15 b. Calculate the value of that maximum average power absorbed by Z_L.

46

sol'n 3. a) $Z_L = Z_{Th}^*$ for max power xfer

Since I_x flows thru it, this looks like inductor with $Z = j15k\Omega$.

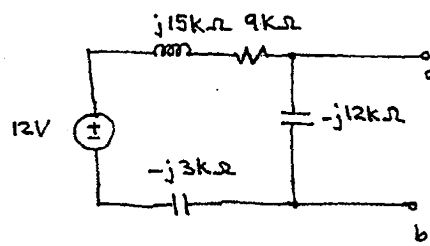


We can put C on left side of Z_L since they are in parallel

Can assume 0° since phase angle doesn't affect ave pwr

not seen in Thevenin equivalent since it is across V source

circuit for determining Thevenin equivalent:



$\Leftarrow Z_{Th}$ = Z seen looking into a, b with 12V source shorted

$$\therefore Z_{Th} = -j12k\Omega \,\|\, (-j3k\Omega + j15k\Omega + 9k\Omega)$$

$$\text{"} = -j12k\Omega \,\|\, (9k\Omega + j12k\Omega)$$

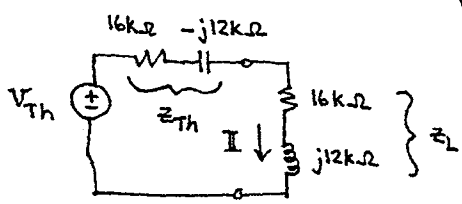
$$\text{"} = \frac{-j12k\Omega\,(9k\Omega + j12k\Omega)}{\cancel{-j12k\Omega} + 9k\Omega + \cancel{j12k\Omega}}$$

$$\text{"} = -j\,\frac{12k\Omega}{9k\Omega}\,(9k\Omega + j12k\Omega)$$

$$\text{"} = -j12k\Omega + 16k\Omega$$

$$\boxed{Z_L = Z_{Th}^* = 16k\Omega + j12k\Omega}$$

b)



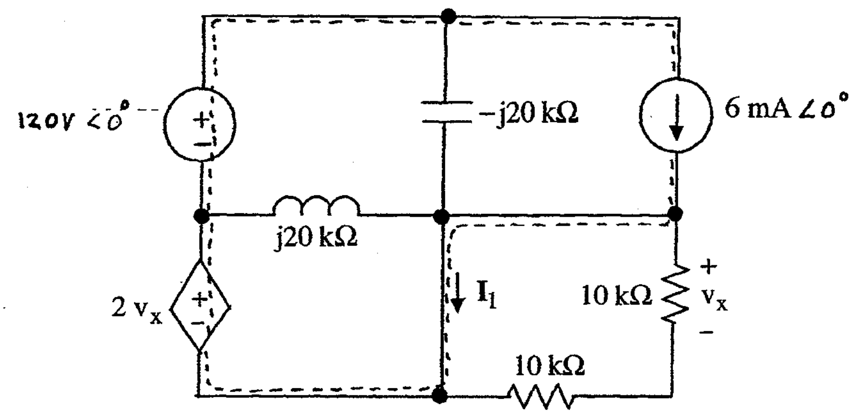
$$P \equiv (\text{ave pwr in } Z_L) = \frac{|I|^2}{2} R \quad \text{where } R = 16k\Omega \text{ in } Z_L$$

$$I = \frac{V_{Th}}{16k\Omega - \cancel{j12k\Omega} + 16k\Omega + \cancel{j12k\Omega}} = \frac{V_{Th}}{32k\Omega}$$

$$V_{Th} = 12V \cdot \frac{-j12k\Omega}{\cancel{-j12k\Omega} + \cancel{-j3k\Omega} + \cancel{j15k\Omega} + 9k\Omega} = 12V\left(\frac{-j12k\Omega}{9k\Omega}\right) = -j16V$$

$$\therefore P = \frac{\left|\frac{-j16V}{32k\Omega}\right|^2 \cdot 16k\Omega}{2} = \frac{|-j|^2 V^2}{|2k\Omega|^2} \cdot \frac{16k\Omega}{2} = \frac{1V^2 \cdot 8}{4k\Omega} \quad \text{or} \quad \boxed{P = 2\text{ mW}}$$

4. (25 points)



Pts

20 a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

5 b. Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

soln: a) Since the two 10kΩ resistors are shorted by wires.

∴ There is no v drop across the 10kΩ resistors,

and $v_x = 0V$.

Thus, the $2v_x$ dependent source = 0V = wire

Superposition Case I: 6mA on, 120V off = wire.

It follows that all of the 6 mA from the independent current source flows in the wires (shown as dashed lines above).

$\therefore \mathbf{I}_{11} = 6\text{ mA} \angle 0°$

Case II: 120V on, 6mA off = open circuit

we observe that the −j20kΩ is directly across the 120V source, given the wires shown as dashed lines.

$\therefore \mathbf{I}_{12} = \dfrac{120V\angle 0°}{-j\,20k\Omega} = j\,6mA = 6mA \angle 90°$

Thus, $\mathbf{I}_1 = \mathbf{I}_{11} + \mathbf{I}_{12} = 6\text{ mA}\cdot(1+j)$

or $\boxed{\mathbf{I}_1 = \sqrt{2}\cdot 6\text{ mA} \angle 45°}$

b) $\boxed{i_1(t) = \sqrt{2}\cdot 6\text{ mA} \cos(\pi t + 45°)}$

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