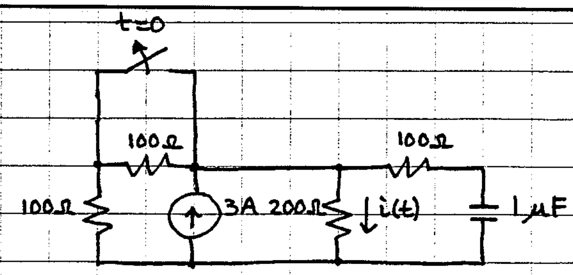
1.



a) Write a numerical expression for $i(t)$, $t \geq 0^+$.

soln: Any $i(t)$ or $v(t)$ will be described by the general form of solution whenever we solve a switching problem:

$$i(t) = i(t=0^+) + \left[i(t\to\infty) - i(t=0^+)\right]\left[1 - e^{-t/RC}\right]$$

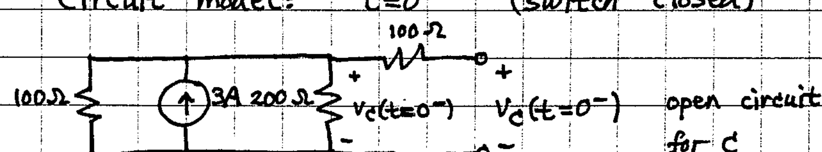
We only need to find $i(t=0^+)$, $i(t\to\infty)$, and RC.

$i(t=0^+)$: To find $i(t=0^+)$, we need the initial conditions for the circuit. Since there is a capacitor, we need to find $v_C(t=0^+)$. (If there were an inductor, we would need to find $i_L(t=0^+)$.

Since the voltage on the capacitor cannot change instantly, $v_C(t=0^+) = v_C(t=0^-)$.

To find $v_C(t=0^-)$, we treat C as an open circuit, (because the switch has been closed for a long time, allowing C to charge to final value where current no longer flows).

Circuit model: $t=0^-$ (switch closed)



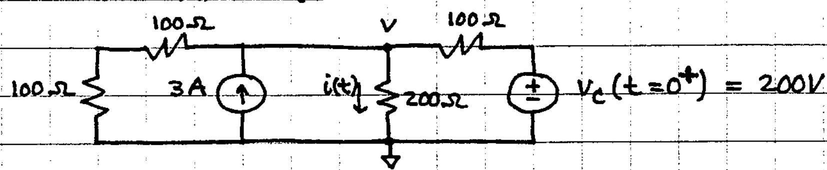
Since no current flows in the 100Ω top right, $v_C(t=0^-)$ also appears across the 200Ω R. (No current in 100Ω ⇒ no v drop.)

1.a) (cont) We have $v_C(t=0^-) = 3A \cdot 100\Omega \| 200\Omega = 3A \cdot 100\Omega \cdot 1 \| 2$

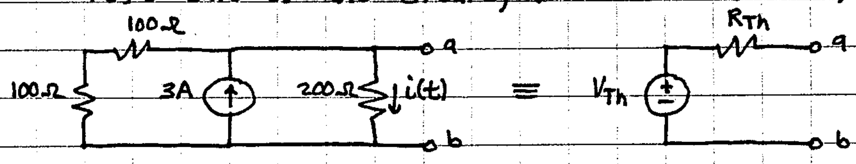
$v_C(t=0^-) = 3A \cdot 100\Omega \cdot \frac{2}{3} = 200V$

$v_C(t=0^+) = v_C(t=0^-)$ (v_C cannot change instantly)

For $t=0^+$ we use a v-source to represent C, and we find $i(t=0^+)$: $t=0^+$ (switch open)

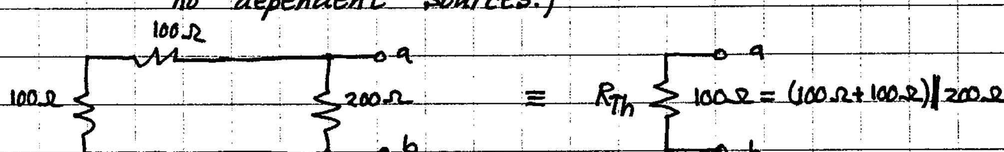


We can find i(t) by any method we prefer. My choice is to take a Thevenin equivalent of the left side of the circuit, (thru the 200Ω R).

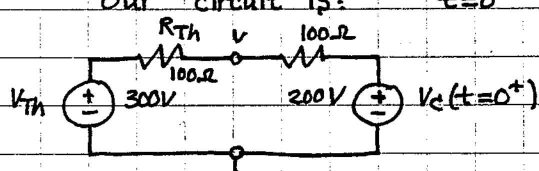


$V_{Th} =$ open circuit $v = 3A \cdot (100\Omega + 100\Omega) \| 200\Omega = 3A \cdot 100\Omega = 300V$

$R_{Th} =$ resistance seen when looking into terminals a-b with independent 3A source set to zero. (This is the same as connecting a 1V source at a-b, measuring current into 'a' and taking $R_{Th} = 1V / i_{1V}$. This simpler approach works whenever there are no dependent sources.)



Our circuit is: $t=0^+$



We see that $v = 250V$, (the two 100Ω R's divide the 100V drop into two equal v-drops of 50V each).

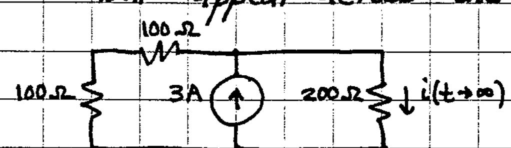
1.a) (cont) Finally, we have $i(t=0^+) = \frac{250V}{200\Omega} = 1.25A$

Note: I used a Thevenin equivalent that had i(t) transformed inside it, but I used it to find the output voltage, v, of the Thevenin equivalent. Then I found $i(t=0^+)$ from the voltage. What I cannot do is try to find i(t) somewhere inside the Thevenin equivalent.

Note: We could use a mesh current, or node-v, or superposition method to find i(t). User's choice. Superposition might be a good choice for finding $i(t=0^+)$.

$i(t\to\infty)$: The capacitor will be charged and acting like an open circuit. Again there will be no current thru the 100Ω top right, and $V_c(t\to\infty)$ will appear across the 200Ω R.

$t\to\infty$ (switch open)



We have a current divider. $i(t\to\infty) = 3A \cdot \frac{100\Omega + 100\Omega}{100\Omega + 100\Omega + 200\Omega}$

$i(t\to\infty) = 1.5A$

RC: Use the Thevenin equivalent found earlier for $t=0^+$ so we have simple RC circuit. Then $R = R_{Th} + 100\Omega = 200\Omega$.
$RC = 200\Omega \cdot 1\mu F = 200\mu s$.

$\therefore\ i(t \geq 0^+) = 1.25A + (1.5 - 1.25A)(1 - e^{-t/200\mu s}) = 1.5A - 0.25A \cdot e^{-t/200\mu s}$

1.b) Calculate energy stored in C at $t=0^+$.

Found in sol'n to 1.a)

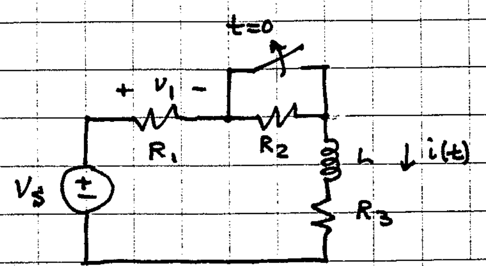
sol'n: $w = \frac{1}{2} C\, v_c^2(t=0^+) = \frac{1}{2} \cdot 1\mu F \cdot (200V)^2 = 0.02\text{ J or } 20\text{ mJ}$

1. c) Calculate energy stored in C as $t \to \infty$.

sol'n: $w = \frac{1}{2} C v_c^2 = \frac{1}{2} \, 1\mu F \cdot (300V)^2 = 0.045 \, J$ or $45 \, mJ$

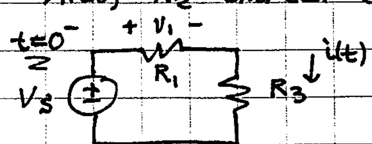
Note: In part (a), we found $v_c(t \to \infty) = 300V$.

2.



a) Write an expression for $v_1(t)$.

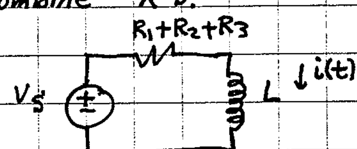
sol'n: For $t=0^-$ the L looks like a short circuit or wire.
Also, R_2 shorted by switch. V-divider gives v_1.



$\therefore \; v_1 = V_s \dfrac{R_1}{R_1 + R_3}, \quad t = 0^-$.

Also, $i(t=0^-) = V_s/(R_1+R_3) = i(t=0^+)$

With switch open, we can find $i(t)$ through inductor.
We combine R's:



At $t \to \infty$ we again have L acting like a wire.

$\therefore \; i(t \to \infty) = \dfrac{V_s}{R_1 + R_2 + R_3}$

Now use general formula to write $i(t)$:

$$i(t) = i(t=0^+) + \left[i(t \to \infty) - i(t=0^+) \right] \left[1 - e^{-t/(L/R')} \right] \qquad \boxed{R' = R_1 + R_2 + R_3}$$

$$i(t) = \frac{V_s}{R_1 + R_3} + \left[\frac{V_s}{R_1+R_2+R_3} - \frac{V_s}{R_1+R_3} \right] \left[1 - e^{-t/(L/R')} \right]$$

$v_1(t) = i(t) \cdot R_1$ since i_1 also flows through R_1.

$$\therefore \; \boxed{v_1(t) = V_s \cdot \left\{ \frac{R_1}{R_1+R_3} + \left[\frac{R_1}{R_1+R_2+R_3} - \frac{R_1}{R_1+R_3} \right] \left[1 - e^{-t/L/R'} \right] \right\}}$$

2.b) Make one consistency check (other than units).

sol'n: Check: If $V_s = 0$, L always discharged so $v_1(t \geq 0^+) = 0$.

Our answer is multiplied by V_s so we get $v_1(t \geq 0^+) = 0$ if $v_s = 0$. ✓

or Check: If $R_1 = 0$, then $v_1 = 0$ for short of $0\,\Omega$.

Our answer gives $$v_1(t \geq 0^+) = V_s \cdot \left\{ \frac{0}{0+R_3} + \left[\frac{0}{0+R_2+R_3} - \frac{0}{0+R_3} \right] \left[1 - e^{-t/(L/(0+R_2+R_3))} \right] \right\}$$
$$= 0 \checkmark$$

Note: We have to be sure we don't get 0/0 (which we have verified above, assuming $R_2, R_3 \neq 0$).

or Check: If $R_2 = 0$, then open switch doesn't add any R, and L continues to act like wire, and $v_1 = v_s \dfrac{R_1}{R_1+R_3}$.

Our answer gives $$v_1(t \geq 0^+) = V_s \cdot \left\{ \frac{R_1}{R_1+R_3} + \underbrace{\left[\frac{R_1}{R_1+0+R_3} - \frac{R_1}{R_1+R_3} \right]}_{\to 0} \left[1 - e^{-t/(L/R')} \right] \right\}$$
$$= V_s \cdot \frac{R_1}{R_1+R_3} \checkmark$$

or Check: If $L = 0\,H$, then we just have wire for L.
Then $v_1 = v_s \cdot \dfrac{R_1}{R_1+R_2+R_3}$ for $t \geq 0^+$.

Our answer gives $$v_1(t \geq 0) = V_s \cdot \left\{ \frac{R_1}{R_1+R_3} + \left[\frac{R_1}{R_1+R_2+R_3} - \frac{R_1}{R_1+R_3} \right] \left[1 - e^{-t/(0/R')} \right] \right\}$$

Now, $e^{-t/0} = e^{-\infty} = 0$.

$$\therefore\ v_1(t \geq 0) = V_s \left\{ \cancel{\frac{R_1}{R_1+R_3}} + \left[\frac{R_1}{R_1+R_2+R_3} - \cancel{\frac{R_1}{R_1+R_3}} \right] \right\} = V_s \frac{R_1}{R_1+R_2+R_3} \checkmark$$