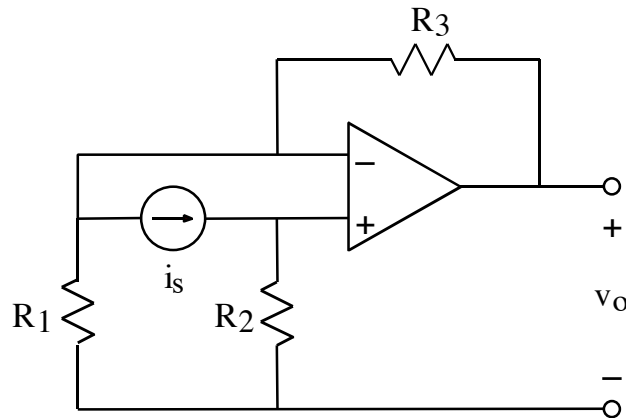


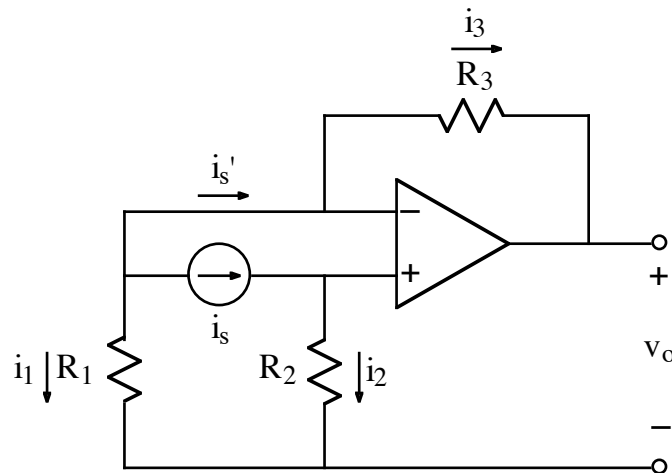
## 4. (30 points)

The op amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for  $v_o$  in terms of not more than  $i_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



**ans:**  $v_o = i_s(R_2 + R_3 + \frac{R_2 R_3}{R_1})$

**sol'n:** Using passive sign convention, label voltage drop and current measurement polarities.



Assume an ideal op-amp since we are in linear mode and we are solving for DC conditions.

Use the standard procedure to find  $v_o$ :

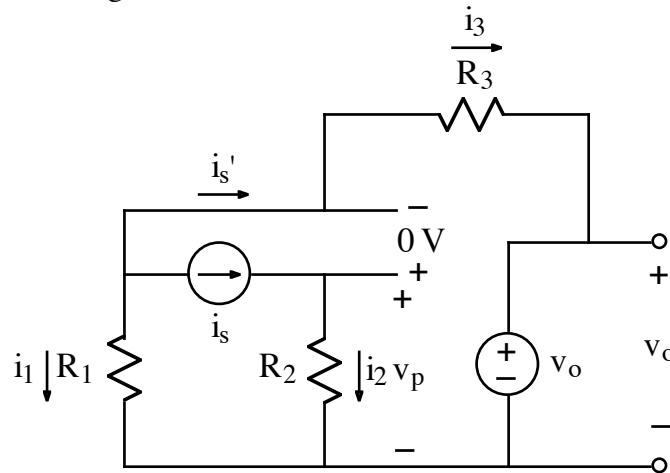
- 1) Calculate the voltage drop,  $v_p$ , from the  $-$  input of the op amp to the bottom wire, (i.e. across  $R_2$ ). We use Kirchhoff's law for summation of currents out of the node next to the  $+$  terminal of the op-amp. Since  $i_p = 0$ , (i.e., no current flows into the  $+$  terminal), we conclude that

$$i_2 = i_s.$$

By Ohm's law, it follows that

$$v_p = i_s R_2$$

- 2) Remove the op amp, put a source called  $v_o$  across the terminals where we measure the output, and assume a 0V drop from  $+$  input to  $-$  input of op amp. (Because of negative feedback, the op-amp output voltage,  $v_o$ , reaches an equilibrium level that results in a nearly 0V drop from  $+$  input to  $-$  input.) Now we have the following model circuit:



Note that we model the  $-$  terminal of the op-amp as a voltage source with zero current flowing into it. We add this constraint on the current with the understanding that  $v_o$  must have a value that results in  $i_n = 0$ .

- 3) Calculate  $i_s'$ , the current flowing toward the  $-$  terminal from the left. Note that current  $i_s'$  has a name similar to the current source only by coincidence.) Now we observe that the voltage drop from the  $-$  terminal of the op amp to the bottom wire is  $v_p - 0V = v_p$ . It is as though we have a voltage source of voltage  $v_p$  from the  $-$  terminal of the op amp to the bottom wire. This voltage source separates the two sides of the circuit. What happens on the right side of the circuit will not affect the value of  $i_s'$ . Thus, we utilize the left side of the circuit to calculate  $i_s'$ .

We use a summation of node currents at the node above  $R_1$  to calculate the value of  $i_s'$ :

$$i_s' + i_1 + i_s = 0 \quad \text{or} \quad i_s' = -(i_1 + i_s)$$

But we can also calculate  $i_1$  because we have  $v_n$  across  $R_1$ :

$$i_1 = v_p/R_1 = i_s R_2/R_1$$

We can now solve for  $i_s'$ :

$$i_s' = i_s(R_2/R_1 + 1) = i_s(1 + R_2/R_1)$$

- 4) Calculate  $i_3$ , the current flowing in the negative feedback path on the right side of the circuit. Again, it is as though we had a voltage source of voltage  $v_p$  from the  $-$  terminal of the op amp to the bottom wire. We utilize this voltage source and a voltage loop around the right side of the circuit to calculate  $v_o$ :

$$v_p - i_3 R_3 - v_o = 0 \quad \text{or} \quad i_3 = (v_o - v_p)/R_3 = (v_o - i_s R_2)/R_3$$

- 5) Since  $i_n = 0$ , set  $i_s' = i_3$ .

$$i_s' = i_s(1 + R_2/R_1) = i_3 = (v_o - i_s R_2)/R_3$$

Equate the 2nd and 4th expressions and solve for  $v_o$ .

$$i_s(1 + R_2/R_1) = (v_o - i_s R_2)/R_3 \quad \text{or} \quad v_o = i_s R_3(1 + R_2/R_1) + i_s R_2 \quad \text{or}$$

$$v_o = i_s(R_2 + R_3 + R_2 R_3/R_1)$$

### Consistency checks:

- 1) By linearity and fact that we have one source,  $i_s$ , we expect  $v_o$  to be directly proportional to  $i_s$ . Agrees with our formula for  $v_o$ . ✓
- 2) Consider  $R_2 = 0$ : Then  $v_p = 0$ . Therefore,  $i_1 = 0$  and  $i_3 = -i_s$ . Then,  $v_o = -i_3 R_3 = i_s R_3$ . Agrees with our formula for  $v_o$  when we plug in  $R_2 = 0$ . ✓
- 3) Consider  $R_1 = \infty$  (open circuit):  $v_p = i_s R_2$ ,  $i_3 = -i_s$ , and  $v_o = v_n - i_3 R_3 = i_s R_2 + i_s R_3$ . Agrees with our formula for  $v_o$  when we plug in  $R_1 = \infty$ . ✓
- 4) Consider  $R_3 = 0$ : Then  $v_o = v_n = v_p = i_s R_2$ . Agrees with our formula for  $v_o$  when we plug in  $R_3 = 0$ . ✓