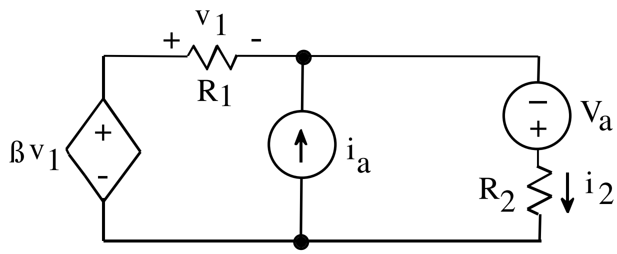
**3.** (30 points)

a. Derive an expression for $i_2$. The expression must not contain more than the circuit parameters ß, $V_a$, $i_a$, $R_1$, and $R_2$.

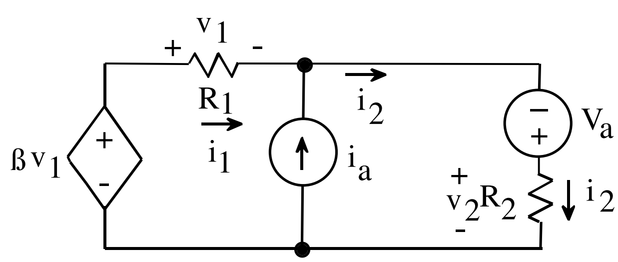



b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

**ans: a)** $i_2 = \dfrac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2}$

**b)** Many possible answers. See solution below.

**sol'n: (a)** Use Kirchhoff's laws to write several equations. Then eliminate unwanted variables.




We sum currents out of top-center node:

$$-i_1 - i_a + i_2 = 0$$

Note that summing currents out of bottom-center node does not give us anything new. By Ohm's law, we also have

$$i_1 = \frac{v_1}{R_1}$$

Now, we sum voltages around a loop. We choose the outer loop because the inner loops have a current source with unknown voltage drop.

$$\beta v_1 - v_1 + V_a - v_2 = 0 \quad \text{or} \quad (\beta - 1)v_1 + V_a - v_2 = 0$$

By Ohm's law, we also have

$$v_2 = i_2 R_2$$

After the Ohm's law substitutions, we have two equations, and we may eliminate $v_1$.

Use the simpler equation first:

$$\frac{v_1}{R_1} + i_a - i_2 = 0 \qquad \text{or} \qquad v_1 = (i_2 - i_a)$$

Substitute for $v_1$ in the second equation:

$$(\beta - 1)R_1(i_2 - i_a) + V_a - i_2 R_2 = 0$$

After some algebra, we get

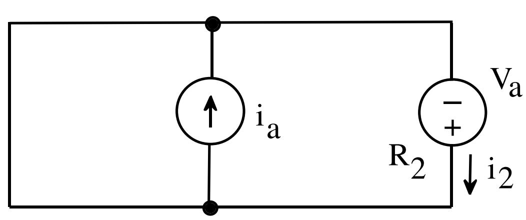
$$i_2 = \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2} \qquad \text{units consistent}$$

**(b)** There are many possible consistency checks.

1) $i_a = 0$ and $R_1 = 0$. Then $v_1 = 0$, $\beta v_1 = 0$, and sum v's around outer loop gives $i_2 = V_a/R_2$. Our formula also gives $V_a/R_2$.

2) Consider $R_1 = 0$ and $R_2 = 0$. As in (1), $v_1 = 0$ and $\beta v_1 = 0$. Since $R_2 = 0$ we also end up with a short across $V_a$:

Short across $V_a$



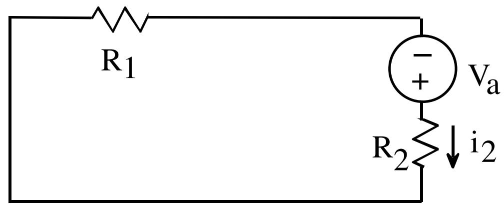

We expect $i_2 \to \infty$ for short across $V_a$

Our formula gives

$$\lim_{R_2 \to 0} \frac{V_a}{R_2} = \infty \qquad \text{for } i_2$$

from (1)

3) Consider $i_a = 0$, $ß = 0$:

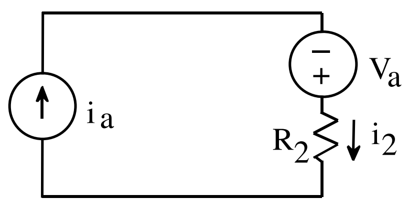



$$i_2 = \frac{V_a}{R_1 + R_2} \text{ by Ohm's law}$$

Our formula gives

$$i_2 = \frac{V_a}{R_1 + R_2} \quad ✓$$

4) Consider $R_1 \to \infty$ (open circuit)




Clearly $i_a = i_2$ since the same current flows through elements in series.

Our formula gives:

$$i_2 = \lim_{R_1 \to \infty} \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2} = \lim_{R_1 \to \infty} \frac{(1-\beta)i_a R_1}{(1-\beta)R_1} \quad ✓$$

$$= i_a$$

5) Consider $R_2 \to \infty$ (open circuit): We have $i_2 = 0$ since no current flows through the open circuit. Our formula gives:

$$i_2 = \lim_{R_1 \to \infty} \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2} = \frac{\text{const}}{\infty} = 0 \quad ✓$$

Many more consistency checks are possible.