1. INTRODUCTION

1.1. Electromyograms

Modern humans are immersed in an environment filled with a background of electromagnetic radiation. Some of the sources are very low in public exposure intensity (TV, radio and other communication stations; 60-Hz power lines), some are of moderate intensity (cell phones when placed against the head), and some are intentionally strong (MRI imagers; radio-frequency tissue ablation for cancer treatment). Around the world, many research teams are studying how exposure to this electromagnetic radiation affects the body, and therefore are interested in modeling the electrical properties of tissue.

Cells are the building blocks of the tissues of the body, and various types and sizes of cells make up a large proportion of the body's volume. A single cell (Fig. 1A) can be viewed as a fluid-like substance with several species of mobile ions (the cytoplasm) contained within a semi-permeable cell wall (the cell membrane). Outside the cell is more fluid (the extracellular fluid), with again several species of mobile ions. Electrically, the cytoplasm can be modeled to first order as a conductive medium—due to the presence of the large concentrations of ions—characterized by a given value of conductivity, σ, (or its inverse, resistivity, ρ). The extracellular fluid can also be modeled as a conductive medium. The cell wall, on the other hand, is relatively insulating and, since it is a thin layer, is modeled as a capacitive medium with a given relative permittivity ε_r.

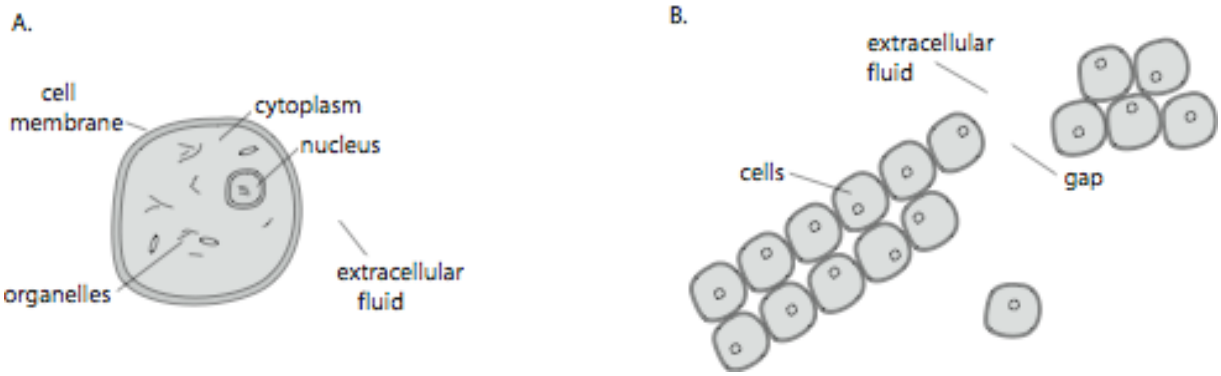

Fig. 1 A) Simplified diagram of a single cell, whose size can vary between 10 μm to nearly 1 mm; B) Tissue is composed of collections of cells. Gaps between the cells allow current to flow through shunt paths.

Tissues are composed of arrangements of cells (Fig. 1B) surrounded by the conductive extracellular fluid. In some places, the cells are tightly bound together and any circulating current must pass through the cell membranes. In other places, there are gaps (shunt paths) through which current can flow.

With cells in mind, consider a region of the body where there is a volume of soft tissue (muscle and fat) covered by skin upon which two opposing electrodes can be attached (e.g., the biceps muscles of the upper arm). A simple equivalent electrical model of the tissue between the two electrodes is shown in Fig. 2A. This is a lumped-element model where all inner-tissue cells are combined together into a single series R_cC_c branch. This branch in turn is in parallel with a resistive branch, R_t, representing the shunt intracellular fluid paths. At both ends of this circuit are parallel R_sC_s segments representing the electrical properties of the thin skin directly underneath the electrodes.

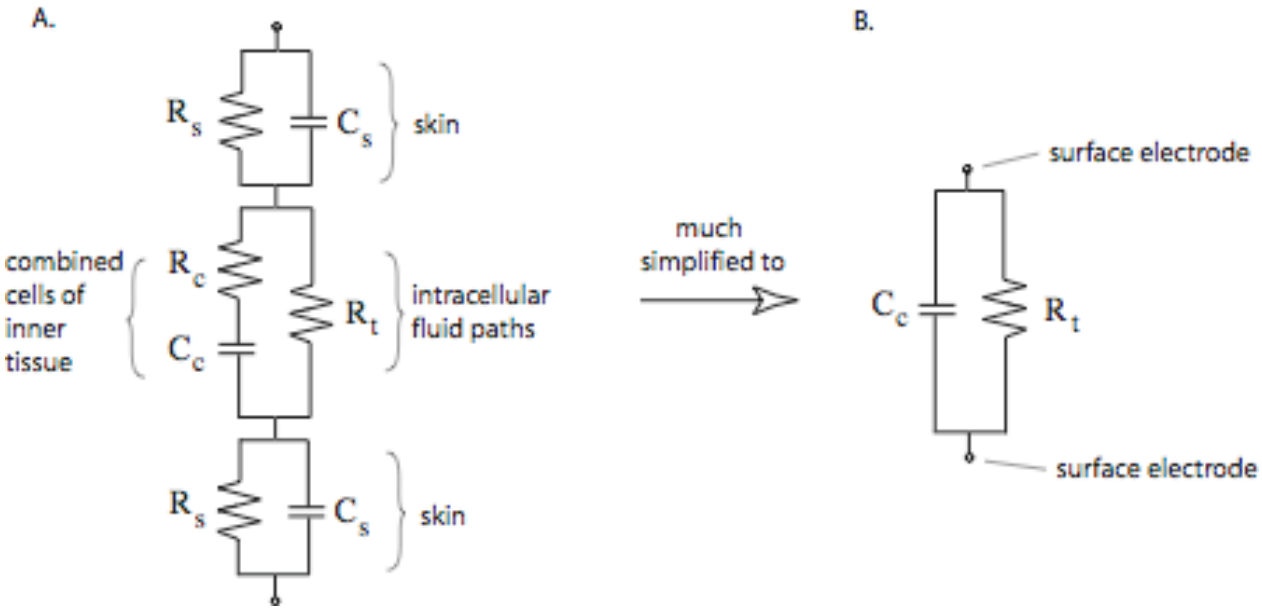

Fig. 2 – A) Lumped-element model of the electrical behavior of tissue bordered by skin; B) After several simplifications, the model reduces to its essence, a parallel RC circuit.

With some reasonable assumptions, this model can be greatly simplified. Assume that the combined resistance of the cell cytoplasm, R_c, is much lower than the reactance, $1/\omega C_c$, due to the cell membranes. Also, assume that the resistance of the thin skin, R_s, is much lower than both the skin reactance, $1/\omega C_s$, and the tissue resistance, R_t. (The validity of these assumptions depends very much upon the particular tissue region being modeled and the frequency of the electrical excitation, but for muscle and skin at a relatively low frequency of 25 kHz, these approximations are appropriate.) Then the

model of Fig. 2A reduces to the simple R_tC_c parallel equivalent circuit shown in Fig. 2B. In the laboratory exercise detailed below, you will make measurements with the aim of finding typical values for the parameters of this simplified circuit.

1.2. The Design Project

Your project is to design an oscillator that will produce a sinusoidal waveform at a frequency of 25 kHz and use that signal to determine the value of resistance and capacitance for a model of tissue impedance based on measurements of your biceps. The oscillator is an op-amp Wein-bridge circuit that produces a sinusoid, as shown in Fig. 3. The sinusoidal output from the oscillator will drive a resistor in series with electrodes placed on both sides of your biceps. The resistor, whose value you will choose, and the tissue in your biceps will form a voltage divider. By measuring the magnitude and phase-shift of the voltage across the resistor relative to the 25 kHz sinusoid, you will be able to determine the value of R and C for a parallel RC model of the tissue. You will use an oscilloscope to make the necessary magnitude and phase-shift measurements.

2. DESIGN OSCILLATOR

2.1. Frequency-Domain Circuit

The Wien-bridge oscillator in Fig. 3 consists of an op-amp and two voltage dividers. One voltage divider consists of components on the left side: R_1, C_1, R_2, and C_2. The other voltage divider consists of components on the right side: R_3 and R_4. These two voltage dividers deliver a fraction of the op-amp output voltage, v_0, to the + and – inputs of the op-amp. Because the circuit has negative feedback, changes in v_0 act to make the voltages at the op-amp inputs equal. Because the circuit also has positive feedback, changes in v_0 also act to make the voltages at both op-amp inputs change in tandem, however. At first glance, the equilibrium value for v_0 would seem to be 0 V. This is true in all cases except the special case when the two voltage dividers output exactly the same voltage. In this case, the bridge is said to be balanced. Though it may be counterintuitive, a balanced bridge leads to oscillation, as we discuss next.

When the bridge is balanced, *any* value of v_0 causes the + and – input voltages of the op-amp to be equal. Because we are dealing with reactive components, the bridge is balanced for one particular frequency of sinusoid. If the bridge is balanced, this sinusoid may have any amplitude. This is equivalent to saying the circuit is an oscillator.

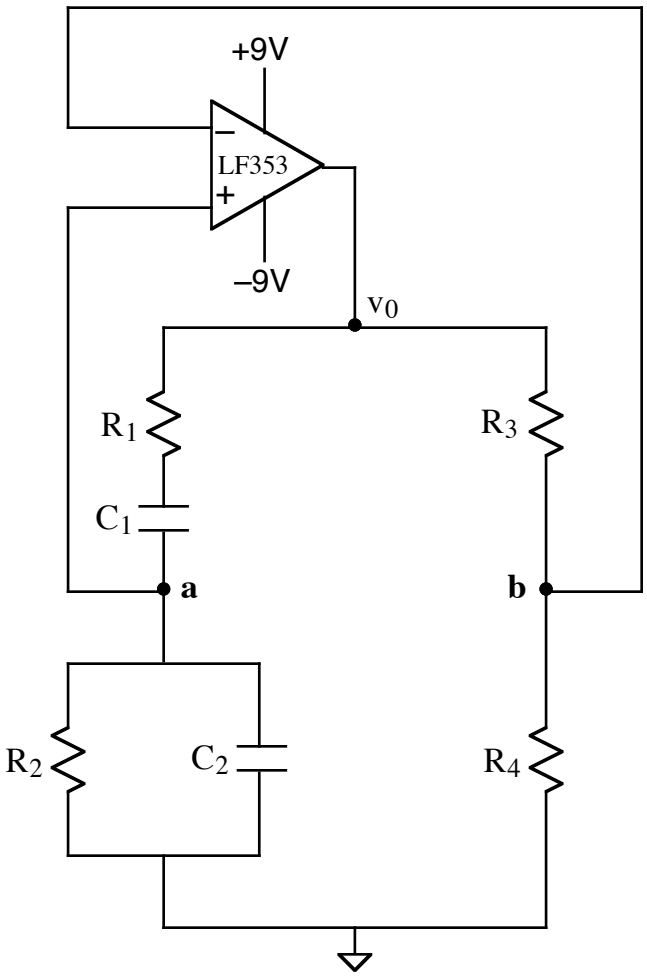

Fig. 3. Schematic diagram of an op-amp Wein-bridge oscillator.

The above discussion establishes that the circuit *may* oscillate. What ensures that the circuit oscillates is the presence of noise at the op-amp inputs. To establish this claim, we use frequency-domain techniques.

Transform the circuit of Fig. 3 to the frequency domain and read the following discussion before proceeding to the next section.

First, we note that, because of negative feedback, the op-amp output, $\mathbf{V}_0$, will move toward a value that makes the voltages at **a** and **b** equal. Using the idea of superposition, we may think of $\mathbf{V}_0$ as a superposition of signals of every possible frequency, with each such signal adjusted to make **a** and **b** equal. Given that the left and right side of the bridge are voltage dividers driven by the same voltage, $\mathbf{V}_0$, there are only two ways to make **a** and **b** equal:

i. The two voltage dividers produce exactly the same output voltage, (which means $\mathbf{V}_0$ may have *any* value). This happens at only one particular frequency that we denote as ω_{ab}.

ii. The two voltage dividers produce different output voltages, (which means $\mathbf{V}_0 = 0$ V). This happens at all frequencies other than ω_{ab}.

Second, we note that, at frequency ω_{ab}, any disturbance such as a noise voltage source across the **a** and **b** inputs will cause $\mathbf{V}_0$ to change in a direction that will make the voltages at **a** and **b** equal again. If the bridge is perfectly balanced, it is possible to show that the voltages at **a** and **b** remain unequal regardless of what value is taken on by $\mathbf{V}_0$.

Whenever the inputs to the op-amp are unequal, the op-amp output will be very large. This means the magnitude of $\mathbf{V}_0$ will be large. In the time domain, the op-amp output, $v_0(t)$, will be a sinusoid at frequency ω_{ab} with large amplitude. This is the oscillatory output we want the circuit to produce.

Third, we note that, at a frequency, ω, close to ω_{ab}, the value of $\mathbf{V}_0$ required to make the voltages at **a** and **b** equal may be quite large. One may show that the magnitude of $\mathbf{V}_0$ is inversely proportional to the difference between ω and ω_{ab}. In theory, these frequencies will also be present in $v_0(t)$. Since this happens only for frequencies close to ω_{ab}, however, $v_0(t)$ will be nearly a sinusoid at frequency ω_{ab}.

Fourth, we note that the output of the op-amp will be so large that it saturates, (i.e., is limited by the power-supply voltage), at frequency ω_{ab}. Thus, the behavior of the oscillator involves difficult nonlinear relations that prevent us from obtaining an exact expression for $v_0(t)$. Nevertheless, we achieve oscillation at frequency ω_{ab}. In this lab, we set $\omega_{ab} = 25$ kHz for measuring tissue impedance.

2.2. Balanced Bridge

Using the frequency-domain model, show that the bridge is balanced, i.e., that the voltages at **a** and **b** are equal, when $Z_1/Z_2 = R_3/R_4$ where Z_1 is the combined impedance of R_1 in series with C_1 and Z_2 is the impedance of R_2 in parallel with C_2.

2.3. Oscillation

Add a noise-voltage source, $\mathbf{V}_n$, in series with a resistor R_n across the input of the op-amp, (i.e., between **a** and **b** in Fig. 3). This simulates the small amount of noise that is always present in a circuit and, in this case, starts the oscillations. Using superposition and assuming the bridge is balanced, find an expression for the voltage drop from **a** to **b**. Note that, when the Wien bridge is balanced, adding $\mathbf{V}_n$ and R_n creates a

condition in which no value of $\mathbf{V}_0$ will make the voltages at **a** and **b** equal. Consequently, the op-amp input voltages are unequal and $\mathbf{V}_0$ grows large. The balanced bridge thus gives rise to a large sinusoidal oscillation that is limited only by the power-supply voltages for the op-amp.

2.4. Oscillation Frequency

From the condition $Z_1/Z_2 = R_3/R_4$, derive an expression for the frequency of oscillation by solving for the value of ω that makes $Z_1/Z_2 = R_3/R_4$. Note that the quantity on the left is complex, whereas the quantity on the right is real. The equation is satisfied only if the real parts are equal and the imaginary parts are equal. This leads to two equations that must be solved. Show that one equation yields the value of ω, and the second equation is satisfied when $R_3/R_4 = R_1/R_2 + C_2/C_1$. (More advanced analysis shows that the condition for oscillation is $R_3/R_4 > R_1/R_2 + C_2/C_1$, but saturation of the op-amp occurs when R_3/R_4 is too high, producing clipping of the waveform.)

2.5. Component Values for Oscillation

Using $C_1 = C_2 = 250$ pF and the equations you derived above, choose component values for the circuit that make it oscillate at frequency $\omega_{ab} = 25$ kHz. Use practical component values that require less than the maximum 10 mA the op-amp can output. When designing for this constraint, consider the instantaneous current flowing out of the op-amp and find its maximum value during one cycle of the square wave.

3. CONSTRUCT AND TEST OSCILLATOR

3.1. Oscillation Frequency for Standard Component Values

Re-calculate the frequency of oscillation for the standard component values chosen for the circuit. That is, assume the components have exactly the standard value shown on their label.

3.2. Oscillation Frequency for Actual Component Values

Measure the values of the actual resistors and capacitors that you used in the circuit and re-calculate the frequency of oscillation for these component values.

3.3. Measured Oscillator Waveforms

Construct the oscillator circuit that you designed. Note that proper performance of the circuit requires that ratios of component values be very accurate. Match capacitor values by padding them with small capacitors added in parallel, if necessary, and use potentiometers instead of resistors to adjust resistor ratios where necessary. In

particular, use a potentiometer for R_4, and adjust it so that the R_3/R_4 value makes $v_0(t)$ a good sine wave.

Record and plot the waveform on the oscilloscope and determine the frequency of the sinusoid.

3.4. Tabulated Values

Make a table showing the following values:

i. Desired oscillator frequency, (25 kHz).
ii. Predicted oscillator frequency for standard component values chosen for the circuit. That is, assume the components have exactly the standard value shown on their label.
iii. Predicted oscillator frequency for measured component values.
iv. Oscillator frequency measured on the oscilloscope.

4. ANALYZE TISSUE IMPEDANCE MODEL

4.1. Circuit for Measuring Tissue Impedance

As discussed in the Introduction, a simplified model of tissue impedance consists of a parallel R and C. Nominal values for R and C are $R_t = 2\ k\Omega$ and $C_c = 2.5$ nF. To determine the measured values of R_t and C_c, we use a voltage divider consisting of a known resistance in series with the tissue that is connected by electrodes. We measure the voltage, $\mathbf{V}_1$, across the total impedance and the voltage, $\mathbf{V}_2$, between the resistor and tissue to obtain two sinusoidal waveforms with different magnitudes and phase shifts. Using this information and some algebra, we can determine the values of R_t and C_c.

Using an external resistor in series with the oscillator output and the tissue model, design a voltage-divider experiment that allows you to determine the impedance of the tissue at 25 kHz. In other words, draw a schematic diagram showing the oscillator output, the external resistor, the tissue model, and connections for two oscilloscope probes with reference inputs. Note that the oscilloscope inputs have a common reference connection. Thus, you must choose a circuit configuration that uses two voltage measurements with a common reference. Furthermore, this reference must be the same as the reference of the oscillator circuit.

Show your measurement-circuit diagram to your TA for approval before proceeding.

4.2. Component Values for Tissue Impedance Model

Derive a symbolic equation relating tissue impedance, z_t, to the two phasor voltages, $\mathbf{V}_1$ and $\mathbf{V}_2$, that you will measure with your circuit. Extend your results to find

equations for R_t and C_c in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$. Note that complex quantities are equal if and only if the real parts are equal and the imaginary parts are equal.

4.3. Choosing Resistance for Impedance Measurement Circuit

Choose an external resistance value, R_1, that yields accurate R_t and C_c values when measured values of $\mathbf{V}_1$ and $\mathbf{V}_2$ include small errors. That is, choose a value of R_1 such that a small measurement error has minimal effect on the calculated values of R_t and C_c. Although an optimal value of R_1 value could be derived by sensitivity analysis, you may use a much more tractable alternative of finding a value of R_1 such that the ratio of the magnitudes of $\mathbf{V}_1$ and $\mathbf{V}_2$ is approximately two-to-one, depending on which is the larger signal. Use the nominal values of R_t and C_c from Subsection 4.1 for this calculation.

5. MEASURE TISSUE IMPEDANCE

5.1. Measurement of Tissue Impedance and Calculation of Component Values

Using an oscilloscope, your oscillator circuit (powered by 9V batteries), your external resistance, R_1, and two electrodes placed halfway up your biceps on the inside and outside edges as shown in Fig. 4 so a line connecting the electrodes runs through the muscle, measure $\mathbf{V}_1$ and $\mathbf{V}_2$ at 25 kHz. Then use your equations from 4.2 to calculate R_t and C_c. Also, measure the approximate distance, *d*, separating the electrodes and the surface area, *A*, of the electrodes.

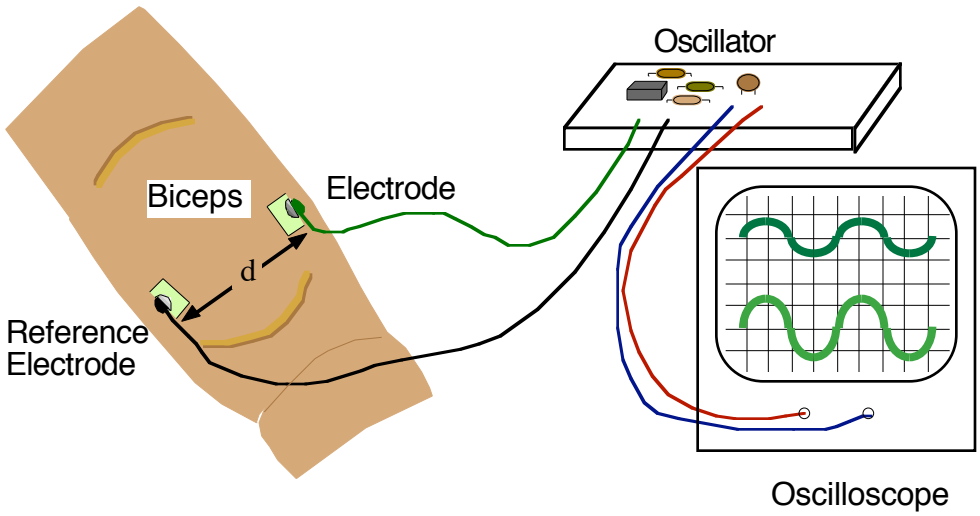

Fig. 4. Tissue measurement configuration.

5.2. Calculation of Conductivity, Relative Permeability, and Power Density

Use your calculated values of R_t and C_c along with *d* and *A* to find estimated values of the conductivity, σ, and relative permeability, ε_r, of your muscle for comparison with

published values. Also calculate the power density, *S*, used in the experiment. The equations you will need come from basic physics and involve *only products or quotients* of physical quantities. By ignoring fringing fields and modeling the tissue between the electrodes as a tube, as shown in Fig. 5, derive equations for the following terms:

i. R_t as a function of *d*, *A*, and σ = tissue conductivity = 1/tissue resistivity in Ωm
ii. C_t as a function of *d*, *A*, and $\varepsilon = \varepsilon_0\varepsilon_r$ where $\varepsilon_0 = 8.85$ pF/m and ε_r = unitless const
iii. Power density *S* (= total power in tissue divided by electrode area) in W/m^2 as a function of *I* = magnitude of electrode current, R_t, and *A*

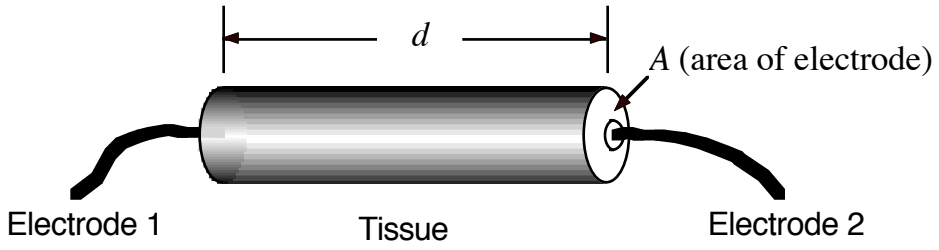

Fig. 5. Tissue modeled as a tube.

To determine whether R_t or C_t are proportional or inversely proportional to *d* or *A*, think of the tube of tissue being subdivided into thinner or shorter tubes connected in parallel or in series. Increasing *A* corresponds to adding components in parallel, while increasing *d* corresponds to adding components in series.

Rearrange your equations for R_t and C_t to solve for σ and ε_r. Use your measured value for $\mathbf{V}_1$ or $\mathbf{V}_2$, along with R_t and C_t, to calculate the magnitude of electrode current, *i*. Then calculate the values of σ, ε_r, and *S*.

5.3. Comparison of Measured Values with Published Values

Make a table comparing your values of σ, ε_r with values for muscle and fat found at

http://www.brooks.af.mil/AFRL/HED/hedr/reports/dielectric/home.html

in an appendix:

http://www.brooks.af.mil/AFRL/HED/hedr/reports/dielectric/Appendix.B/AppendixB1.html

5.4. Comparison of Power Density with FDA Limit

Compare your value of power density, *S*, with the FDA limit of 100 mW/cm^2 (for higher frequency) published on the web at

http://www.fcc.gov/Bureaus/Engineering_Technology/Documents/bulletins/oet65/oet65.pdf (see p. 67, Table I(B) value for 0.3 MHz).

6. WRITE FORMAL REPORT

Write a formal report describing your work on this project. See instructions in "Course Procedures" about how to write the report. (Also, look for detailed point breakdowns for Lab 3 grading on the course web site.) Include at least the following in your report:

i. An abstract. The abstract is a one paragraph succinct summary of the entire results. It should describe the key result of the experiments performed.

ii. A short introduction. You may attach this handout to the report in the appendix and refer to it so that you don't have to copy the information in it. Your introduction, however, must introduce your report and will be unique to your report. The introduction gives the motivation for the experiments performed and describes the organization of the report.

iii. A careful description of the work that you did in Sections 2 through 5, above.

   a. Discuss and give appropriate quantitative results for each of the numbered subsections in Sections 2 through 5. Every subsection corresponds to a specific task with a specific quantitative result that must be described in your report. To facilitate grading, number the subsections of your report with the same numbers used in this handout.

   b. Give clear derivations of mathematical expressions, including explanations in words for every equation in every derivation. Include consistency checks of final results whenever possible.

   c. Explain how you chose the values of circuit components and include a schematic diagram showing component values for the final circuit.

   d. Explain all measurements carefully and include data appropriately in clearly labeled tables and graphs in the body of the report.

   e. Include listings of all your Matlab® programs in an appendix, and explain how the code works in comments.

   f. Show plots of your oscillator output and voltage waveforms for the tissue impedance measurement.

iv. A succinct conclusion. The conclusion must list the most salient quantitative results of this laboratory project. As a guide to what the conclusion should say, consider what information would be most useful to a student about to start the lab.