HOMEWORK #9 solution

EX: The following equation describes the voltage, v_L, across an inductor as a function of time. Find an expression for the current, $i_L(t)$, through the inductor as a function of time. Assume that $i_L(t = 0) = 0$ A.

$$v_L(t) = 2 + 6(1 - e^{-t/12.5\mu s})\text{ kV}$$

SOL'N: We use the defining equation for an inductor and solve for i in terms of v.

$$v_L = L\frac{di_L}{dt}$$

First, we multiply both sides by dt.

$$v_L dt = L di_L$$

Second, we integrate both sides and use limits that correspond to the variable of integration for each side and are evaluated at the same points in time for both sides.

$$\int_0^t v_L dt = \int_{i_L(t=0)}^{i_L(t)} L di_L$$

The integral on the right side simplifies nicely.

$$\int_0^t v_L dt = Li_L\Big|_{i_L(t=0)}^{i_L(t)} = L\left[i_L(t) - i_L(t=0)\right]$$

or

$$i_L(t) = \frac{1}{L}\int_0^t v_L dt + i_L(t=0)$$

The above expression applies to any inductor in any circuit.

We now substitute the formula given for $v_L(t)$ and the value given for $i_L(t = 0)$ to find $i_L(t)$:

$$i_L(t) = \frac{1}{L}\int_0^t \left[2 + 6(1 - e^{-t/12.5\mu s})\text{kV}\right] dt + 0A$$

or

$$i_L(t) = \frac{1}{L}\int_0^t \left[8 - 6e^{-t/12.5\mu s})\text{kV}\right] dt + 0A$$

or

$$i_L(t) = \frac{1}{L}\left[8t\Big|_0^t + 6\cdot 12.5\mu s\cdot e^{-t/12.5\mu s}\Big|_0^t\right]\text{kV}$$

or

$$i_L(t) = \frac{1}{L}\left[8t + 75\mu s\cdot\left(e^{-t/12.5\mu s} - 1\right)\right]\text{kV}$$

or

$$i_L(t) = \frac{1}{L}\left[8\text{kV}\cdot t + 75\text{mV}\cdot\left(e^{-t/12.5\mu s} - 1\right)\right]$$