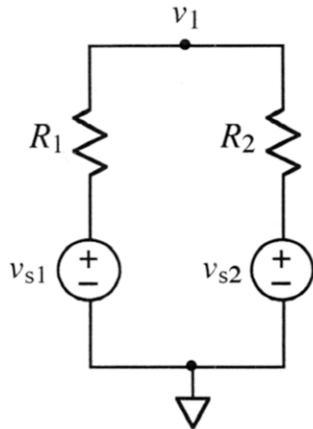


Ex:



Use the node-voltage method to find a formula for v_1 . Write your answer as a sum of a voltage divider for v_{s1} and a voltage divider for v_{s2} .

SOL'N: We sum the currents out of the v_1 node:

$$\frac{v_1 - v_{s1}}{R_1} + \frac{v_1 - v_{s2}}{R_2} = 0A$$

Note: We can use the node-voltage method even for a point in the circuit that is not the meeting point for three or more wires.

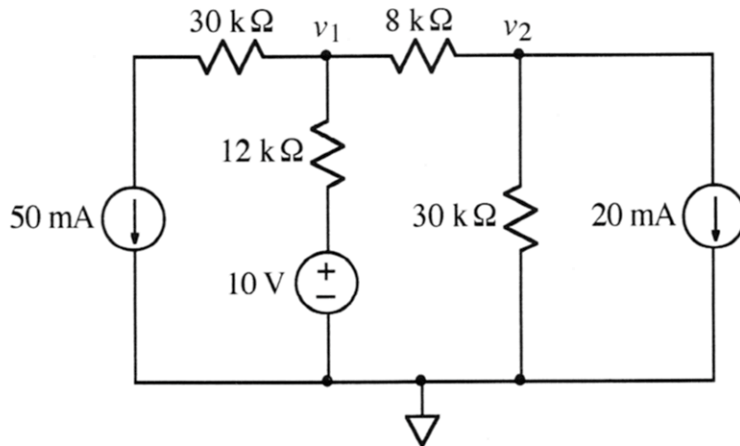
Simplifying our equation reveals that it is the sum of two voltage dividers:

$$v_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2}$$

or (after multiplying both sides by $\frac{R_1 R_2}{R_1 + R_2}$)

$$v_1 \frac{(R_2 + R_1)}{R_1 + R_2} = v_{s1} \frac{R_2}{R_1 + R_2} + v_{s2} \frac{R_1}{R_1 + R_2}$$

Ex:



Use the node-voltage method to find v_1 and v_2 .

SOL'N: We sum the currents out of the v_1 node and then the v_2 node.

$$v_1 \text{ node: } 50 \text{ mA} + \frac{v_1 - 10 \text{ V}}{12 \text{ k}\Omega} + \frac{v_1 - v_2}{8 \text{ k}\Omega} = 0 \text{ A}$$

$$v_2 \text{ node: } \frac{v_2 - v_1}{8 \text{ k}\Omega} + \frac{v_2}{30 \text{ k}\Omega} + 20 \text{ mA} = 0 \text{ A}$$

For each equation, one approach to the algebra is to group terms multiplying v_1 and v_2 and move constant terms to the right side.

$$v_1 \left(\frac{1}{12 \text{ k}\Omega} + \frac{1}{8 \text{ k}\Omega} \right) - v_2 \frac{1}{8 \text{ k}\Omega} = \frac{10 \text{ V}}{12 \text{ k}\Omega} - 50 \text{ mA}$$

$$-v_1 \frac{1}{8 \text{ k}\Omega} + v_2 \left(\frac{1}{8 \text{ k}\Omega} + \frac{1}{30 \text{ k}\Omega} \right) = -20 \text{ mA}$$

The next ^{step} is to multiply by common denominators.

$$24\text{k}\Omega \left[v_1 \left(\frac{1}{12\text{k}\Omega} + \frac{1}{8\text{k}\Omega} \right) - v_2 \frac{1}{8\text{k}\Omega} \right] = 24\text{k}\Omega \left(\frac{10\text{V} - 50\text{mA}}{12\text{k}\Omega} \right)$$

$$\text{or } v_1(2+3) - v_2(3) = 20\text{V} - 1200\text{V}$$

Now for the second equation:

$$120\text{k}\Omega \left[(-v_1) \frac{1}{8\text{k}\Omega} + v_2 \left(\frac{1}{8\text{k}\Omega} + \frac{1}{30\text{k}\Omega} \right) \right] = 120\text{k}\Omega (-20\text{mA})$$

or

$$-v_1(15) + v_2(15+4) = -2400\text{V}$$

Summary of our two eq'ns:

$$v_1(5) - v_2(3) = -1180\text{V}$$

$$-v_1(15) + v_2(19) = -2400\text{V}$$

One way to solve these equations is to multiply the first eq'n by 3 and add the result to the second eq'n, cancelling the v_1 term:

$$\begin{array}{r} 3v_1(5) - 3v_2(3) = 3(-1180\text{V}) \\ + \quad -v_1(15) + v_2(19) = -2400\text{V} \\ \hline v_2(10) = -5940\text{V} \end{array}$$

or

$$v_2 = \frac{-5940\text{V}}{10} = -594\text{V}$$

Using an eq'n above, we find v_1 :

$$v_1 = \frac{-1180\text{V} + (-594\text{V})(3)}{5} = -592.4\text{V}$$