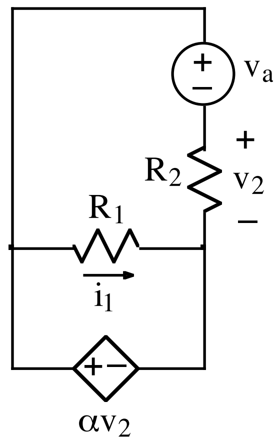
HOMEWORK #4 solution

Ex:



a) Derive an expression for i_1. The expression must not contain more than the circuit parameters α, v_a, R_1, and R_2. **Note:** $\alpha \neq 0$.

b) Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

SOL'N:

a) We have two inner v-loops:

$-v_a + -v_2 + i_1 R_1 = 0\text{V}$ (top v-loop)
using Ohm's law here

$-i_1 R_1 + \alpha v_2 = 0\text{V}$ (bottom v-loop)

From the 2nd eq'n, $\alpha v_2 = i_1 R_1$

or $v_2 = \dfrac{i_1 R_1}{\alpha}$.

From 1st eq'n, $-v_a - \dfrac{i_1 R_1}{\alpha} + i_1 R_1 = 0\text{V}$

or $i_1 \left(R_1 - \dfrac{R_1}{\alpha} \right) = v_a$

or $i_1 = \dfrac{v_a}{R_1 (1 - 1/\alpha)}$

b) For the consistency check, we choose values of sources and R's that yield a simpler circuit for which solution is obvious. Many checks may be possible. Only one is required here.

ex: $v_a = 0V$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $\alpha = 3$.

The circuit has no independent power source. Thus, all currents and voltages = 0. So $i_1 = 0A$.

Now we try our formula from (a):

$$i_1 = \frac{0}{1\Omega(1 - 1/3)} = 0 \quad \checkmark \text{ (Consistent)}$$

ex: $R_1 = \infty\,\Omega$ (open circuit), $R_2 = 2\Omega$, $\alpha = 3$, $v_a = 12V$

If R_1 is open circuit, then $i_1 = 0A$.

Now we try our formula from (a):

$$i_1 = \frac{12V}{\infty\,\Omega(1 - 1/3)} = \frac{12V}{\infty\,\Omega} = 0A$$

Note: Some consistency checks might lead to invalid circuits such as v-sources shorted out. Avoid these. This particular circuit is prone to that problem.