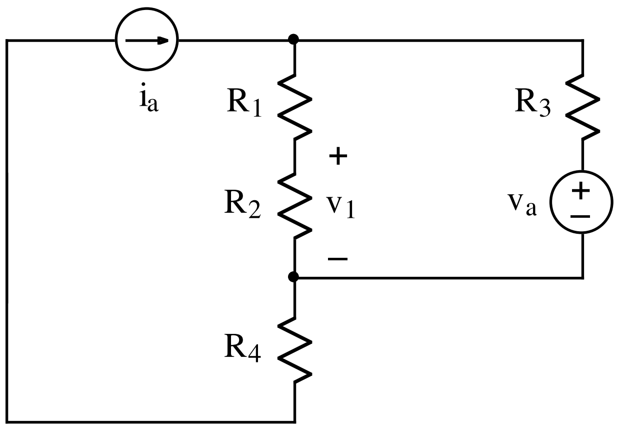
HOMEWORK #4 solution

EX:



Derive an expression for v_1. The expression must not contain more than the circuit parameters v_a, i_a, R_1, R_2, and R_3.

SOL'N:

We have the same current in R_1 and R_2, namely i_1.

Current i_a must the current in R_4. $v_4 = i_a R_4$.

From a current sum at the top-center node:

$$-i_a + i_1 + i_3 = 0\text{A} \quad \text{or} \quad i_1 + i_3 = i_a$$

The current sum at the other node is redundant.

We have a v-loop on the right. (Other v-loops pass thru i_a source and are omitted.)

$$+v_1 + i_1 R_1 - i_3 R_3 - v_a = 0\text{ V}$$

Note: We have used Ohm's law as we go, although we must also use eq'n
$v_1 = i_1 R_2$.

We could solve for i_1, but the solution approach taken here finds v_1.

First, $i_3 = i_q - i_1$ from current sum.

Second, $i_1 = \dfrac{v_1}{R_2}$

Third, we substitute into the v-loop eq'n:

$$v_1 + i_1 R_1 - (i_q - i_1) R_3 - v_q = 0\,V$$

or $$v_1 + \frac{v_1}{R_2} R_1 - \left(i_q - \frac{v_1}{R_2}\right) R_3 - v_q = 0\,V$$

or $$v_1 \left(1 + \frac{R_1}{R_2} + \frac{R_3}{R_2}\right) = i_q R_3 + v_q$$

or $$v_1 = \frac{i_q R_3 + v_q}{1 + \frac{R_1}{R_2} + \frac{R_3}{R_2}} = \frac{(i_q R_3 + v_q) \cdot R_2}{R_1 + R_2 + R_3}$$

Consistency check: $v_q = 0$, $R_3 = 0\,\Omega \Rightarrow v_1 = 0\,V$
R_2 bypassed

our eq'n gives $v_1 = \dfrac{(i_q \cdot 0\,\Omega + 0\,V)\, R_2}{R_1 + R_2 + 0\,\Omega} = 0\,V$ ✓