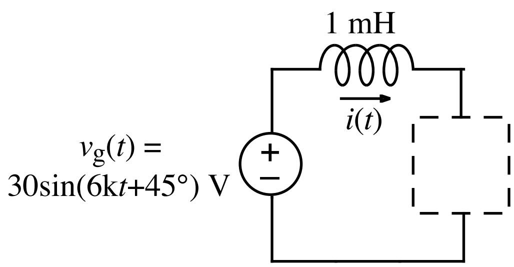
HOMEWORK #16 solution

Ex:



a) Choose an R, an L, or a C to be placed in the dashed-line box to make
$$i(t) = 0.5\cos(6\text{k}t - 135°)\text{ A}.$$

b) State the value of the component you chose for Problem 2. Note that the value of the component cannot be negative.

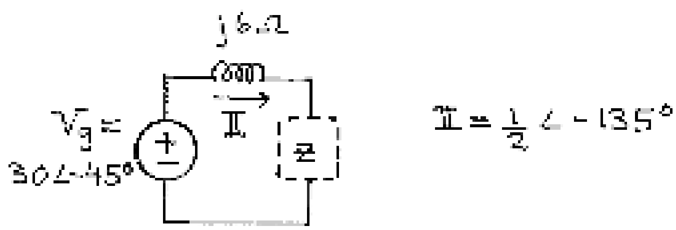
sol'n: We first transform the circuit to the frequency domain.

$$\mathbf{V}_g = 30 \cdot (-j) \cdot 1\angle 45° = 30\angle 45° - 90° = 30\angle -45°$$

Note: $-j = 1\angle -90°$

$$j\omega L = j\,6\text{K} \cdot 1\text{mH} = j6\,\Omega$$

$$\mathbf{I} = \tfrac{1}{2}\angle -135°\text{ A}$$



By Ohm's law, $\mathbf{I} = \dfrac{\mathbf{V}_g}{Z + j6\Omega}$.

or $z + j6\,\Omega = \dfrac{V_g}{I}$

We could solve for z directly in this case, but it is instructive to solve the problem using phase and magnitude separately.

$$\angle(z + j6\,\Omega) = \angle V_g - \angle I$$

Notes: $\angle\left(\dfrac{V_g}{I}\right) = \angle V_g - \angle I$

$$\angle(z + j6\,\Omega) = -45^\circ - -135^\circ = 90^\circ$$

We conclude that $z + j6\,\Omega$ must be purely imaginary and positive.

This means that z might be an L or a C. If it is a C, however, it must be large enough that z is smaller in magnitude than $j6\,\Omega$.

In either case, (L or C), we can write

$$z + j6\,\Omega = jk$$

where k is a positive constant.

Now we consider magnitude.

$$|z + j6\Omega| = \left|\frac{V_g}{I}\right| = \frac{|V_g|}{|I|} = \frac{30}{\frac{1}{2}} = 60$$

Since we know $z + j6 = jk$, we have

$$|z + j6| = k = 60$$

$$z = j60 - j6 = j54$$

We must use an L to get a positive imaginary z:

$$z = j\omega L = j \cdot 6k \cdot L = j54$$

$$L = \frac{54}{6k} = 9 \text{ mH}$$