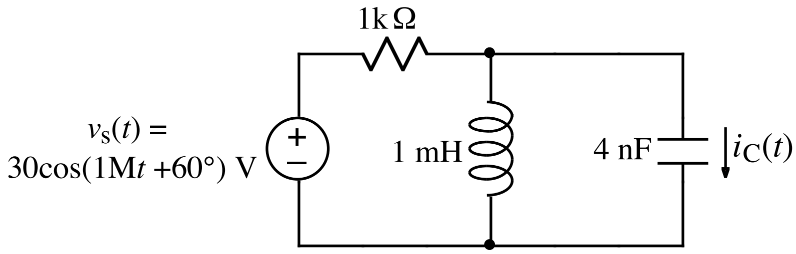
HOMEWORK #15 solution

Ex:



a) Find the phasor value for $v_S(t)$.

b) Draw the frequency-domain circuit diagram, including the phasor value for $v_S(t)$ and impedance values for components.

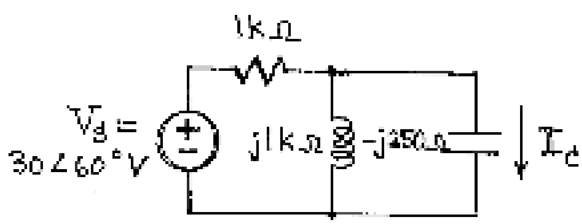
c) Find the phasor value for $i_C(t)$.

soln: a) $P\{30\cos(1Mt + 60°)\ V\} = 30e^{j60°}$ V or $30\angle 60°$ V

b) We calculate the impedances:

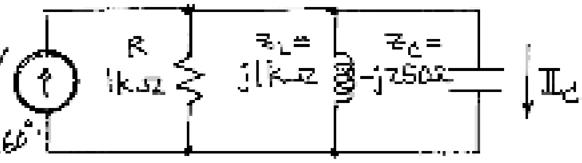
$$Z_L = j\omega L = j\,1M \cdot 1\,mH = j1k\,\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{1M\,4nF} = \frac{-j}{4m}\,\Omega = -j250\,\Omega$$



c) One way to solve this problem is to use a Norton equivalent for V_s and the 1kΩ resistor.

$$\mathbb{I} = \frac{30\angle 60^\circ\,\text{V}}{1\,\text{k}\Omega} = 30\,\text{mA}\angle 60^\circ$$



Using the current divider formula, we have

$$\mathbb{I}_C = \mathbb{I}\,\frac{R\|Z_L}{R\|Z_L + Z_C}$$

$$= \mathbb{I}\,\frac{1}{1 + \dfrac{Z_C}{R\|Z_L}}$$

$$= \mathbb{I}\,\frac{1}{1 + Z_C\left(\dfrac{1}{R} + \dfrac{1}{Z_L}\right)}$$

$$= \mathbb{I}\,\frac{1}{1 + \dfrac{Z_C}{R} + \dfrac{Z_C}{Z_L}}$$

$$= 30\,\text{mA}\angle 60^\circ\,\frac{1}{1 + \dfrac{-j250\,\Omega}{1\,\text{k}\Omega} + \dfrac{-j\,250\,\Omega}{j\,1\,\text{k}\Omega}}$$

$$= 30\,\text{mA}\angle 60^\circ\,\frac{1}{1 - \frac{1}{4} - j\,\frac{1}{4}}$$

$$= \frac{30\,\text{mA}\angle 60^\circ}{\frac{3}{4} - j\,\frac{1}{4}}$$

$$= \frac{120\ mA \angle 60^\circ}{3 - j}$$

$$= \frac{120\ mA \angle 60^\circ}{3 - j} \quad \frac{3+j}{3+j}$$

$$= \frac{120\ mA \angle 60^\circ \ (3+j)}{3^2 + 1^2}$$

$$= \quad 12\ mA \angle 60^\circ \ (3+j)$$

$$= \quad 12\ mA \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(3+j)$$

$$= \quad 12\ mA \left[\frac{3}{2} - \frac{\sqrt{3}}{2} + j\left(\frac{1}{2} + \frac{3\sqrt{3}}{2}\right)\right]$$

$$\mathbb{I}_c = \quad 6\ mA \left[3 - \sqrt{3} + j\left(1 + 3\sqrt{3}\right)\right]$$

Note: This is an exact answer, but an approximate polar answer is more useful.

$$\mathbb{I}_c = \quad 12\ mA \angle 60^\circ \ (3+j)$$

$$= \quad 12\ mA \angle 60^\circ \cdot \sqrt{3^2+1^2} \angle \tan^{-1} \frac{1}{3}$$

$$= \quad 12\ mA \angle 60^\circ \cdot \sqrt{10} \angle 18.4^\circ$$

$$\mathbb{I}_c = \quad 12\sqrt{10}\ mA \angle 78.4^\circ$$

Note: $i_c(t) = 12\sqrt{10}\ mA \cos(1Mt + 78.4^\circ)$