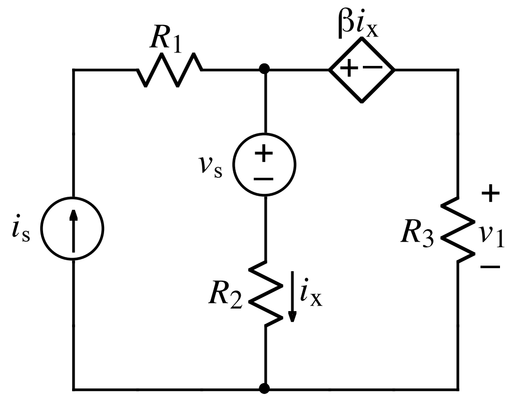
HOMEWORK #12 solution

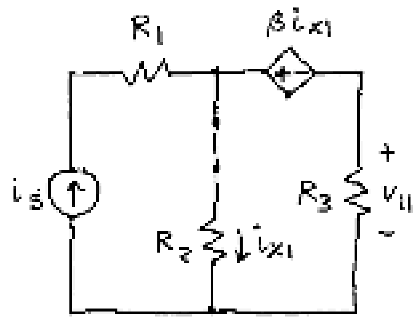
Ex:



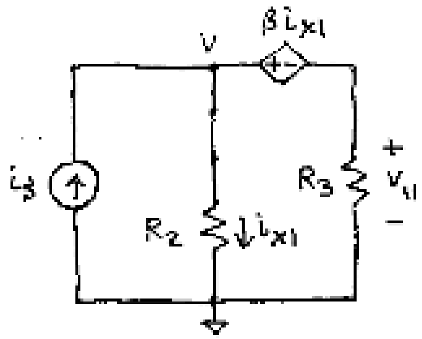
Using superposition, derive an expression for v_1 that contains no circuit quantities other than i_s, v_S, R_1, R_2, R_3, and β, where $\beta > 0$.

soln: We turn on one independent source at a time. (We never turn off independent sources, since they act like R's.)

case I: i_s on, v_s off (v src off = wire)



Note: Since R_1 is in series with a current source and the problem does not ask about i or v for R_1, we may ignore R_1 and replace it with a wire.



We add a reference and use the node-v method:

$$-i_s + \frac{v}{R_2} + \frac{\left(v - \beta \overbrace{\frac{v}{R_2}}^{i_{x1}}\right)}{R_3} = 0\,A$$

$$v\left(\frac{1}{R_2} + \frac{1}{R_3} - \frac{\beta}{R_2 R_3}\right) = i_s$$

$$v = \frac{i_s}{\frac{1}{R_2} + \frac{1}{R_3} - \frac{\beta}{R_2 R_3}} = i_s \cdot R_2 \| R_3 \| \left(-\frac{R_2 R_3}{\beta}\right)$$

For v_{11}, we use v and the dependent src

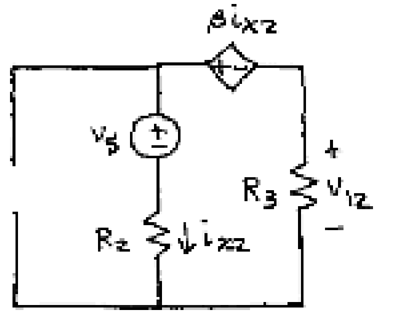
$$v_{11} = v - \beta \overbrace{\frac{v}{R_2}}^{i_x} = v\left(1 - \frac{\beta}{R_2}\right)$$

$$v_{11} = i_s \cdot R_2 \| R_3 \| \left(\frac{-R_2 R_3}{\beta}\right) \cdot \left(1 - \frac{\beta}{R_2}\right)$$

With some effort doing algebra, this may also be written as follows:

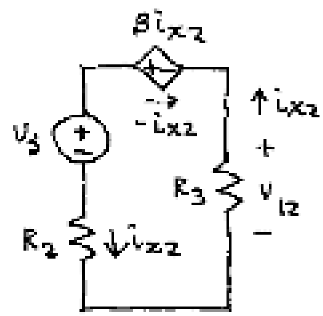
$$v_{11} = i_s \cdot (R_2 - \beta) \| R_3$$

case II: i_s off (i src off = open), v_s on



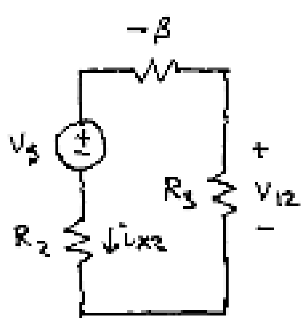
Again, we may ignore R_1 in series with i src.

or



Here, we may replace the dependent source with an $R_{eq} = v/i$:

$$R_{eq} = \frac{\beta i_{x2}}{-i_{x2}} = -\beta$$



Now we use a v-divider:

$$v_{12} = v_s \cdot \frac{R_3}{R_2 - \beta + R_3}$$

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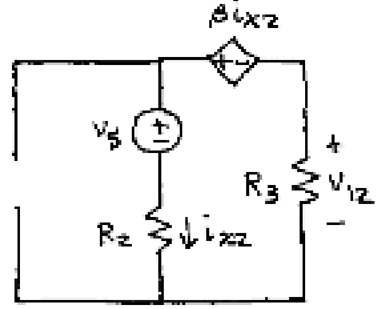

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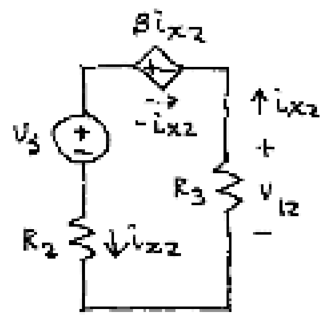

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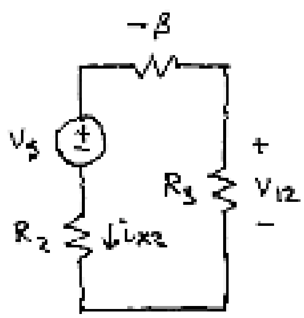

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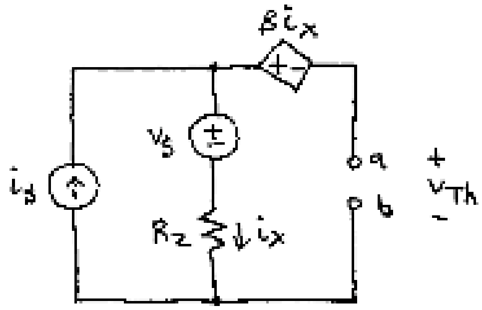
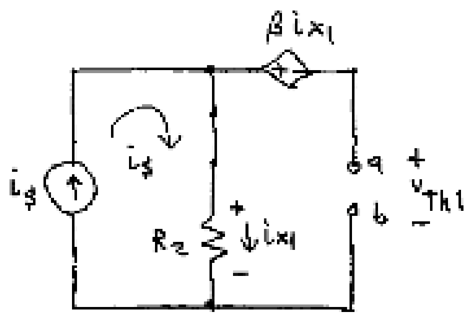


Now we use superposition or any other method of our choosing to find v_{Th}. As always, v_{Th} equals the open circuit voltage from a to b.

Here, we use superposition as directed in the problem.

case I: i_S on, v_S off = wire

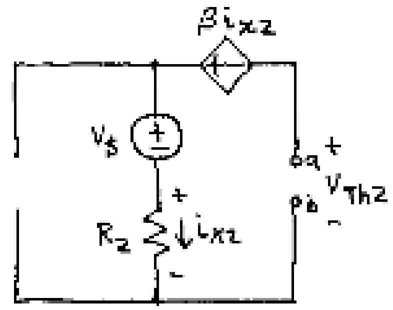


We observe that $i_{x1} = i_S$, and

$$v_{Th1} = i_{x1} R_2 - \beta i_{x1} = i_S (R_2 - \beta)$$

(This is significantly simpler that the earlier superposition approach. There is more to be done before we are finished here, however.)

case II: i_s off = open, v_s on



Here, $i_{x2} = 0$ since there is no complete circuit. Thus, we 0V across R_2 and across the dependent source. It follows (from a v-loop on the right side) that

$$v_{Th2} = v_s$$

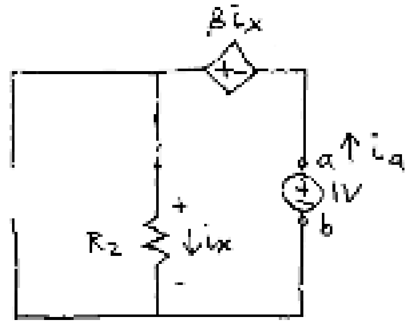
Summing results, we have

$$v_{Th} = v_{Th1} + v_{Th2} = i_s(R_2 - \beta) + v_s$$

To find R_{Th}, we turn off independent sources and attach a 1V source to a, b. We measure the current, i_a, flowing into the 'a' terminal and use Ohm's law:

$$R_{Th} = \frac{1V}{i_a} \qquad \text{(independent src's off)}$$

Note: this method simplifies calculations for circuits with multiple independent sources.



From v-loop on the right side, we have

$$i_x R_2 - \beta i_x - 1V = 0V$$

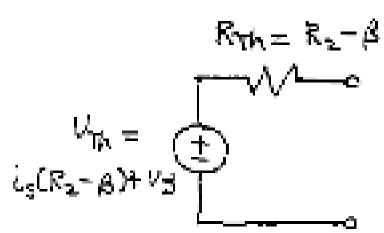
or $$i_x = \frac{1V}{R_2-\beta}$$

We also have that $i_x = i_a$.

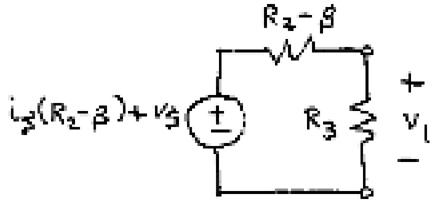
Thus, $$i_a = \frac{1V}{R_2-\beta}$$

and
$$R_{Th} = \frac{1V}{i_a} = \frac{1V}{\frac{1V}{R_2-\beta}} = R_2-\beta_2$$

Our Thevenin equivalent without R_3 is



Now we add R_3 and find v_1:



$$v_1 = \left[i_s (R_2 - \beta) + v_s \right] \frac{R_3}{R_2 - \beta + R_3}$$

This is the same result as earlier but with dramatically less algebra.