

Ex:



- a) Find an expression for $i_{\rm L}(t)$ for $t \ge 0$.
- b) Find the energy stored in the inductor at time $t = 30 \,\mu s$.
- **SOL'N:** a) The following general form of solution applies to any RL circuit with a single inductor:

$$i_L(t \ge 0) = i_L(t \to \infty) + [i_L(t = 0^+) - i_L(t \to \infty)]e^{-t/(L/R_{\text{Th}})}$$

The Thevenin resistance, R_{Th} , is for the circuit after t = 0 (with the *L* removed) as seen from the terminals where the *L* is connected. In the present case, we have $R_{\text{Th}} = 10 \text{ k}\Omega$.

 $L/R_{\rm Th} = 2 \text{ nH}/10 \text{ k}\Omega = 0.2 \text{ ps}$

The value of $i_L(t=0)$ is given in the problem as 0 V. Note that the *L* could not have any current before t = 0 in this circuit since there is no closed path to carry current.

As time approaches infinity, the *L* current will converge to its final value, and the voltage across the *L* will cease to change. Thus, $di_L/dt = 0$ and $v_L = 0$, meaning the *L* will act like a wire. It follows that the current through the *L* will equal the current through the *R*, which will equal 2.3 V/10 k $\Omega = 0.23$ mA.

 $i_L(t \rightarrow \infty) = 0.23 \text{ mA}$

Substituting values, we have the following result:

 $i_L(t \ge 0) = 0.23 \text{ mA} + [0 - 0.23 \text{ mA}]e^{-t/0.2 \text{ps}}$

b) The energy in an inductor is given by the following formula:

$$w_L = \frac{1}{2}Li_L^2$$

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We use the solution to (a) to evaluate $i_L(t)$ at $t = 30 \ \mu s$.

$$i_L(t = 30 \mu s) = 0.23 \text{ mA V} - 0.23 \text{ mA} \cdot e^{-30 \mu s / 0.2 \text{ ps}} \approx 0.23 \text{ mA}$$

Using this voltage, we evaluate the energy on the capacitor.

$$w_L = \frac{1}{2} 2 \text{nH} \cdot (0.23 \text{mA})^2 = 0.053 \text{ fJ} = 53 \text{ aJ}$$