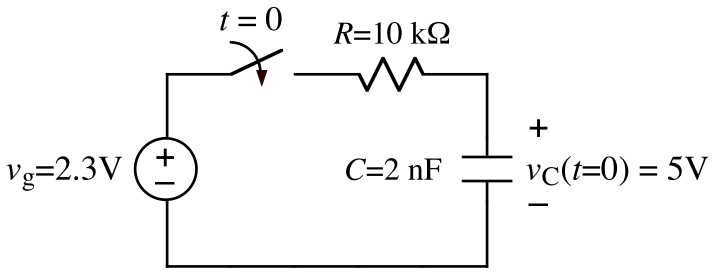
HOMEWORK #10 solution

Ex: After being open for a long time, the switch closes at $t = 0$.



a) Find an expression for $v_C(t)$ for $t \geq 0$.
b) Find the energy stored in the capacitor at time $t = 30$ μs.

Sol'n: a) The following general form of solution applies to any RC circuit with a single capacitor:

$$v_C(t \geq 0) = v_C(t \to \infty) + [v_C(t = 0^+) - v_C(t \to \infty)]e^{-t/R_{Th}C}$$

The Thevenin resistance, R_{Th}, is for the circuit after $t = 0$ (with the C removed) as seen from the terminals where the C is connected. In the present case, we have $R_{Th} = 10$ kΩ.

$$R_{Th}C = 10\text{ k}\Omega \cdot 2\text{ nF} = 20\ \mu\text{s}$$

The value of $v_C(t=0)$ is given in the problem as 5 V. Note that the C could have any voltage before $t = 0$ in this circuit if the value were not specified. The voltage would stay on the ideal C indefinitely prior to $t = 0$.

As time approaches infinity, the C will charge to its final value, and current will cease to flow in the C. Thus, the C will become an open circuit. It follows that the current through the R, which is the same as the current through the C, will become zero. By Ohm's law, this in turn means that the voltage drop across the R will become zero, and the voltage across the C will be the same as the source voltage, 2.3 V.

$$v_C(t \to \infty) = 2.3\text{ V}$$

Substituting values, we have the following result:

$$v_C(t \geq 0) = 2.3\text{ V} + [5\text{ V} - 2.3\text{ V}]e^{-t/20\mu\text{s}} = 2.3\text{ V} + 2.7\text{ V} \cdot e^{-t/20\mu\text{s}}$$

b) The energy in a capacitor is given by the following formula:

$$w_C = \frac{1}{2} C v_C^2$$

We use the solution to (a) to evaluate $v_C(t)$ at $t = 30$ μs.

$$v_C(t = 30\mu s) = 2.3 \text{ V} + 2.7 \text{ V} \cdot e^{-30\mu s / 20\mu s} = 2.90 \text{ V}$$

Using this voltage, we evaluate the energy on the capacitor.

$$w_C = \frac{1}{2} 2\text{nF} \cdot (2.90\text{V})^2 = 8.42 \text{ nJ}$$