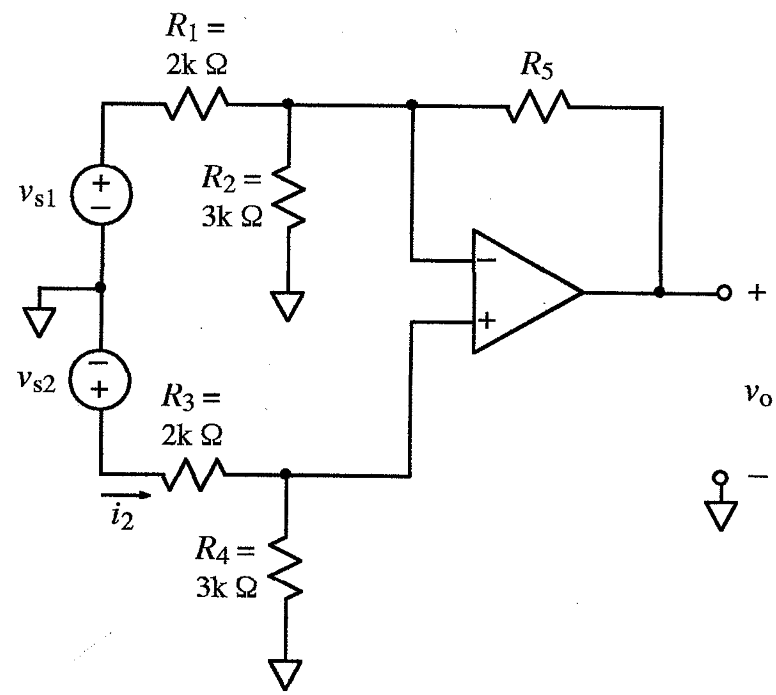
ECE 1270

HW 9 and 10

THE UNIVERSITY OF UTAH

1.



a) The above circuit operates in linear mode. Derive a symbolic expression for v_o. The expression must contain not more than the parameters v_{s1}, v_{s2}, R_1, R_2, R_3, R_4, and R_5.

2. If $v_{s1} = 0$ V and $v_{s2} = 1$ V, find the value of R_5 that will yield an output voltage of $v_o = 1$ V.

~~c) Derive a symbolic expression for v_o in terms of common mode and differential input voltages:~~

$$v_{cm} \equiv \frac{(v_{s2} + v_{s1})}{2} \quad \text{and} \quad v_{dm} \equiv v_{s2} - v_{s1}$$

~~The expression must contain not more than the parameters v_{cm}, v_{dm}, R_1, R_2, R_3, R_4, and R_5. Write the expression as v_{cm} times a term plus v_{dm} times a term. Hint: start by writing v_{s1} and v_{s2} in terms of v_{cm} and v_{dm}:~~

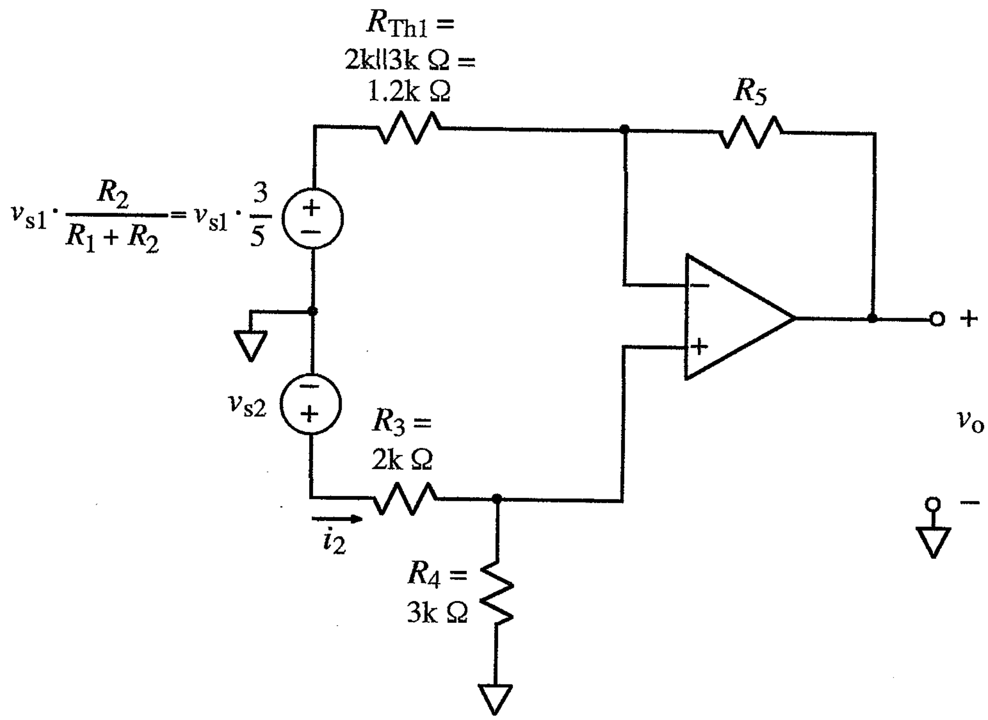
$$v_{s1} = v_{cm} - \frac{v_{dm}}{2} \quad \text{and} \quad v_{s2} = v_{cm} + \frac{v_{dm}}{2}$$

3. Find the numerical value of the circuit's input resistance, R_{in}, as seen by source v_{s2}. In other words, write a formula for voltage, v_{s2}, divided by i_2:

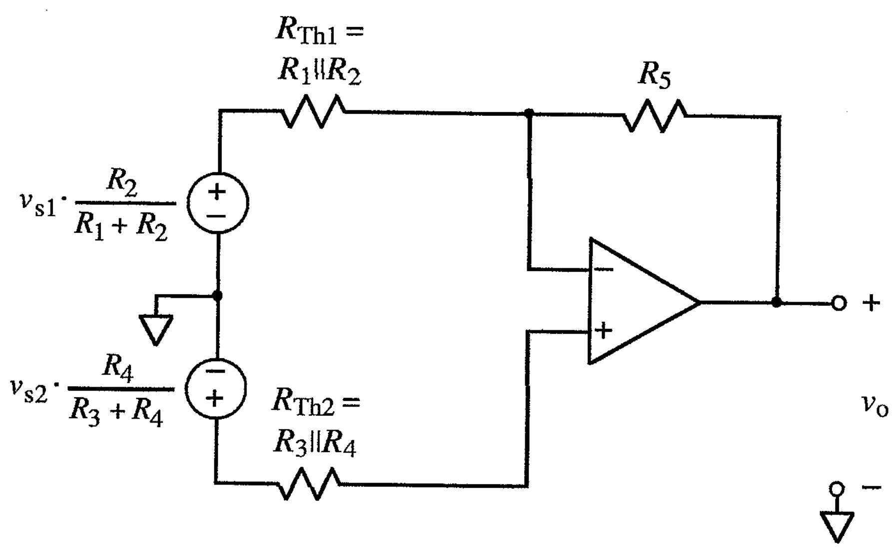
$$R_{in} \equiv \frac{v_{s2}}{i_2}$$

Write R_{in} in terms of not more (and possibly less) than R_1, R_2, R_3, R_4, and R_5.

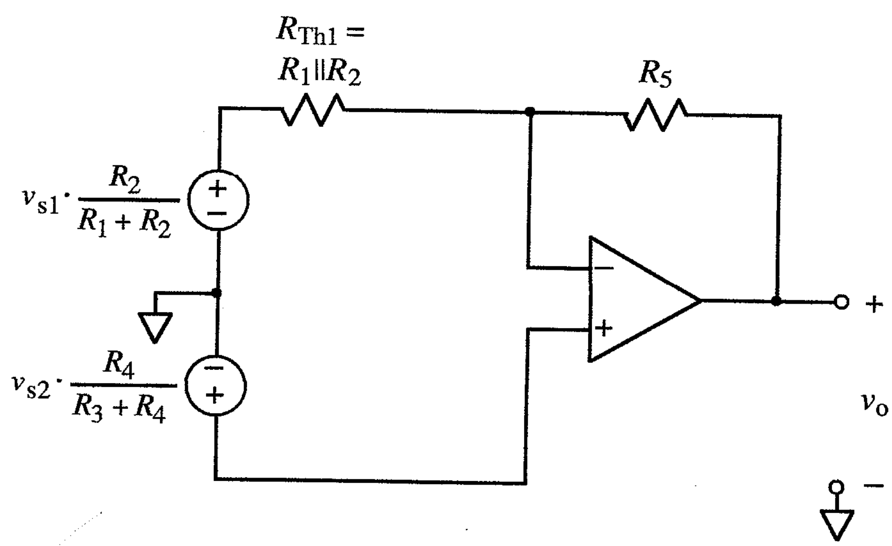
SOL'N: a) Converting the circuitry comprised of v_{s1}, R_1, and R_2 into a Thevenin equivalent yields a standard differential amplifier.



Likewise, converting the circuitry comprising v_{s2}, R_3, and R_4 into a Thevenin equivalent yields further simplification:

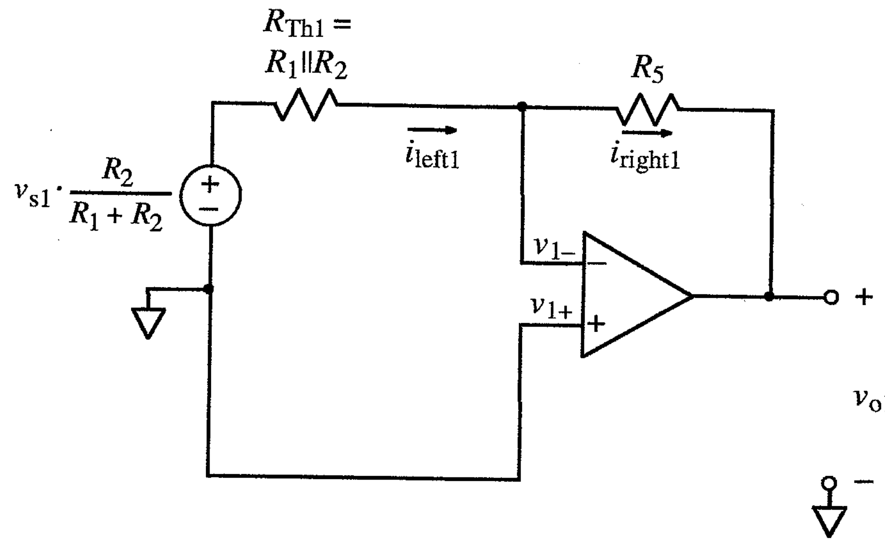


Since no current flows into the + input of the op-amp, R_{Th2} has no voltage drop and may be removed.



Now, applying superposition reveals that this circuit is a combination of a standard negative-gain circuit and positive-gain circuit.

case I: (v_{s1} on, v_{s2} off = wire)



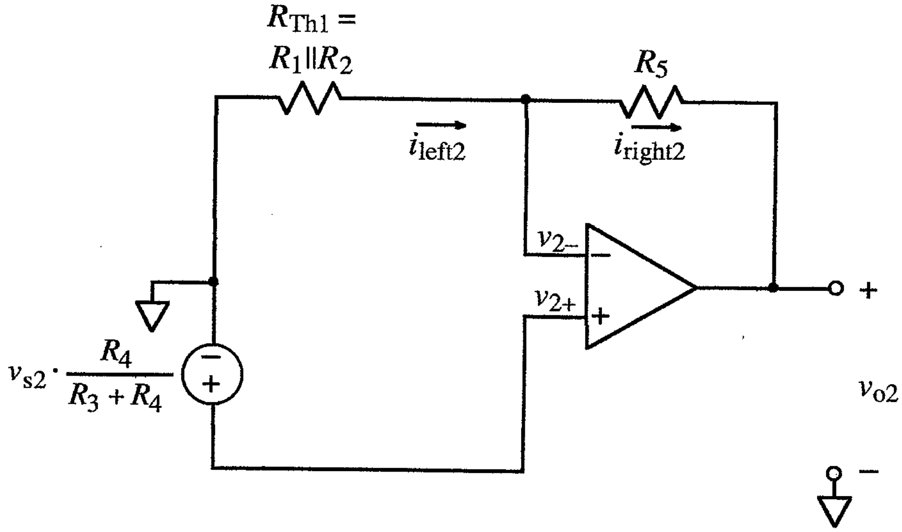
For this circuit, $v_{1-} = v_{1+} = 0$ V. (The "1" stands for "case I".) Using v_{1-} to calculate i_{left1} and i_{right1} gives the following equation:

$$i_{\text{left1}} = \frac{v_{s1} \cdot \dfrac{R_2}{R_1 + R_2}}{R_1 \,||\, R_2} = \frac{-v_{o1}}{R_5} = i_{\text{right1}}$$

or

$$v_{o1} = -v_{s1} \frac{R_5}{R_1}$$

case II: (v_{s1} off = wire, v_{s2} on)



For this circuit, $v_{2-} = v_{2+} = v_{s2} \dfrac{R_4}{R_3 + R_4}$. (The "2" stands for "case II".)

Using v_{2-} to calculate i_{left2} and i_{right2} gives the following equation:

$$i_{\text{left2}} = \frac{v_{s2} \cdot \dfrac{R_4}{R_3 + R_4}}{R_1 \,||\, R_2} = \frac{v_{s2} \cdot \dfrac{R_4}{R_3 + R_4} - v_{o2}}{R_5} = i_{\text{right2}}$$

or

$$v_{o2} = v_{s2} \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_5}{R_1 \,||\, R_2}\right)$$

Summing results gives the final result.

$$v_o = -v_{s1} \frac{R_5}{R_1} + v_{s2} \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_5}{R_1 \,||\, R_2}\right)$$

Node voltage may also be used. For the v_- node:

$$\frac{v_+}{R_4} + \frac{v_+ - v_{s2}}{R_3} = 0\,\mathrm{A}$$

For the v_+ node:

$$\frac{v_- - v_{s1}}{R_1} + \frac{v_-}{R_2} + \frac{v_- - v_o}{R_5} = 0\,\mathrm{A}$$

Setting $v_- = v_+ = 0$ V and solving for v_o yields the answer obtained earlier.

2. The conditions match the superposition case II described in (a) but with $v_{s2} = 1$ V.

$$v_o = 1\mathrm{V} = v_{s2}\frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \,||\, R_2}\right) = 1\mathrm{V}\frac{3\mathrm{k}}{2\mathrm{k} + 3\mathrm{k}}\left(1 + \frac{R_5}{2\mathrm{k}\Omega \,||\, 3\mathrm{k}\Omega}\right)$$

or

$$1\mathrm{V} = 1\mathrm{V}\frac{3}{5}\left(1 + \frac{R_5}{1.2\mathrm{k}\Omega}\right)$$

or

$$R_5 = 1.2\mathrm{k}\Omega\left(\frac{5}{3} - 1\right) = 1.2\mathrm{k}\Omega \cdot \frac{2}{3} = 0.8\mathrm{k}\Omega$$

~~c) We make the suggested substitutions for v_{s1} and v_{s2}:~~

$$v_o = -\left(v_{cm} - \frac{v_{dm}}{2}\right)\frac{R_5}{R_1} + \left(v_{cm} + \frac{v_{dm}}{2}\right)\frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \,||\, R_2}\right)$$

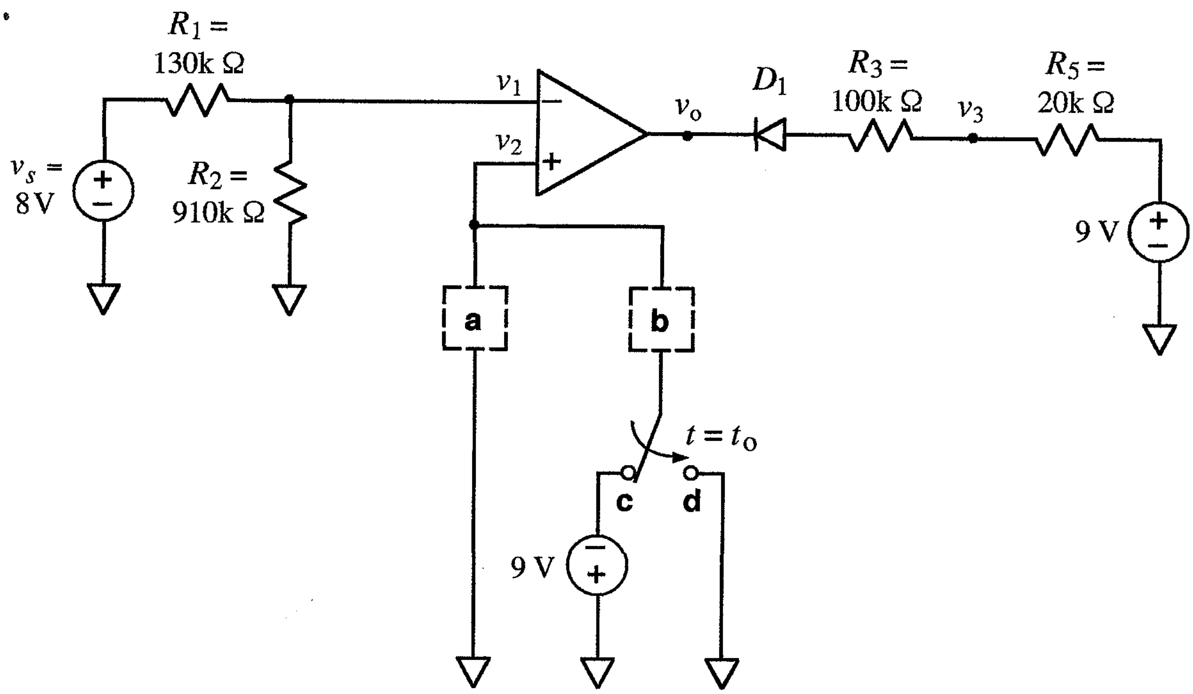
or

$$v_o = v_{cm}\left[-\frac{R_5}{R_1} + \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \,||\, R_2}\right)\right]$$
$$+ \frac{v_{dm}}{2}\left[\frac{R_5}{R_1} + \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \,||\, R_2}\right)\right]$$

3.

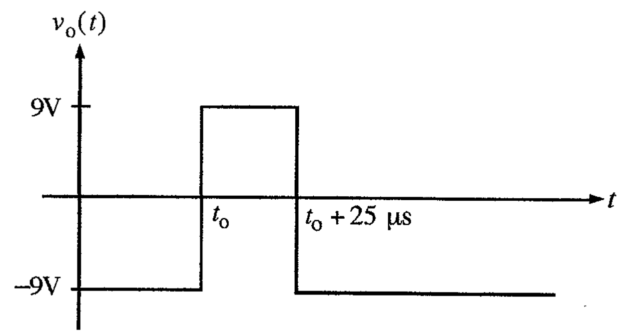
$$R_{\text{in}} = \frac{v_{s2}}{\dfrac{v_{s2}}{R_3 + R_4}} = R_3 + R_4 = 2\text{k}\,\Omega + 3\text{k}\,\Omega = 5\text{k}\,\Omega$$

4.



Rail voltages = ±9 V

After being at **c** for a long time, the switch moves to **d** at time $t = t_o$.



a) Choose either an R or C to go in box **a** and either an R or C to go in box **b** to produce the v_o(t) shown above. Use at least one R, and use 2 kΩ for the R value or values. Also, note that v_o stays low forever after t_o + 25 μs. Specify which element goes in each box and its value.

5. Sketch $v_1(t)$, showing numerical values appropriately.

6. Sketch $v_2(t)$, showing numerical values appropriately.

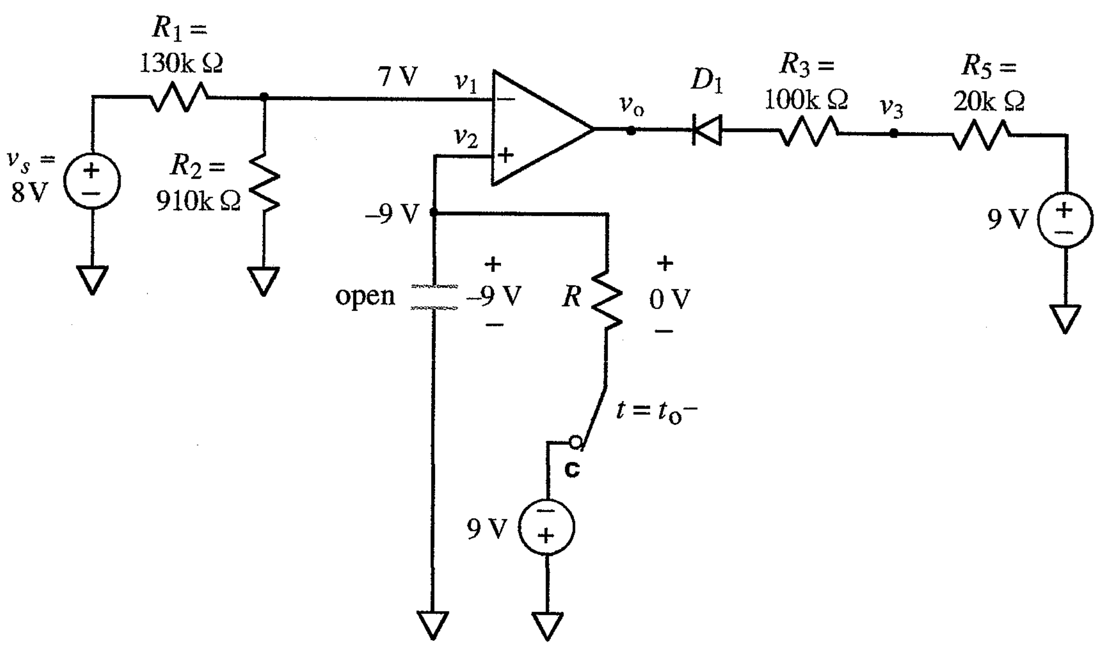
7. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 25$ μs, and for $t > t_o + 25$ μs. Use the ideal model of the diode: when forward biased, its resistance is zero, (a wire); when reverse biased, its resistance is infinite, (an open).

SOL'N: a) Since no current flows into the op-amp, we have a voltage-divider on the left side that produces a voltage of 7 V at the – input. This voltage remains constant for all time.

$$v_{-} = v_s \frac{R_2}{R_1 + R_2} = 8\,\text{V} \frac{910\,\text{k}\Omega}{130\,\text{k}\Omega + 910\,\text{k}\Omega} = 7\,\text{V}$$

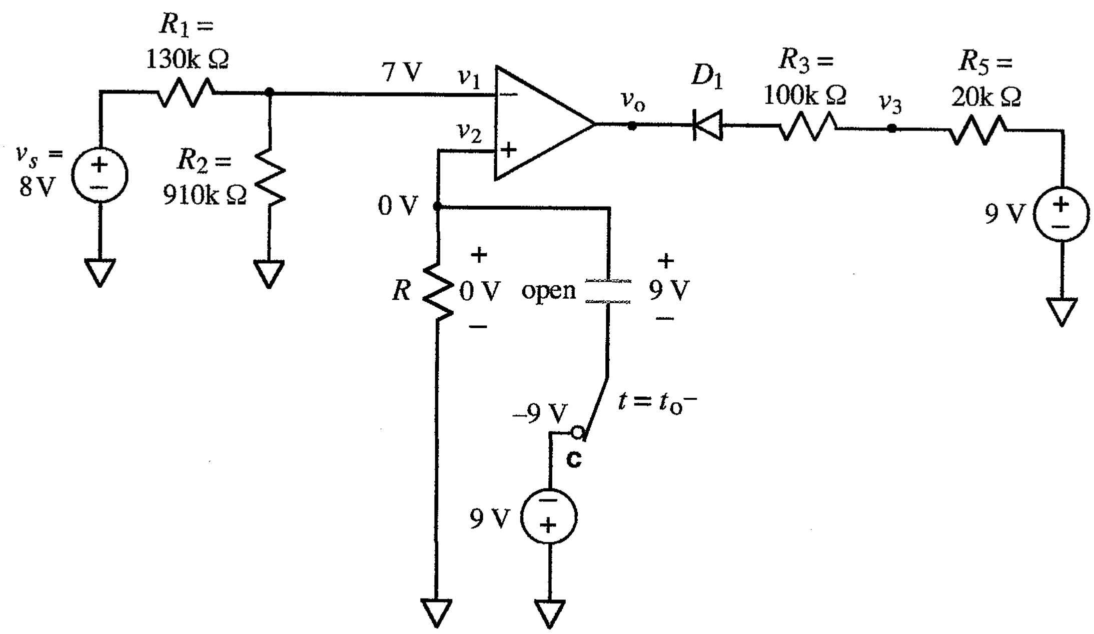
To understand which box should contain C and which box should contain R, trying each possibility reveals the appropriate solution. At t_o the waveform given for v_o dictates that $v_2 < v_1$, as v_o is negative (equal to $-v_{\text{rail}}$) when $v_2 < v_1$. In other words, the polarity of the v_o output matches the polarity of the voltage measured across the inputs on the op-amp.

If box **a** contains C, then the initial voltage on the C and the voltage v_2 is found by treating C as an open circuit. As shown below, the circuit gives the desired result that $v_o = -v_{\text{rail}}$ since $v_2 = -9\text{ V} < v_1 = 7\text{ V}$.

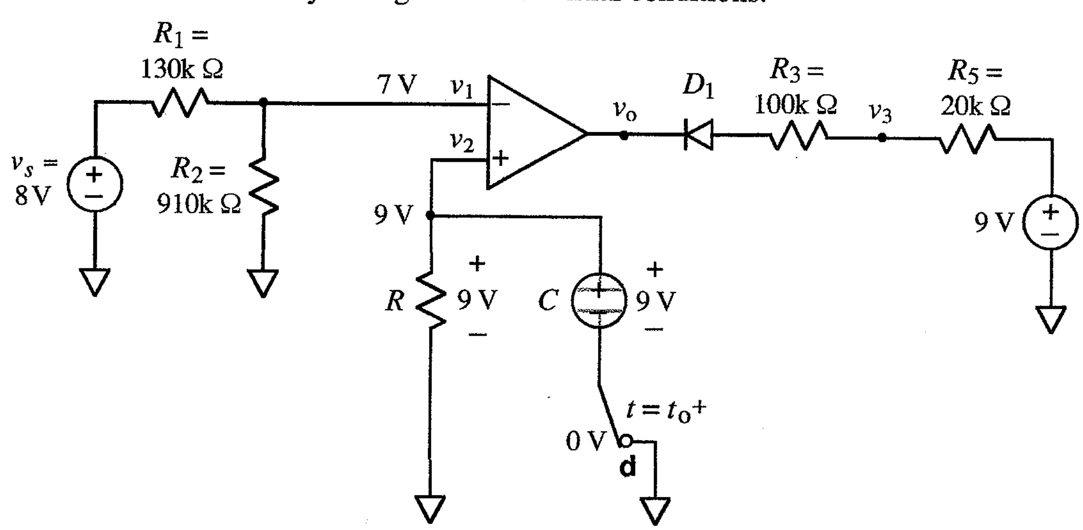


At time $t = 0^+$, however, the voltage across the capacitor and voltage v_2 remain the same, namely –9 V. Thus, v_o would not immediately change to $+v_{\text{rail}}$ as required in the statement of the problem, and this is an incorrect configuration.

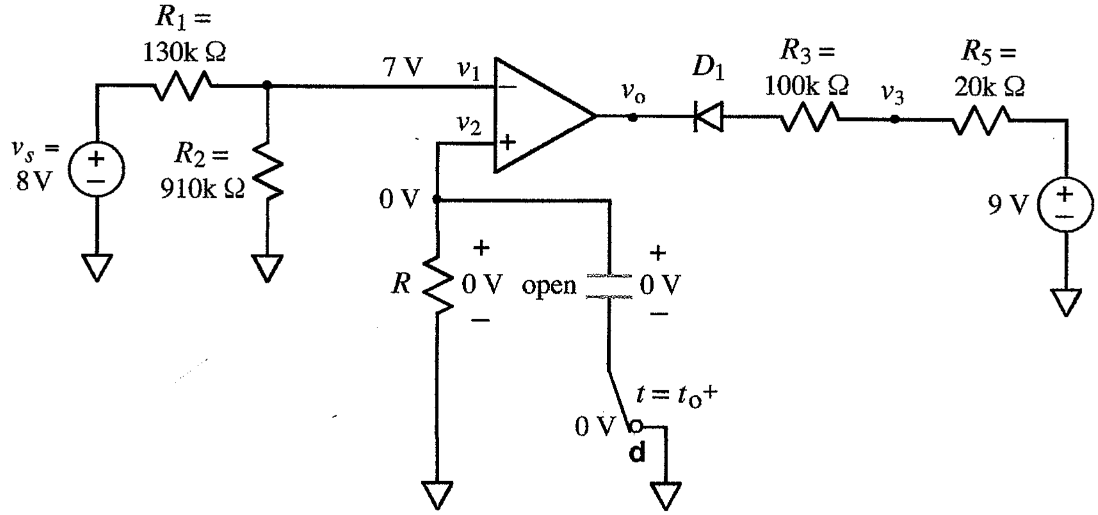
If box **b** contains C, then the initial voltage on the C and the voltage v_2 is again found by treating C as an open circuit. As shown below, this circuit also gives the desired result that $v_o = -v_{rail}$ since $v_2 = 0$ V $< v_1 = 7$ V.



At time $t = 0^+$, the voltage across the capacitor remains the same, namely 9 V. As shown below, v_2 will jump up to 9 V. This means $v_2 = 9$ V $> v_1 = 7$ V and $v_o = -v_{rail}$, as desired in the problem. This is a satisfactory configuration for initial conditions.



The next step is to examine the final conditions in the circuit, to determine whether $v_2 < v_1 = 7$ V as time approaches infinity so that v_o will be negative as time approaches infinity. As shown in the diagram below, v_2 will fall to 0 V, thus yielding the appropriate circuit behavior. Thus, this circuit can satisfy the problem requirements.



box $\boxed{a = R}$ box $\boxed{b = C}$

The output of the op-amp will switch from high to low when the op-amp input voltages are equal: $v_2 = v_1 = 7$ V. The value of C is chosen to cause switching at time $t_o + 25$ μs using the formula $v_2(t)$.

The general form of RC solution describes $v_2(t)$ for $t > 0$.

$$v_2(t > 0) = v_2(t \to \infty) + [v_2(0^+) - v_2(t \to \infty)]e^{-t/R_{Th}C}$$

Above, it was shown that $v_2(0^+) = 9\text{V}$ and $v_2(t \to \infty) = 0\text{V}$.

$$v_2(t > 0) = 9e^{-t/2\text{k}\Omega\cdot C}\ \text{V}$$

Since no current flows into the op-amp, $R_{Th} = R = 2$ kΩ. Without loss of generality, we assume $t_o = 0$. Thus, switching occurs at time $t = 25$ μs when $v_2 = v_1 = 7$ V.

$$v_2(25\mu\text{s}) = 7\text{V} = 9Ve^{-25\mu\text{s}/2\text{k}\Omega\cdot C}$$

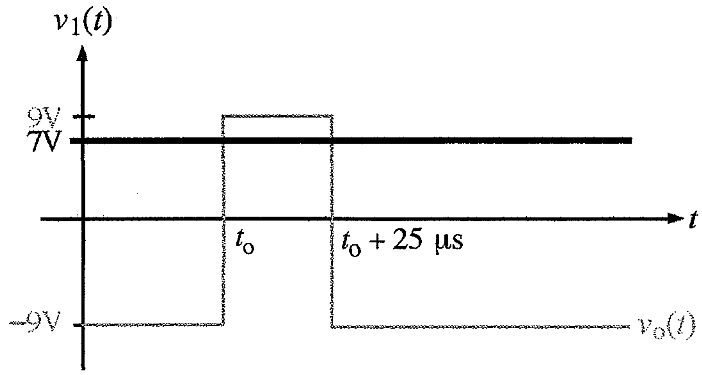
or

$$\ln\frac{7\text{V}}{9\text{V}} = -t/2\text{k}\Omega\cdot C$$

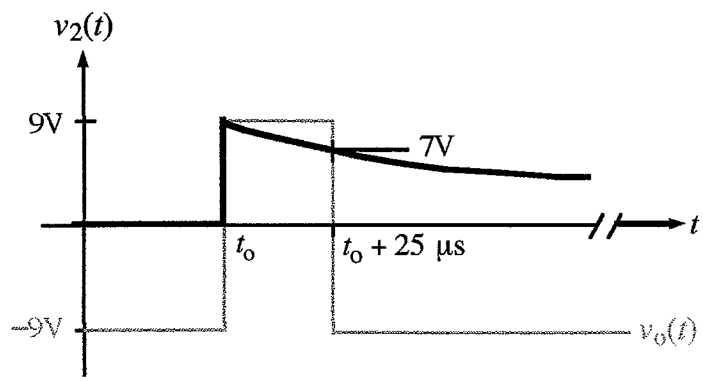
or

$$C = \frac{-25\mu\text{s}/2\text{k}\Omega}{\ln\frac{7\text{V}}{9\text{V}}} \approx 50\text{nF}$$

5. As shown in part (a), $v_1(t)$ is constant and equal to 7 V.



6. As shown in part (a), $v_2(t > t_o) = 9e^{-t/2\text{k}\Omega\cdot C}$ V and $v_2(t < t_o) = 0\text{V}$.



7. When v_o is low, the current will try to flow from right (where the +9 V supply is located) to the left (where v_o = –9 V). The diode is forward biased in this case and allows the current to flow. (The diode allows current to flow in the direction of the "arrow" in the diode symbol.) The diode acts like a wire and R_3 and R_5 form a voltage-divider. There are

several ways to solve for v_3: superposition, voltage-divider formula to find voltage across R_5 that is subtracted from 9 V, source transformations to Norton equivalents followed by combining current sources and parallel resistances, or node-voltage. Here, the latter is used:

$$\frac{v_3 - v_o}{R_3} + \frac{v_3 - 9\text{V}}{R_5} = 0\,\text{A}$$

or

$$\frac{v_3 - -9\text{V}}{100\text{k}\Omega} + \frac{v_3 - 9\text{V}}{20\text{k}\Omega} = 0\,\text{A}$$

or

$$v_3\left(\frac{1}{100\text{k}\Omega} + \frac{1}{20\text{k}\Omega}\right) = \frac{-9\text{V}}{100\text{k}\Omega} + \frac{9\text{V}}{20\text{k}\Omega}$$

or

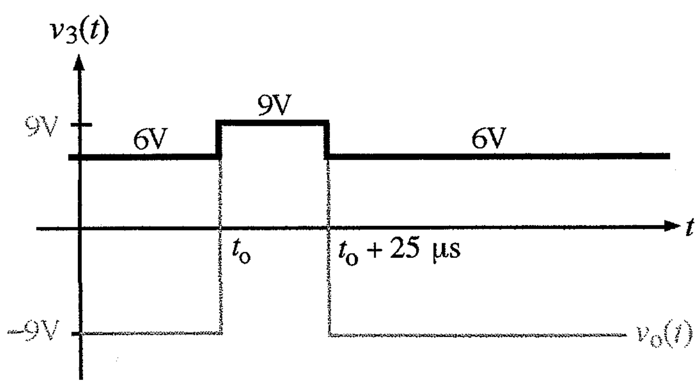
$$v_3(1+5) = -9\text{V} + 9\text{V}(5) = 36\text{V}$$

or

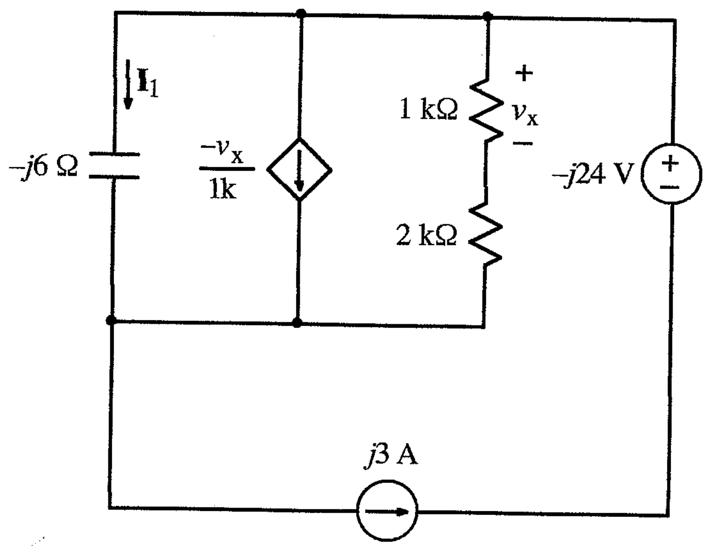
$$v_3 = 6\text{V}$$

When v_o is high (+9 V), the circuitry connected to the output of the op-amp has 9V on both ends, and there is no net voltage drop to drive current through the diode, R_3, or R_5. Thus, $v_3 = 9$ V. Note that the diode may be modeled as either a wire or an open circuit and the same result will be obtained.

The complete graph of $v_3(t)$ is shown below.



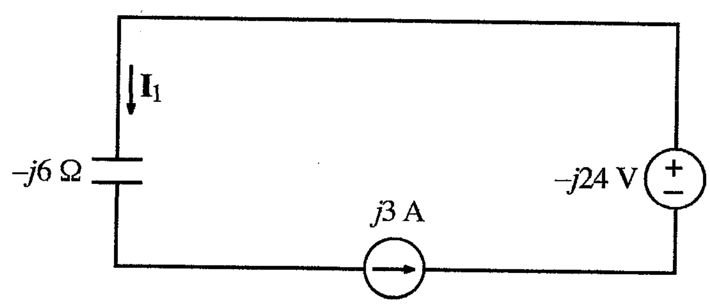
8.



a) A frequency-domain circuit is shown above. Write the value of phasor current $\mathbf{I}_1$ in rectangular form.

9. Given ω = 25k rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

SOL'N: a) The current in the 1 kΩ resistor (and the 2 kΩ resistor) is v_x/1 kΩ, which is exactly the opposite of the dependent source current. Since current sources in parallel sum, the dependent source cancels the effect of the 1 kΩ and 2 kΩ resistors in series. This cancellation leaves the circuit shown below.



Since $\mathbf{I}_1$ is in series with the $j3$ A current source, $\mathbf{I}_1 = j3$ A, which is in rectangular form.

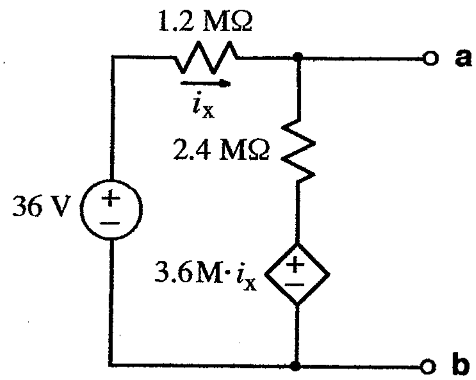
9. The j is equivalent to a 90° phase shift, and $\omega = 25\text{k}$ rad/s is given.

$$i_1(t) = 3\cos(25\text{k}t + 90°)\text{ A}$$

An equivalent expression employs a sine function:

$$i_1(t) = -3\sin(25\text{k}t)\text{ A}$$

10.

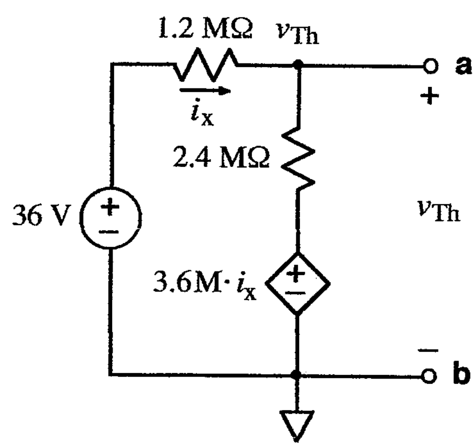


a) Find the Thevenin equivalent of the above circuit relative to terminals **a** and **b**.

b) If we attach R_L to terminals **a** and **b**, find the value of R_L that will absorb maximum power.

c) Calculate the value of that maximum power absorbed by R_L.

SOL'N: a) Node-voltage is one method for finding v_{Th}, which equals the voltage across **a** and **b** with no components connected across **a** and **b**.



First, we define the dependent variable, i_x, in terms on node voltage v_{Th}.

$$i_x = \frac{36\text{V} - v_{Th}}{1.2\text{M}\Omega}$$

Second, we sum the currents out the v_{Th} node.

$$\frac{v_{Th} - 36\text{V}}{1.2\text{M}\Omega} + \frac{v_{Th} - 3.6\text{M}\left(\dfrac{36\text{V} - v_{Th}}{1.2\text{M}\Omega}\right)}{2.4\text{M}\Omega} = 0\text{A}$$

or, after multiplying both sides by 2.4 MΩ,

$$2(v_{Th} - 36\text{V}) + v_{Th} - 3(36\text{V} - v_{Th}) = 0\text{A}$$

or

$$6v_{Th} = 72\text{V} + 3(36)\text{V} = 180\text{V}$$

or

$$v_{Th} = 30\text{V}$$

An alternative approach is to observe that i_x flows through the dependent source, making it possible to define the equivalent resistance for the dependent source:

$$R_{eq} = \frac{v}{i} = \frac{3.6\text{M}i_x}{i_x} = 3.6\text{M}\Omega$$

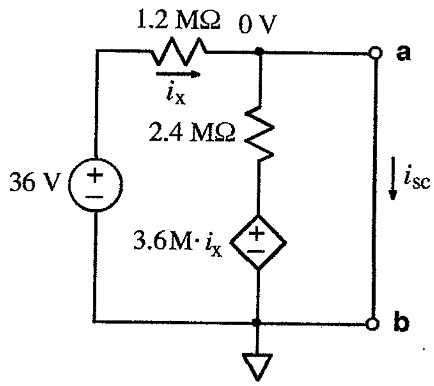
Replacing the dependent source with this resistance yields a simple voltage-divider circuit.

$$v_{Th} = 36\text{V}\frac{2.4\text{M}\Omega + 3.6\text{M}\Omega}{1.2\text{M}\Omega + 2.4\text{M}\Omega + 3.6\text{M}\Omega} = 36\text{V}\left(\frac{5}{6}\right) = 30\text{V}$$

To find R_{Th} there are two possible approaches: finding the short-circuit current from **a** to **b**, or turning off the 36 V source and connecting a 1 V source to **a** and **b**.

Using the short-circuit method, the current i_x is determined by the outer voltage loop:

$$i_x = \frac{36\text{V}}{1.2\text{M}\Omega} = 30\mu\text{A}$$



The current contributed to i_{sc} by the dependent source is given by the voltage loop on the right:

$$i_{dep} = \frac{3.6\text{M}i_x}{2.4\text{M}\Omega} = \frac{3}{2}i_x = 45\mu\text{A}$$

Summing the currents, we have i_{sc}:

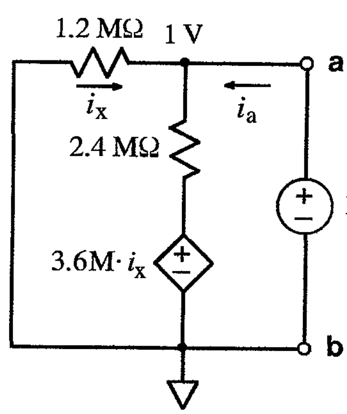
$$i_{sc} = 75\mu\text{A}$$

R_{Th} is the ratio of v_{Th} to i_{sc}:

$$R_{Th} = \frac{30\text{V}}{75\mu\text{A}} = 0.4\text{M}\Omega = 400\text{k}\Omega$$

Using the external-source method, current i_x is given by the outer voltage loop:

$$i_x = -\frac{1\text{V}}{1.2\text{M}\Omega} = -\frac{5}{6}\mu\text{A}$$



The current contributed to i_a by the middle branch is also given by a voltage-loop equation:

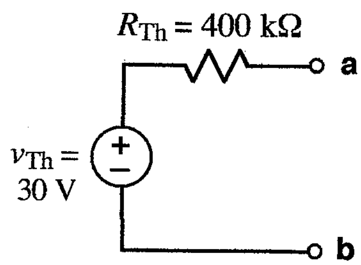
$$i_{\text{middle}} = \frac{1\text{V} - 3.6\text{M}i_x}{2.4\text{M}\Omega} = \frac{1\text{V} - 3.6\text{M}\left(-\frac{5}{6}\mu\text{A}\right)}{2.4\text{M}\Omega} = \frac{1\text{V} + 3\text{V}}{2.4\text{M}\Omega} = \frac{5}{3}\mu\text{A}$$

A sum of currents gives the value of i_a:

$$i_a = -i_x + i_{\text{middle}} = \frac{5}{6}\mu\text{A} + \frac{5}{3}\mu\text{A} = \frac{5}{2}\mu\text{A}$$

R_{Th} is the ratio of 1 V to i_a:

$$R_{\text{Th}} = \frac{1\text{V}}{i_a} = \frac{1\text{V}}{\frac{5}{2}\mu\text{A}} = 0.4\text{M}\Omega = 400\text{k}\Omega$$



b) For maximum power transfer, $R_L = R_{\text{Th}} = 400$ kΩ.

c) The maximum power transferred is

$$p_{\max} = \frac{v_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(30\text{V})^2}{4(400\text{k}\Omega)} = \frac{3^2}{16}\text{mW} = \frac{9}{16}\text{mW} = 0.5625\text{ mW}.$$