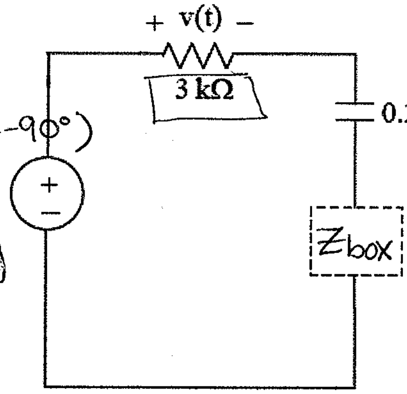
1.

$V_g(t) = 6\cos(1kt - 90°)$

$v_g(t) = 6\sin(1kt)\text{V}$

$\boxed{V_g = 6e^{-j90°}}$



$\omega = 1K$

$\Rightarrow \frac{-j}{\omega C} = \frac{-j}{1K(.2\mu)} = \boxed{-5{,}000j}$

Choose an R, an L, or a C to be placed in the dashed-line box to make $v(t) = V_0\cos(1kt - 45°)$ where I_0 is a positive, (i.e., nonzero), real constant. State the value of the component you choose.

2. With your component from problem 1 in the circuit, calculate the resulting value of I_0.

① Note what the final angle needs to be.

$\angle V = -45°$

② Change circuit to frequency domain values & label box Z_{box}.

③ Write an equation for known variable (v(t) in this case).

- Use ohm's laws and Kirchoff's laws or node-voltage, current mesh techniques. (Also check for current or voltage divider)
- $$V = \frac{V_g(3k)}{3k - 5kj + Z_{box}}$$

④ Replace eq. with angles and plug in known value for V.

$$\angle V = \frac{\angle V_g(3k)}{\angle(3k - 5kj + Z_{box})} \Rightarrow \angle V = \angle V_g(3k) - \angle(3k - 5kj + Z_{box})$$

$$\angle -45° = \angle -90° - \angle(3k - 5kj + Z_{box})$$

$$\angle\underbrace{(3k - 5kj + Z_{box})}_{} = \angle -90° - \angle -45° = \angle -45°$$

To get this to be −45° which means $\boxed{\frac{Im}{Real} = -1}$

1. (cont.)

If $Z_{box} = R$ then $\frac{-5k}{3k+R} = -1$ and solving for R:

$$-5k = -3k - R$$

$$R = -3k + 5k = \boxed{2k\Omega}$$

Checking:

$$V = \frac{(6e^{-j90^\circ})(3k)}{3k - 5kj + 2k} = \frac{18ke^{-j90^\circ}}{5k - 5kj} = \frac{18ke^{-j90^\circ}}{\sqrt{5k^2+5k^2}\, e^{j\tan^{-1}\left(\frac{-5k}{5k}\right)}}$$

$$V = \frac{18k}{5k\sqrt{2}} \cdot e^{-j90^\circ + 45^\circ} = \frac{18}{5\sqrt{2}} e^{-j45^\circ} \checkmark \text{ correct form}$$

$$\therefore \boxed{V_o = \frac{18}{5\sqrt{2}}}$$

If C, then $Z_{box} = \frac{-j}{1kC}$ and $\frac{\left(-5k - \frac{1}{1kC}\right)}{3k} = -1$

$$-5k - \frac{1}{1kC} = -3k$$

$$-5k + 3k = \frac{1}{1kC}$$

$$-2k(1kC) = 1 \implies C = -\frac{1}{2k(1k)} \times$$ Not possible! negative C?

If L, then $Z_{box} = j(1k)L$ and $\frac{-5k + 1kL}{3k} = -1$

$$-5k + 1kL = -3k$$

$$+2k = +1kL$$

$$\therefore \boxed{L = 2H}$$

checking: $V = \frac{18ke^{-j90^\circ}}{3k - 5kj + 2kj}$

$$V = \frac{18ke^{-j90^\circ}}{\sqrt{3k^2+3k^2}\, e^{j\tan^{-1}\left(\frac{-3k}{3k}\right)}} = \frac{18\cancel{k}}{3\cancel{k}\sqrt{2}} e^{-j90^\circ + 45^\circ} = \boxed{\frac{6}{\sqrt{2}}} e^{-j45^\circ}$$

$$\underset{V_o}{\shortparallel}$$

2. Give numerical answers to each of the following questions:

a. Rationalize $\frac{-30k\cdot(j1k)}{30k+j1k}$. Express your answer in rectangular form.

b. Find the polar form of $\left(e^{j45^\circ}\right)^*\left(\sqrt{1+\frac{5}{4}}-j\sqrt{1-\frac{5}{4}}\right)^*$ (Note: The asterisk means conjugate.)

c. Find the following phasor: $\mathrm{P}\left[8\sin(3kt+115^\circ)\right]$.

d. Find the magnitude of $\frac{(1-4j)2e^{-j50^\circ}}{2+2e^{j90^\circ}}$.

e. Find the imaginary part of $\frac{1-5j}{e^{-j60^\circ}}$.

a. $\frac{-30k(1k)j(30k-j1k)}{(30k+j1k)(30k-j1k)} = \frac{(1k)(1k)[-30(30k)j+30kj^2]}{(1k)(1k)(30^2+1^2)} = \frac{-900kj-30k}{901}$

$\cong \boxed{-33-999j}$

b. $(e^{-j45^\circ})(\sqrt{1+\frac{5}{4}}+j\sqrt{1-\frac{5}{4}}) = (e^{-j45^\circ})(1.5+j(0.5j)) = e^{-j45^\circ}(1.5-0.5) = \boxed{e^{-j45^\circ}}$

c. $8\sin(3kt+115^\circ) = 8\cos(3kt+115^\circ-90^\circ) = \boxed{8e^{j25^\circ}}$

d. $\frac{|(1-4j)||2e^{j-50}|}{|2+\underbrace{2e^{j90}}_{2j}|} = \frac{[\sqrt{1^2+(-4)^2}][2]}{\sqrt{2^2+2^2}} = \boxed{\frac{2\sqrt{17}}{\sqrt{8}}}$

e. $\mathrm{Im}\left[\frac{1-5j}{e^{-j60^\circ}}\right] = \frac{\sqrt{1^2+(-5^2)}e^{j\tan^{-1}(\frac{-5}{1})}}{e^{-j60^\circ}} \cong \sqrt{26}e^{-j79^\circ+60^\circ} = \sqrt{26}e^{-j19^\circ}$

$\mathrm{Im}: \sqrt{26}\sin(-19^\circ) \cong \boxed{-1.7}$

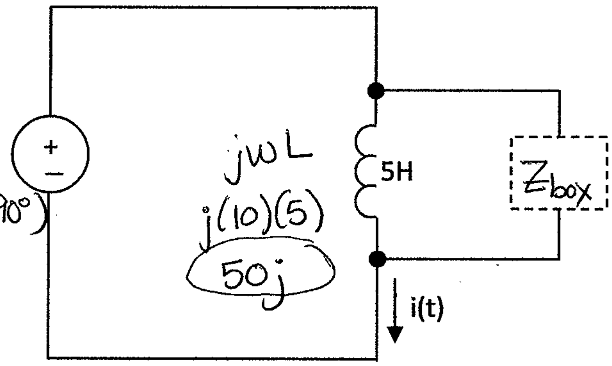
3.

$\omega = 10$

$v_g(t) = 2\sin(10t+180^\circ)\text{V}$

$V_g(t) = 2\cos(10t+180-90^\circ)$

$V_g = 2e^{j90^\circ}$



Choose an R, an L, or a C to be placed in the dashed-line box to make i(t) = $I_o\cos(10t+45^\circ)$A where I_o is a real constant. State the value of the component you choose. $I = I_o e^{j45^\circ}$

4. **With your component from problem 3 in the circuit, calculate the resulting value of I_o.**

$$I = \frac{V}{Z_{eq}}$$

$$Z_{eq} = 50j \parallel Z_{box}$$

$$I = \frac{2e^{j90^\circ}}{\frac{50j(Z_{box})}{50j + Z_{box}}} = \frac{2e^{j90^\circ}(50j + Z_{box})}{50j\, Z_{box}}$$

(Try to remove as many Zbox as possible):

$$I = \frac{\cancel{Z_{box}}\,(2\cancel{e^{j90^\circ}})\left(\frac{50j}{Z_{box}} + 1\right)}{50\cancel{j}\,\cancel{Z_{box}}} = \frac{1}{25}\left(\frac{50j}{Z_{box}} + 1\right)$$

$\cancel{j} \to e^{j90^\circ}$

In order to get $\angle 45^\circ$, then Im. = Real

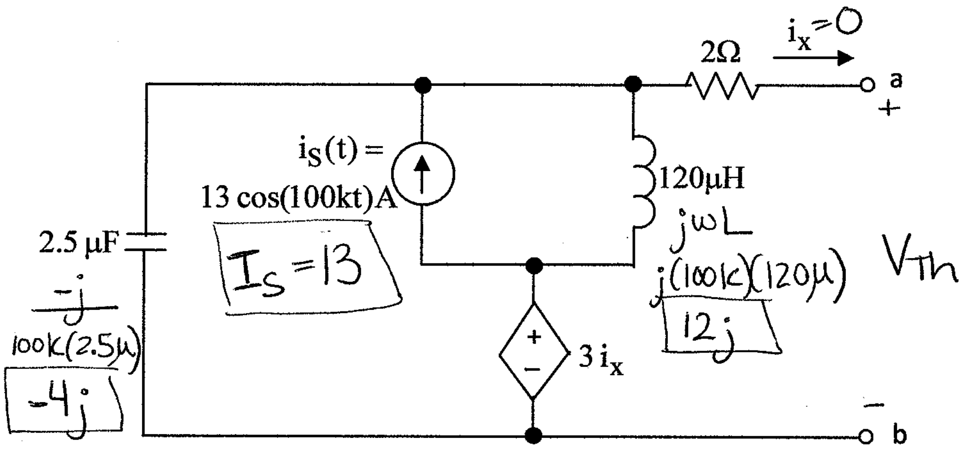
$\therefore\ Z_{box} = \boxed{R = 50\,\Omega}$ {Note: if $C \Rightarrow \frac{-j}{10C} \Rightarrow \frac{1}{25}\left(\frac{50\cancel{j}\omega C}{-\cancel{j}} + 1\right)$} No j

(If $L \Rightarrow j\omega L \Rightarrow \frac{1}{25}\left(\frac{50\cancel{j}}{\cancel{j}\omega L} + 1\right)$ again no j)

If $R = 50$ then $\frac{1}{25}(j+1) = \frac{1}{25}\sqrt{2}\,e^{j45^\circ}$ and

$$I_o\cancel{e^{j45^\circ}} = \frac{1}{25}\sqrt{2}\,\cancel{e^{j45^\circ}} \Rightarrow \boxed{I_o = \frac{\sqrt{2}}{25}}$$

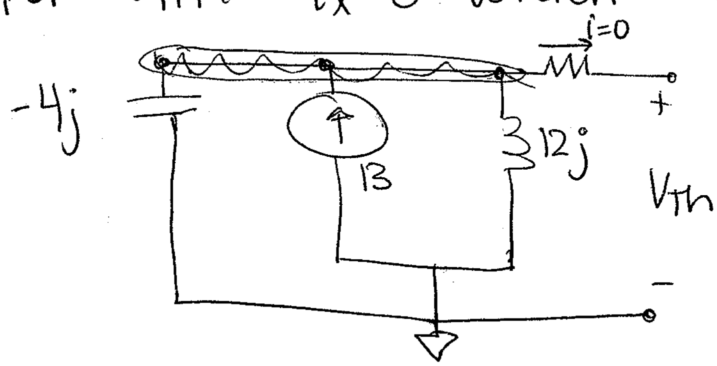
5.



Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

6. Find the Thevenin equivalent (in the frequency domain) for the circuit from Problem 6. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

① For V_{Th}: $i_x = 0$ which shorts the $3i_x$ source $= 0$



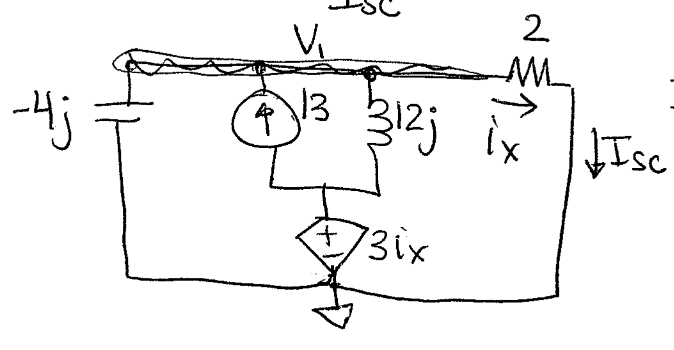
Using node-voltage:

$$\frac{-V_{Th}(3)}{+4j(3)} - 13 + \frac{V_{Th}}{12j} = 0$$

$$\frac{-3V_{Th} + V_{Th}}{12j} = 13$$

$$V_{Th}(\cancel{-2}) = \frac{13(12j)}{-2} \Rightarrow \boxed{V_{Th} = -78j}$$

② Find $Z_{Th} = \frac{V_{Th}}{I_{sc}}$ (Redraw circuit)



$$I_{sc} = i_x = \frac{V_1}{2}$$

$$\frac{V_1}{-4j} - 13 + \frac{(V_1 - 3i_x)}{12j} + \frac{V_1}{2} = 0$$

$$V_1\left(\frac{-3(2)}{(2)4(3)j} + \frac{1(2)}{12j(2)} - \frac{3}{2(12j)} + \frac{12j}{2(12j)}\right) = 13$$

5. (cont.)

$$V_1\left(\frac{-6+2-3+12j}{24j}\right)=13$$

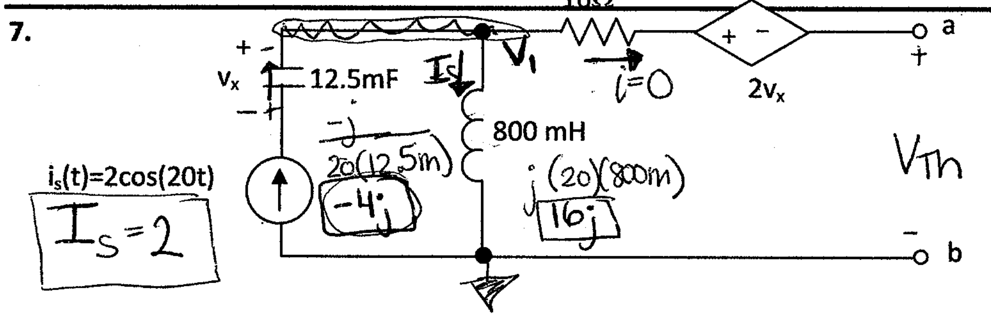
$$V_1=\frac{13(24j)}{-7+12j}$$

$$I_{SC}=\frac{V_1}{2}=\frac{312j}{2(-7+12j)}\cdot\frac{(-7-12j)}{(-7-12j)}=\frac{312j(-7)-312j(12j)}{2(7^2+12^2)}=\frac{-2184j+3744}{386}$$

$$Z_{Th}=\frac{V_{Th}}{I_{SC}}=\frac{-78j(386)}{3744-2184j}=\frac{30,108j(3744+2184j)}{3744^2+2184^2}=1.6mj(3744+2184j)$$

$$Z_{Th}\approx 6j+3.5j^2\approx -3.5+6j$$

7.



Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

8. Find the Thevenin equivalent (in the frequency domain) for the circuit from Problem 8. Give the numerical phasor value for $\mathbf{V_{Th}}$ and the numerical impedance value of z_{Th}.

① Find V_{Th}

Taking a voltage loop: $+V_1 - 2V_x - V_{Th} = 0$

$\therefore V_{Th} = V_1 - 2V_x$

where $V_x = -2(-4j) = +8j$

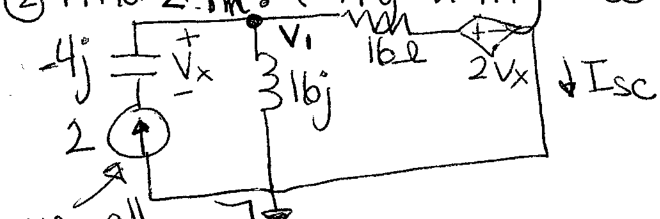
Using the fact that at V_1, the current is zero towards V_{Th}

so $+2(16j) - 2V_x - V_{Th} = 0$

$2(16j) - 2(8j) = V_{Th}$

$\therefore V_{Th} = 32j - 16j = \boxed{16j}$

② Find Z_{Th}: (Try using I_{sc}-short circuit current) $\Rightarrow Z_{Th} = \frac{V_{Th}}{I_{sc}}$



Leave all sources for this method

$$-2 + \frac{V_1}{16j} + \frac{V_1 - 2V_x}{16} = 0$$

$$V_1\left(\frac{1}{16j} + \frac{j}{16j}\right) = 2 + \frac{2(8j)}{16}$$

$$V_1 = \left(\frac{32 + 16j}{16}\right)\frac{16j}{(1+j)} = \frac{(32j - 16)(1-j)}{(1+j)(1-j)}$$

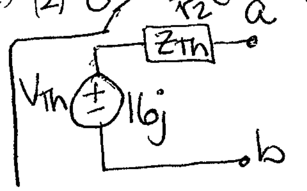
$$V_1 = \frac{32j - 32j^2 - 16 + 16j}{1^2 + 1^2} = \frac{48j + 16}{2}$$

$$I_{sc} = \frac{V_1 - 2V_x}{16} = \frac{(48j + 16)}{2(16)} - \frac{2(8j)}{16}$$

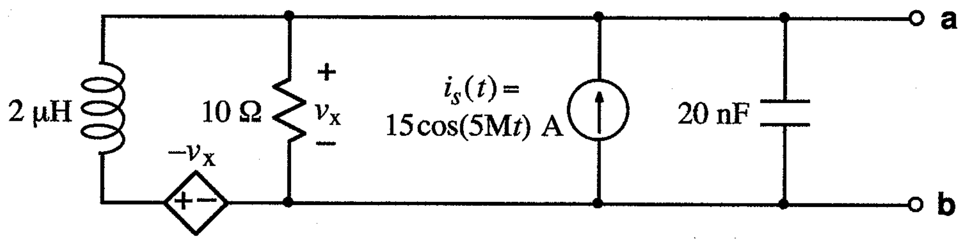
$$I_{sc} = \frac{3}{2}j + \frac{1}{2} - \frac{2}{2}j = \frac{1}{2}j + \frac{1}{2}$$

$$Z_{Th} = \frac{16j}{\frac{1}{2}j + \frac{1}{2}} = \frac{16e^{j90°}}{\sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2}\, e^{j45°}} \quad \frac{32}{\sqrt{2}}e^{j45°}$$

$$\boxed{Z_{Th} = \frac{32}{\sqrt{2}}e^{j45°}}$$



9.



a) Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_S(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b) Find the Thevenin equivalent (in the frequency domain) for the above circuit relative to terminals **a** and **b**. Give the numerical phasor value for $\mathbf{V}_{Th}$ and the numerical impedance value of z_{Th}.

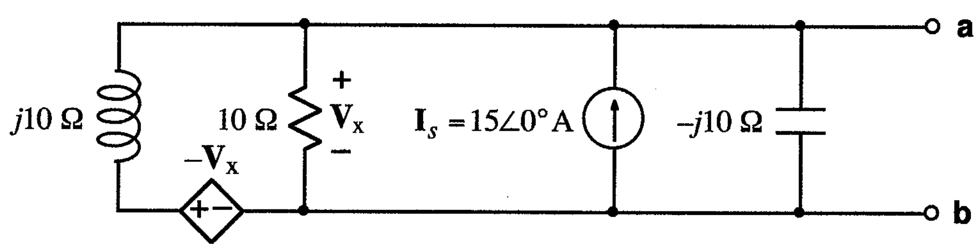
SOL'N: a) The frequency-domain has phasor values for the voltages and currents and impedances for R's, L's, and C's. For the value of ω, we look at the independent source and find that $\omega = 5$ M r/s.

$$\mathbf{I}_s = 15\angle 0° \text{ A}$$

$$j\omega L = j(5\text{M})2\mu\Omega = j10\Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{5\text{M}\cdot 20\text{n}}\Omega = -j10\Omega$$

Frequency-domain diagram:



b) We find the Thevenin equivalent the same way we would find it if we were dealing with a DC circuit. The only difference is that the values we are dealing with are complex.

9. (cont.)

The Thevenin equivalent voltage may be found by the node-voltage method. The Thevenin voltage is the drop from **a** to **b**, and is the same as $\mathbf{V}_X$. We sum the currents out of the top node.

$$\frac{\mathbf{V}_X - -\mathbf{V}_X}{j10\Omega} + \frac{\mathbf{V}_X}{10\Omega} - \mathbf{I}_s + \frac{\mathbf{V}_X}{-j10\Omega} = 0\,\text{A}$$

or

$$\mathbf{V}_X\left(\frac{2}{j10\Omega} + \frac{1}{10\Omega} + \frac{1}{-j10\Omega}\right) = \mathbf{I}_s = 15\,\text{A}$$

or

$$\mathbf{V}_X\left(\frac{-j2}{10\Omega} + \frac{1}{10\Omega} + \frac{j}{10\Omega}\right) = 15\,\text{A}$$

or

$$\mathbf{V}_X\left(\frac{1}{10\Omega} - \frac{j}{10\Omega}\right) = 15\,\text{A}$$

or

$$\mathbf{V}_X\left(\frac{1}{10\Omega} - \frac{j}{10\Omega}\right) = 15\,\text{A}\frac{10\Omega}{1-j} = 15\frac{10}{1-j}\cdot\frac{1+j}{1+j}\text{A} = 15\frac{10(1+j)}{2}\text{V}$$

or

$$\boxed{\mathbf{V}_{Th} = \mathbf{V}_X = 75 + j75\text{ V}}$$

Another approach would be to observe that, from a voltage loop on the left side, we have voltage drop $2\mathbf{V}_X$ across the L. Thus, we may calculate the current through the L and the dependent source in terms of $\mathbf{V}_X$.

$$\mathbf{I}_L = \frac{2\mathbf{V}_X}{j10\Omega}$$

We calculate an equivalent impedance for the dependent source:

$$z_{eq} = \frac{-\mathbf{V}_X}{\dfrac{2\mathbf{V}_X}{j10\Omega}} = -j5\,\Omega$$

9. (cont.)

Adding this impedance to the impedance of the L, we have an impedance on the left side of $j5\ \Omega$. We may use this value in place of the L and dependent source. This substitution is valid regardless of what is connected across the **a** and **b** terminals. Thus, we may use the equivalent impedance and find the z_{Th} by turning off the independent source and looking in from the **a** and **b** terminals. This yields the following answer:

$$z_{Th} = -j10\Omega \,\|\, 10\Omega \,\|\, j5\Omega = \frac{1}{\dfrac{1}{-j10} + \dfrac{1}{10} + \dfrac{1}{j5}}\Omega = \frac{1}{\dfrac{j}{10} + \dfrac{1}{10} + \dfrac{-2j}{10}}\Omega$$

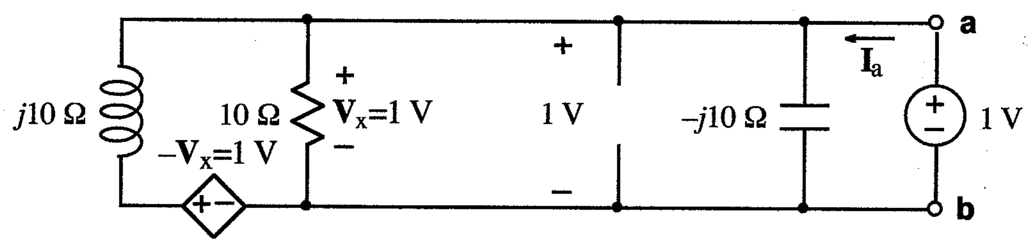
or

$$z_{Th} = \frac{10}{1-j}\Omega = \frac{10}{1-j} \cdot \frac{1+j}{1+j}\Omega = \frac{10(1+j)}{2}\Omega$$

or

$$\boxed{z_{Th} = 5 + j5\Omega}$$

We obtain the same answer if we use the dependent source, turn off the independent current source, and attach a 1 V source to the **a** and **b** terminals.



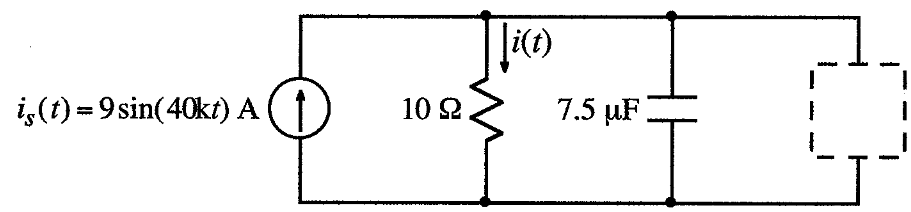
We find i_a by summing currents through the components.

$$\mathbf{I}_a = \frac{2V}{j10\Omega} + \frac{1V}{10\Omega} + \frac{1V}{-j10\Omega} = \frac{1}{10\Omega} + \frac{-j}{10\Omega}A$$

This yields the same value of z_{Th} as before.

$$z_{Th} = \frac{1V}{\mathbf{I}_a} = \frac{10}{1-j}\Omega = \frac{10}{1-j} \cdot \frac{1+j}{1+j}\Omega = \frac{10(1+j)}{2}\Omega = 5 + j5\Omega$$

10.



a) Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = I_0 \cos(40kt - 45°)$$

where I_0 is a positive, (i.e., nonzero and non-negative), real constant with units of Amps. State the value of the component you choose.

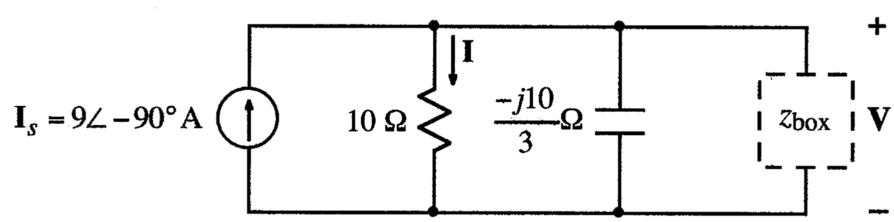
b) With your component from part (a) in the circuit, calculate the resulting value of I_0.

SOL'N: a) The first step is to convert the circuit to the frequency domain. Note that the value of ω comes from the current source on the left.

$$z_C = \frac{1}{j\omega C} = \frac{-j}{(40k)7.5\mu}\Omega = \frac{-j}{300m}\Omega = \frac{-j10}{3}\Omega$$

$$\mathbf{I}_s = -j9\,\text{A} = 9\angle -90°\,\text{A}$$

$$\mathbf{I} = I_0\angle -45°\,\text{A}$$



A label for the voltage, **V**, from the top to the bottom rail has been added on the right, to assist in calculations. The value of **V** is equal to the current from the source times the total impedance in the circuit.

$$\mathbf{V} = \mathbf{I}_s z_{tot}$$

From **V**, we may obtain the value of **I** by Ohm's law.

$$\mathbf{I} = \frac{\mathbf{V}}{10\Omega} = \frac{\mathbf{I}_s z_{tot}}{10\Omega}$$

10. (cont.)

At this point, we may work with angles alone.

$$\angle \mathbf{I} = \angle \mathbf{I}_s + \angle z_{tot} - \angle 10\Omega$$

or

$$-45° = -90° + \angle z_{tot} - 0°$$

or

$$\angle z_{tot} = -45° + 90° = 45°$$

Matters are simplified by considering the admittance, (i.e., $1/z$), form of parallel impedance.

$$z_{tot} = \frac{1}{\dfrac{1}{10\Omega} + \dfrac{3}{-j10\Omega} + \dfrac{1}{z_{box}}}$$

Taking this a step further, if we consider the total admittance we obtain a summation of admittances.

$$\frac{1}{z_{tot}} = \frac{1}{10\Omega} + \frac{3}{-j10\Omega} + \frac{1}{z_{box}}$$

The angle of admittance is the negative of the angle of impedance, (because we subtract the angle of the denominator from the angle of the numerator when we divide). This allows us to determine the angle of the total admittance.

$$\angle \frac{1}{z_{tot}} = -\angle z_{tot} = -45°$$

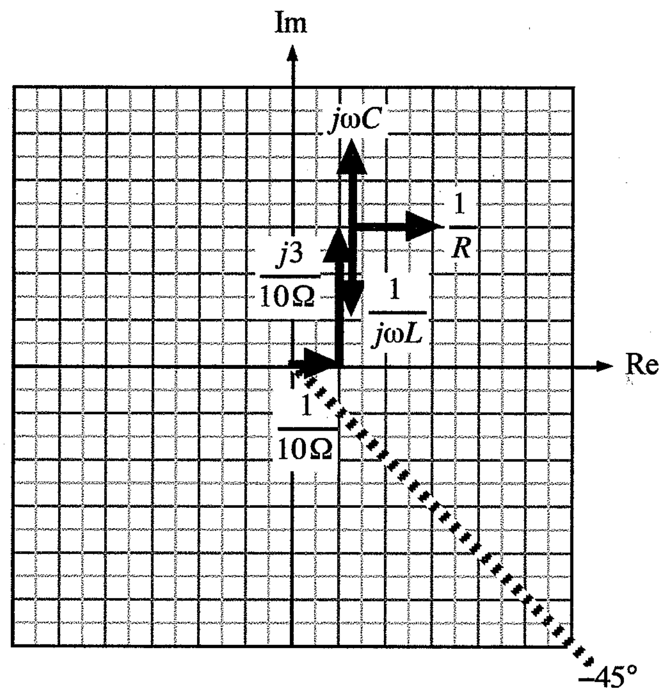
We now have an equation that allows us to use a graphical approach to solve for the admittance of z_{box}.

$$\frac{1}{z_{tot}} = -45° = \angle \left(\frac{1}{10\Omega} + \frac{j3}{10\Omega} + \frac{1}{z_{box}} \right)$$

The graph, below, shows the –45° line on which the total admittance must lie and the sum of the first two admittances on the right side of the above equation. The admittances, being complex, sum as vectors. The vectors are placed end-to-end. The third admittance, $(1/z_{box})$, placed at the end of

10. (cont.)

the second vector, must take the total admittance down to the –45° line. The diagram shows the directions each type of component would move the total impedance.



The solution for $1/z_{\text{box}}$ must be an inductor that moves the total down to $\frac{1}{10} - j\frac{1}{10}\Omega$.

$$\frac{1}{j\omega L} = -j\frac{4}{10}\Omega$$

Solving for L, we have the following derivation.

$$\omega L = \frac{10}{4}\Omega$$

or

$$\boxed{L = \frac{10}{4\omega}\Omega = \frac{10}{4(40\text{k})}\text{H} = \frac{1}{16}\text{mH} = 62.5\mu\text{H}}$$

We can check our solution to verify that it yields the correct result.

10. (cont.)

$$j\omega L = j(40\text{k})62.5\mu\Omega = j2.5\,\Omega$$

and

$$z_{\text{tot}} = \frac{1}{\frac{1}{10\Omega} + \frac{3}{-j10\Omega} + \frac{1}{j2.5\Omega}} = \frac{1}{\frac{1}{10\Omega} + \frac{j3}{10\Omega} + \frac{-j4}{10\Omega}} = \frac{1}{\frac{1}{10\Omega} - j\frac{1}{10\Omega}}$$

or

$$z_{\text{tot}} = \frac{1}{\frac{1}{10\Omega}(1-j)} = \frac{10}{(1-j)}\Omega = \frac{10}{(1-j)} \cdot \frac{1+j}{1+j}\Omega = \frac{10(1+j)}{2} = 5 + j5\ \Omega$$

We can verify that the angle for **I** is –45°, as desired:

$$\mathbf{I} = \frac{\mathbf{I}_s z_{\text{tot}}}{10\Omega} = \frac{9\angle -90^\circ\,\text{A} \cdot (5 + j5)\,\Omega}{10\Omega} = \frac{9\angle -90^\circ\,\text{A} \cdot 5\sqrt{2}\angle 45^\circ\,\Omega}{10\angle 0^\circ\,\Omega}$$

or

$$\mathbf{I} = \frac{9\sqrt{2}}{2}\angle -45^\circ\,\text{A}$$

The angle is correct.

b) The above verification gives the value for I_0. If only the magnitude of **I** is of concern, then we may perform a more efficient calculation.

$$\boxed{|\mathbf{I}| = \text{I}_o = \left|\frac{\mathbf{I}_s z_{\text{tot}}}{10\Omega}\right| = \frac{|\mathbf{I}_s||z_{\text{tot}}|}{|10\Omega|} = \frac{9\,\text{A} \cdot |5 + j5|}{10} = \frac{9(5\sqrt{2})}{10}\text{A} = \frac{9\sqrt{2}}{2}\text{A}}$$