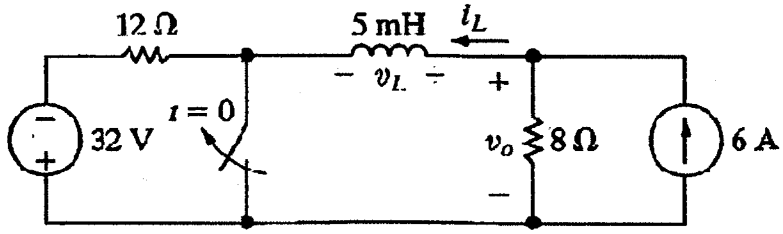
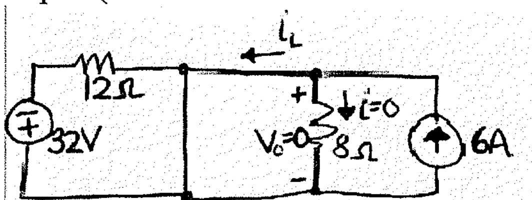
UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ECE 1270 **HOMEWORK #6 Solution** Summer 2010

1. After being closed a long time, the switch opens at t = 0. Find $v_o(t)$ or t > 0.

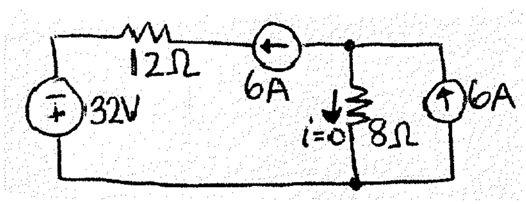


Step 1: (Redraw circuit at **t=0⁻** and solve for i_L. Inductor acts as a wire since it has sat for a long time)



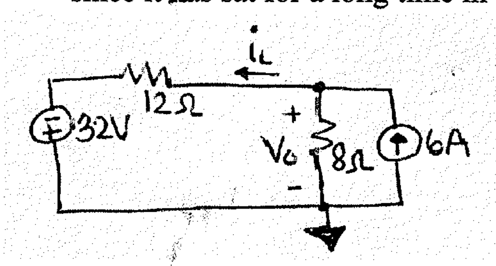
Since there is 0 volts across v_o then the 8 ohm resistor has no current through it. Therefore i_L=6A.

Step 2: **Initial Value** (Redraw circuit at **t=0⁺** where switch is now open and solve for unknown variable v_o. Inductor acts as a current source since the current in the inductor has to remain the same.)



A current summation at top node shows that v_o=0 since i=0.

Step 3: **Final Value**(Redraw circuit at **t=∞** and solve for unknown variable. Inductor acts as a wire since it has sat for a long time in this position.)

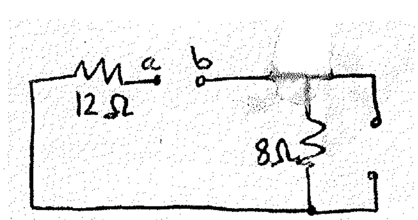


$$\left(\frac{v_o}{8}\right)-6+\left(\frac{v_o+32}{12}\right)=0$$

$$v_o\left(\frac{1}{8}+\frac{1}{12}\right)=6-\left(\frac{32}{12}\right)$$

$$v_o\left(\frac{3}{24}+\frac{2}{24}\right)=\left(\frac{72}{12}\right)-\left(\frac{32}{12}\right)$$

$$v_o=\left(\frac{40}{12}\right)\cdot\left(\frac{24}{5}\right)=16V$$



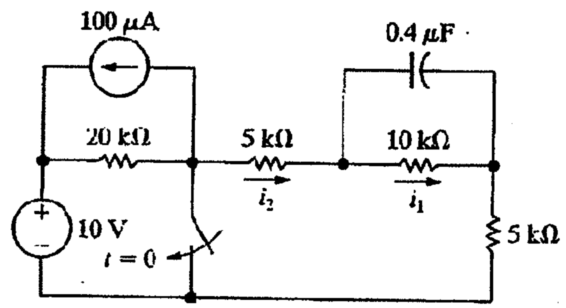
To find R_{eq} the inductor is removed from the final circuit to find path from top of inductor to bottom of inductor(sources are removed):

$$\tau=\frac{L}{R_{eq}}=\frac{5m}{12+8}=250\mu\text{sec}$$

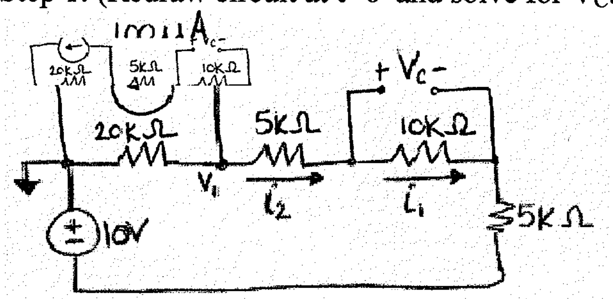
Step 4: Plug values into general equation:

$$v_o(t)=16+[0-16]e^{-t/250\mu\text{sec}}V=16-16e^{-t/250\mu\text{sec}}V$$

2. After being open for a long time, the switch closes at $t = 0$. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.



Step 1: (Redraw circuit at $t=0^-$ and solve for V_C. Capacitor acts as an open since it has been a long time)



To find Vc: $V_c = i_1 * 10k$ where $i_1 = V_1/20k$

$$100\mu + \left(\frac{V_1}{20k}\right) + \left(\frac{V_1+10}{20k}\right) = 0$$

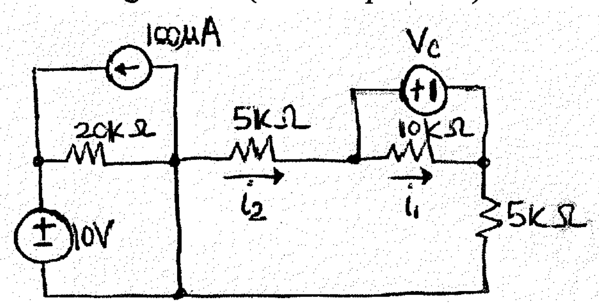
$$V_1\left(\frac{1}{20k} + \frac{1}{20k}\right) = -100\mu - \left(\frac{10}{20k}\right)$$

$$V_1\left(\frac{2}{20k}\right) = -\left(\frac{2}{20k}\right) - \left(\frac{10}{20k}\right)$$

$$V_1 = -\left(\frac{12}{20k}\right)\cdot\left(\frac{20k}{2}\right) = -6V$$

$$V_c = \left(\frac{-6}{20k}\right)10k = -3V$$

Step 2: **Initial Value** (Redraw circuit at $t=0^+$ and solve for unknown variable. Capacitor acts as a voltage source(which equals 3V) since the voltage across capacitor has to remain the same.)



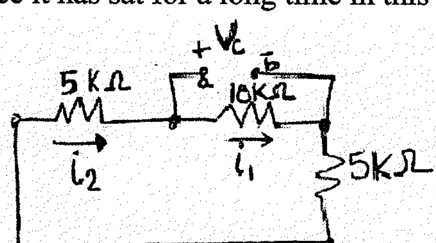
Taking a voltage loop:

$$i_2 \cdot 5k - 3 + i_2 \cdot 5k = 0$$

$$i_2 = \left(\frac{+3}{10k}\right) = +300\mu A$$

$$i_1 = -\left(\frac{3}{10k}\right) = -300\mu A$$

Step 3: **Final Value**(Redraw circuit at $t=\infty$ and solve for unknown variable. Capacitor acts as an open since it has sat for a long time in this position.)



There are no sources to Vc=0; $i_1=i_2=0$

To find R_{eq} the capacitor is removed from the final circuit(same circuit) to find path from top to bottom of capacitor. Independent sources are removed and the equivalent resistance is found:

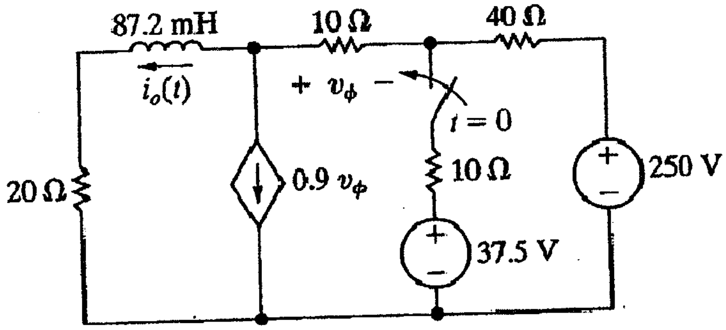
$$\tau = R_{eq}\cdot C = (10k\|10k)\cdot 0.4\mu = \left(\frac{1}{\frac{1}{10k}+\frac{1}{10k}}\right)\cdot 0.4\mu = 2m\sec$$

Step 4: Plug values into general equation:

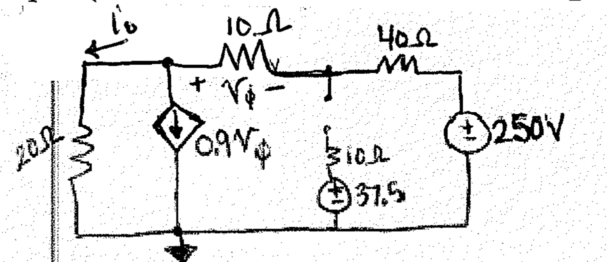
$$i_1(t) = 0 + [-300\mu + 0]e^{-t/2m\sec}A = -300\mu e^{-t/2m\sec}A$$

$$i_2(t) = 0 + [+300\mu + 0]e^{-t/2m\sec}A = +300\mu e^{-t/2m\sec}A$$

3. After being open for a long time, the switch closes at t = 0. Find $v_\phi(t)$ for t > 0.



Step 1: (Redraw circuit at **t=0⁻** and solve for i_L. Inductor acts as a wire since it has sat for a long time)



When the circuit contains a dependent source, an extra step is needed to determine the value of the dependent variable:

Using node voltage:

$$\left(\frac{V_1}{20}\right)+0.9v_\phi+\left(\frac{V_1-250}{50}\right)=0$$

$$v_\phi=\left(\frac{V_1-250}{50}\right)\cdot 10$$

$$\left(\frac{V_1}{20}\right)+9\cdot\left(\frac{V_1-250}{50}\right)+\left(\frac{V_1-250}{50}\right)=0$$

$$V_1\left(\frac{5}{100}+\frac{9\cdot 2}{100}+\frac{2}{100}\right)=\left(\frac{2250}{50}+\frac{250}{50}\right)$$

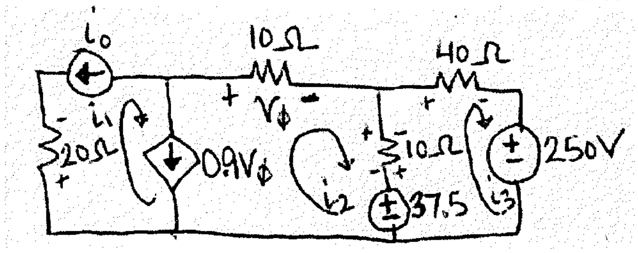
$$V_1\left(\frac{25}{100}\right)=\left(\frac{2500}{50}\right)$$

$$V_1=\left(\frac{2500}{50}\right)\cdot\left(\frac{100}{25}\right)=200V$$

$$v_\phi=\left(\frac{V_1-250}{50}\right)10=-10V$$

$$i_o=\left(\frac{V_1}{20}\right)=-0.5A$$

Step 2: **<u>Initial Value</u>** (Redraw circuit at **t=0⁺** and solve for unknown variable. Inductor acts as a current source since the current in the inductor has to remain the same.)



Mesh-current method: (note: i_o=-0.5A)

With dependent sources: solve for dependent variable in terms of the mesh currents:

$$v_\phi=i_2\cdot 10$$

There is a supermesh:

$$0.9v_\phi=i_1-i_2$$

$$i_1=+0.5A$$

$$i_2+0.9(i_2 10)=+0.5$$

$$i_2=\frac{0.5}{10}=50m$$

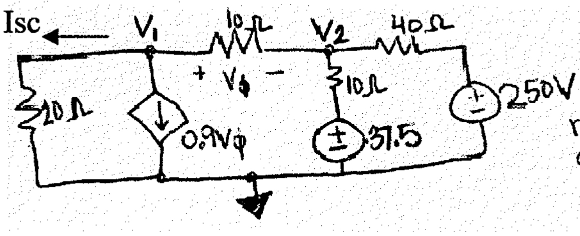
$$+37.5+10(i_2-i_3)-40i_2-250=0$$

$$10i_3=-30(50m)+37.5-250$$

$$i_3=-21.4A$$

$$v_\phi=10i_2=10\cdot 50m=0.5V$$

Step 3: **Final Value**(Redraw circuit at t=∞ and solve for unknown variable. Inductor acts as a wire since it has sat for a long time in this position.)



$$v_\phi = V_1 - V_2$$

node-V at V1: $\left(\frac{V_1}{20}\right)+0.9(V_1-V_2)+\left(\frac{V_1-V_2}{10}\right)=0$

① $V_1\left(\frac{1}{20}+\frac{9\cdot 2}{10\cdot 2}+\frac{1\cdot 2}{10\cdot 2}\right)-V_2\left(\frac{1}{10}+\frac{9}{10}\right)=0$

node-V at V2: $-\left(\frac{V_1-V_2}{10}\right)+\left(\frac{V_2-37.5}{10}\right)+\left(\frac{V_2-250}{40}\right)=0$

$$V_2\left(\frac{1\cdot 4}{10\cdot 4}+\frac{1\cdot 4}{10\cdot 4}+\frac{1}{40}\right)=V_1\left(\frac{1}{10}\right)+\frac{37.5\cdot 4}{10\cdot 4}+\frac{250}{40}$$

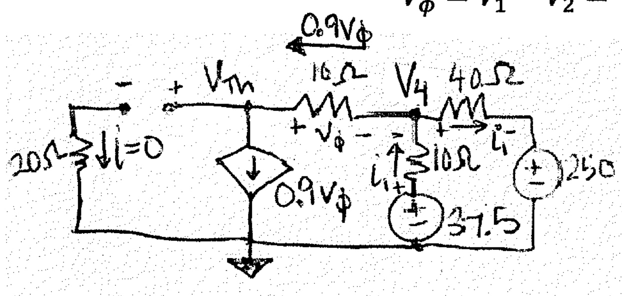
② $V_2=V_1\left(\frac{1}{10}\right)\cdot\left(\frac{40}{9}\right)+\frac{400}{40}\cdot\left(\frac{40}{9}\right)$

② into ①: $V_1\left(\frac{21}{20}\right)-V_1\left(\frac{1}{10}\right)\cdot\left(\frac{40}{9}\right)-\frac{400}{40}\cdot\left(\frac{40}{9}\right)=0$

$$V_1=\frac{400}{9}\frac{20(9)}{21(9)-40(2)}=\frac{8000}{109}$$

$$V_2=\frac{8000}{109}\left(\frac{4}{9}\right)+\frac{400(109)}{109(9)}=\frac{75600}{981}$$

$$V_\phi=V_1-V_2=\frac{8000}{109}\left(\frac{9}{9}\right)-\frac{75600}{109(9)}=\frac{-3600}{981}$$



To find R_{eq} the inductor is removed from the final circuit to find path from top of inductor to bottom of inductor(Thevenin Resistance):

With dependent source, find the open circuit voltage, Vth and short circuit current Isc (figure above): $R_{th}=\frac{V_{Th}}{I_{sc}}$

$V_\phi = -0.9V_\phi(10) \Rightarrow V_\phi(1+0.9(10))=0 \Rightarrow V_\phi=0 \quad \therefore V_{Th}=V_4$

$+37.5-i_1(10)-i_1(40)-250=0$

$i_1=\frac{250-37.5}{50}=4.25 \Rightarrow \quad +37.5-i_1(10)-V_4=0 \quad V_4=V_{Th}=37.5-4.25(10)$

$V_{Th}=-5V$

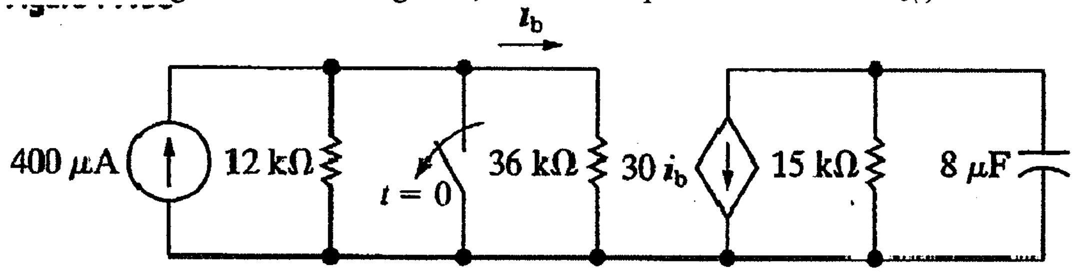
$I_{sc}(\text{above figure})=\frac{V_1}{20}=\frac{-3600}{981}$

$\therefore R_{Th}=R_{eq}=\frac{+5(981)}{+3600}\cong 1.36\Omega \Rightarrow \tau=\frac{L}{R_{eq}}=\frac{87.2m}{1.36}=64msec.$

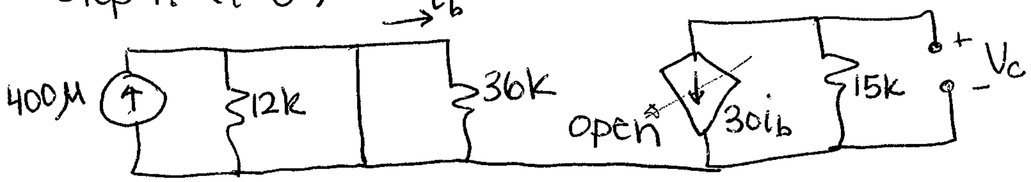
Step 4: Plug values into general equation:

$$\boxed{V_\phi(t)\cong -3.67+[0.5+3.67]e^{-t/64m}\ V}$$

4. After being closed for a long time, the switch opens at t = 0. Find $i_b(t)$ for t > 0.



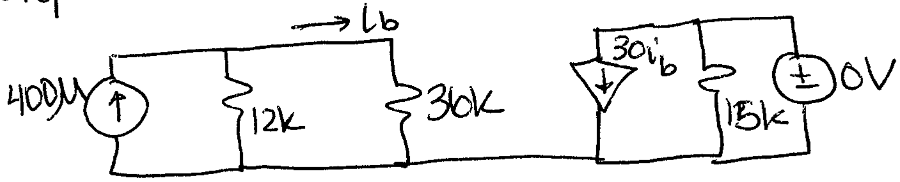
Step 1: $(t=0^-)$



$i_b = 0$

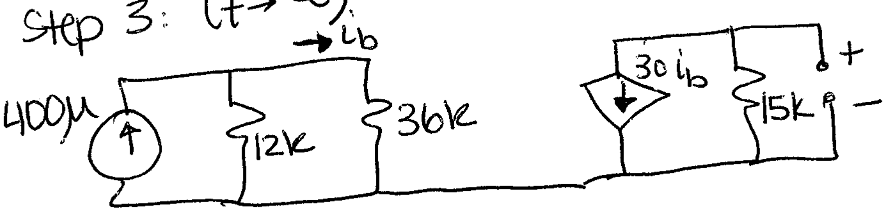
$\therefore V_c = 0$

Step 2: $(t=0^+)$



Current Divider: $i_b = \dfrac{400\mu(12k)}{12k+36k} = \underline{100\mu A}$ ← Initial Value

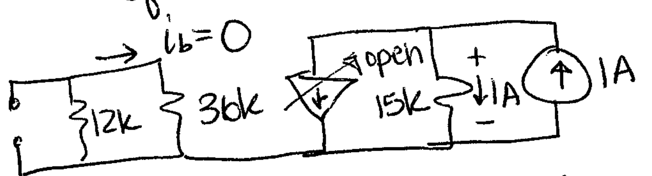
Step 3: $(t \to \infty)$



current-divider to find i_b:

$i_b = 100\mu A$

To find Req:



$V_{test} = 1(15k)$

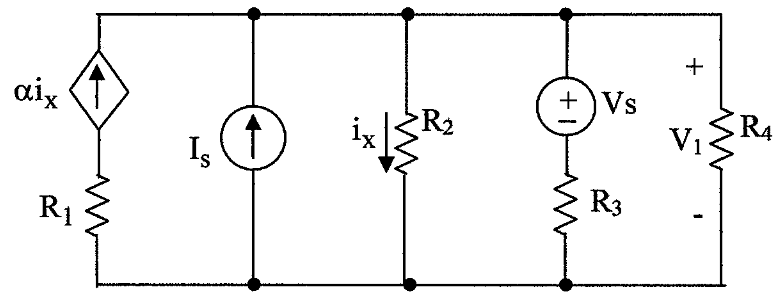
$R_{Th} = R_{eq} = \dfrac{15k}{1} = \underline{15k\,\Omega}$

For $t>0$:

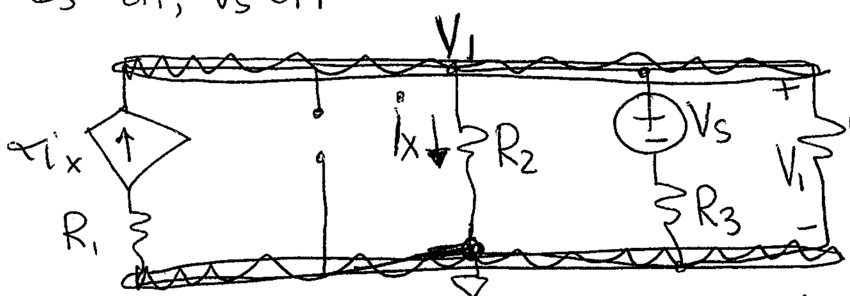
$i_b(t) = 100\mu + (\overset{0}{\cancel{100\mu - 100\mu}})e^{-t/15k(8\mu)} = \boxed{100\mu A}$

(correct since i_b does not have the cap in its circuit)

5. Using superposition, derive an expression for V_1 that contains no circuit quantities other than $i_s, R_1, R_2, R_3, R_4, \alpha$, or V_s.



① I_s off, V_s on



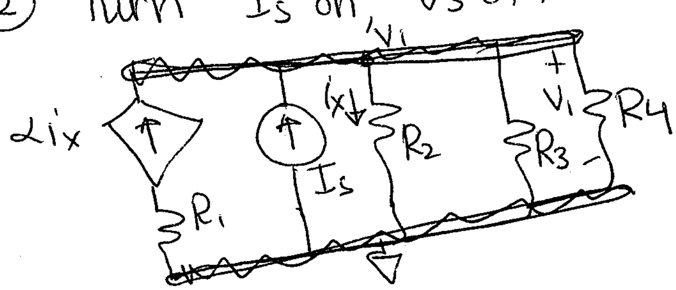
$$i_x = \frac{V_1}{R_2}$$

Using node-Voltage: $-\alpha\left(\frac{V_1}{R_2}\right) + \frac{V_1}{R_2} + \frac{V_1 - V_s}{R_3} + \frac{V_1}{R_4} = 0$

$$V_1\left(\frac{-\alpha}{R_2} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{V_s}{R_3}$$

$$V_{1_1} = \frac{V_s}{R_3\left[\frac{(1-\alpha)}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right]}$$

② Turn I_s on, V_s off



node-Voltage: $-\alpha\left(\frac{V_1}{R_2}\right) - I_s + \frac{V_1}{R_2} + \frac{V_1}{R_3} + \frac{V_1}{R_4} = 0$

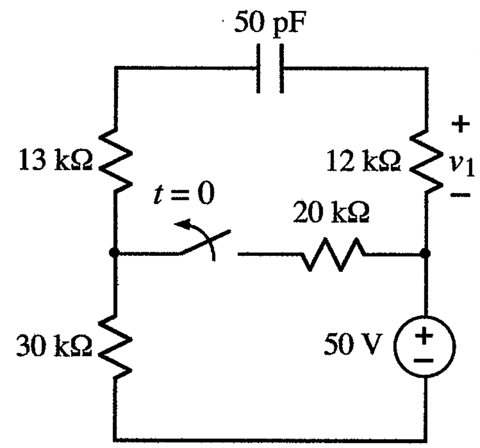
$$V_1\left(\frac{-\alpha}{R_2} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) = I_s$$

$$V_{1_2} = \frac{I_s}{\frac{(1-\alpha)}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

Total:

$$\boxed{V_1 = \frac{V_s + I_s R_3}{R_3\left[\frac{(1-\alpha)}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right]}}$$

6.



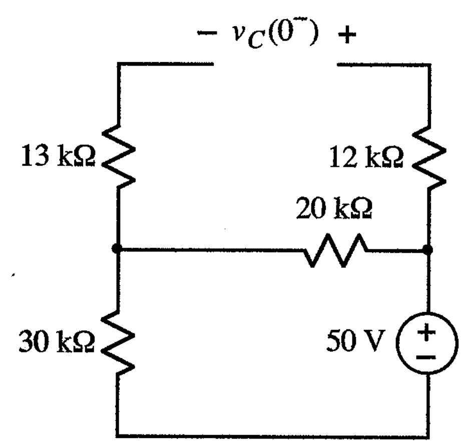
After being closed for a long time, the switch opens at $t = 0$.

a) Calculate the energy stored on the capacitor at $t = 0^+$.

b) Write a numerical expression for $v_1(t)$ for $t > 0$.

SOL'N: a) At $t = 0^-$, the switch is closed and the C acts like an open. The 12 kΩ and 13 kΩ are in series with the open circuit for C and carry no current. Therefore, they have no voltage drop.

$t = 0^-$:



The 20 kΩ and the 30 kΩ are effectively in series, since the 12 kΩ and 13 kΩ carry no current. Thus, we have a voltage divider formed by the 50 V source and the 20 kΩ and the 30 kΩ resistors. The voltage across the C will be the same as the voltage across the 20 kΩ:

$$v_C(t=0^-) = 50\text{ V}\frac{20\text{ k}\Omega}{20\text{ k}\Omega + 30\text{ k}\Omega} = 20\text{ V}$$

Since v_C is an energy variable, it stays the same as the switch moves:

$$v_C(t=0^+) = v_C(t=0^-) = 20\text{ V}$$

The stored energy is a function of the square of the voltage across the capacitor:

$$w_C(t=0^+) = \frac{1}{2}Cv_C^2(t=0^+) = \frac{1}{2}50\text{p}\cdot(20\text{ V})^2\text{ J} = 10\text{ nJ}$$

b) We use the general form of solution for RC problems:

$$v_1(t) = [v_1(t=0^+) - v_1(t\to\infty)]e^{-t/R_{\text{Th}}C} + v_1(t\to\infty)$$

From part (a), we have $v_C(t=0^+) = 20\text{ V}$. To find $v_1(0^+)$, we treat the C as a voltage source of 20 V, (with + sign on right side). The switch is open, and we have two voltage sources and three resistors around the outside of the circuit. We may sum the voltage sources and use the voltage-divider formula to calculate $v_1(0^+)$:

$$v_1(t=0^+) = (20\text{ V} - 50\text{ V})\frac{12\text{ k}\Omega}{12\text{ k}\Omega + 13\text{ k}\Omega + 30\text{ k}\Omega} = -30\text{ V}\cdot\frac{12}{55}$$

or

$$v_1(t=0^+) = -\frac{72}{11}\text{ V} \approx -6.55\text{ V}$$

For $v_1(t\text{->}\infty)$, the C acts like an open circuit, and no current will flow through the 12 kΩ resistor. Thus, $v_1(t\text{->}\infty) = 0$ V.

$$v_1(t\to\infty) = 0\text{ V}$$

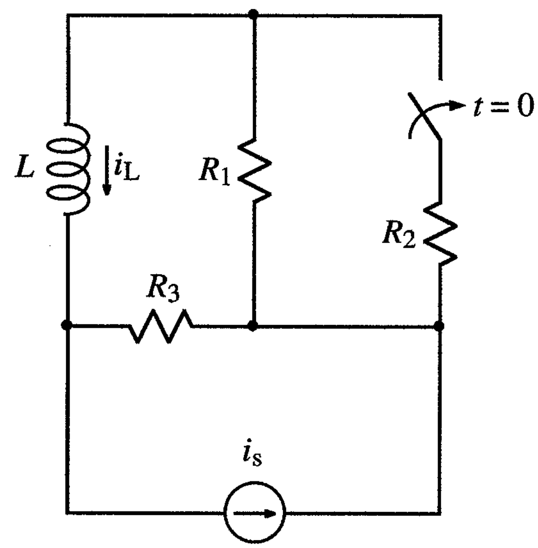
To find the Thevenin equivalent seen from the capacitor terminals, we remove the capacitor, turn off the independent 50 V source, and look into the circuit from the capacitor terminals.

$$R_{\text{Th}} = 12\text{ k}\Omega + 30\text{ k}\Omega + 13\text{ k}\Omega = 55\text{ k}\Omega$$

Now we use the general form of solution for *RL* problems:

$$v_1(t) = [-6.55\text{ V} - 0\text{ V}]e^{-t/55\text{k}50\text{ps}} + 0\text{ V} = -6.55\text{ V}e^{-t/2.75\mu\text{s}}$$

7.



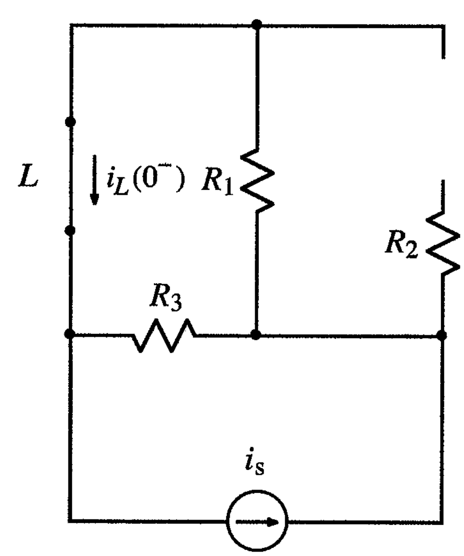
After being open for a long time, the switch closes at $t = 0$.

a) Write an expression for $i_L(t = 0^+)$.

b) Write an expression for $i_L(t > 0)$ in terms of R_1, R_2, R_3, i_s, and L.

SOL'N: a) We first find the current through the inductor for $t = 0^-$. The switch is open, eliminating R_2 from consideration. The inductor looks like a wire.

$t = 0^-$:



We are left with the i_s source driving R_1 and R_3 in parallel. The inductor is in series with R_1 and so has the same current as R_1. We use a current divider to find the current in the inductor:

$$i_L(0^-) = i_s \frac{R_3}{R_1 + R_3}$$

At $t = 0^+$, the inductor has the same current as at $t = 0^-$.

$$i_L(0^+) = i_L(0^-) = i_s \frac{R_3}{R_1 + R_3}$$

b) As $t \to \infty$, the switch is closed, the L acts like a wire, and we have a current divider with R_1 and R_2 in parallel on one side and R_3 on the other side. The current through R_1 parallel R_2 is the same as the current through L:

$$i_L(t \to \infty) = i_s \frac{R_3}{R_1 \parallel R_2 + R_3} = i_s \frac{R_1 R_3 + R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

For the time constant of the circuit, we take the Thevenin resistance from the terminals where the L is attached with the switch closed for $t > 0$. Since we have only an independent source, we turn off the source, i_s, and look into the circuit from the terminals where L is attached (but without the L). The current source looks like an open source, and we see R_3 in series with R_1 parallel R_2.

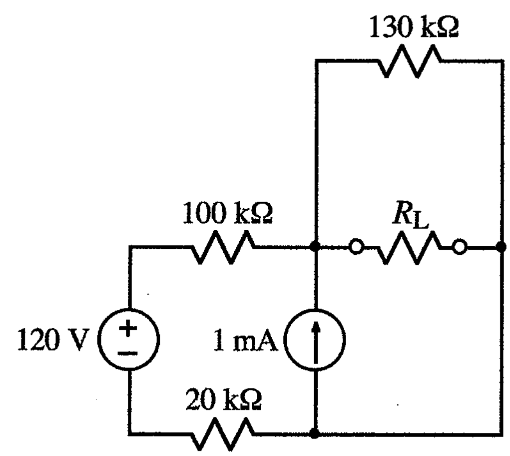
$$R_{\mathrm{Th}} = R_1 \parallel R_2 + R_3 = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_2}$$

Now we use the general formula for RL circuit solutions:

$$i_L(t > 0) = i_L(t \to \infty) + [i_L(0^+) - i_L(t \to \infty)]e^{\frac{-t}{L/R_{\mathrm{Th}}}}$$

or

$$i_L(t > 0) = i_s \frac{R_3}{R_1 \parallel R_2 + R_3} + i_s\left(\frac{R_3}{R_1 + R_3} - \frac{R_3}{R_1 \parallel R_2 + R_3}\right)e^{\frac{-t}{L/(R_3 + R_1 \parallel R_2)}}$$



8. Calculate the value of R_L that would absorb maximum power.

9. Calculate that value of maximum power R_L could absorb.

SOL'N: 8. Use $R_L = R_{Th}$ for maximum power transfer. To find R_{Th}, we turn off the independent sources and look in from the terminals where R_L is attached (with R_L removed). The voltage source becomes a wire, and the current source becomes an open circuit.

$$R_{Th} = 130\text{ k}\Omega \,\|\, (100\text{ k}\Omega + 20\text{ k}\Omega) = 10\text{ k}\Omega \cdot 13 \,\|\, 12 = 62.4\text{ k}\Omega$$

We use this Thevenin resistance value for R_L:

$$R_L = 62.4\text{ k}\Omega$$

9 The maximum power transferred is

$$p_{max} = \frac{v_{Th}^2}{4R_{Th}}$$

The Thevenin equivalent voltage is the voltage across R_L without R_L. We can use superposition to calculate v_{Th}. First, we turn on the 120 V source and turn off the 1 mA source. The 1 mA source turns into an open source. The voltage at the terminals for R_L is the same as the voltage across the 130 kΩ, and we can find this voltage using a voltage-divider.

$$v_{\text{Th1}} = 120\text{ V} \cdot \frac{130\text{ k}\Omega}{130\text{ k}\Omega + 100\text{ k}\Omega + 20\text{ k}\Omega} = 120\text{ V} \cdot \frac{13}{25} = 62.4\text{ V}$$

Second, we turn on the 1 mA source and turn off the 120 V source. The voltage across R_L is the same as the voltage across the 130 kΩ in parallel with the 100 kΩ and 130 kΩ in series. By Ohm's law, we find the voltage by multiplying the current by the total parallel resistance.

$$v_{\text{Th2}} = 1\text{ mA} \cdot 130\text{ k}\Omega \,\|\, (100\text{ k}\Omega + 20\text{ k}\Omega) = 62.4\text{ V}$$

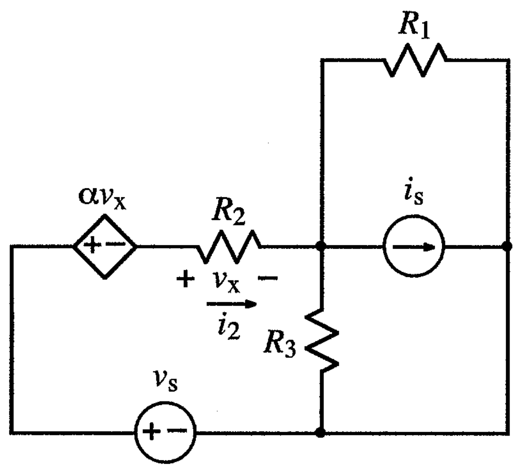
The sum of the Thevenin voltages gives the total Thevenin voltage.

$$v_{\text{Th}} = v_{\text{Th1}} + v_{\text{Th2}} = 62.4\text{ V} + 62.4\text{ V} = 124.8\text{ V}$$

Now we use the formula for maximum power transferred:

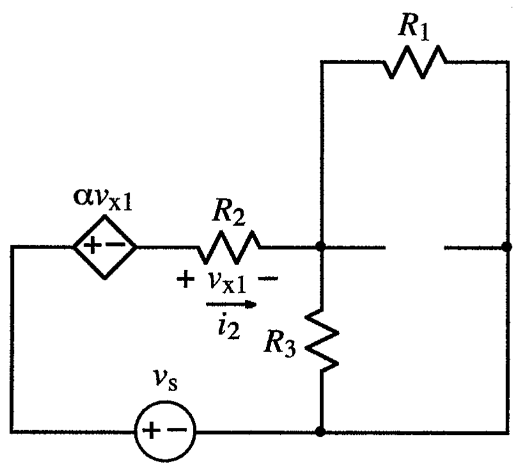
$$p_{\max} = \frac{v_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(2 \cdot 62.4)^2}{4 \cdot 62.4\text{ k}}\text{ W} = 62.4\text{ mW}$$

10.

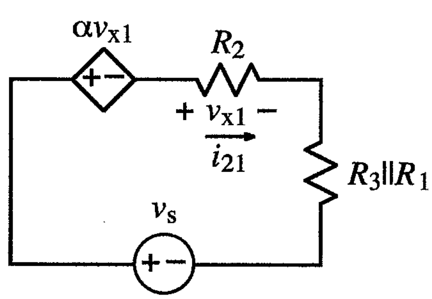


Using superposition, derive an expression for i_2 that contains no circuit quantities other than i_s, v_s, R_1, R_2, R_3, and α.

SOL'N: For the first case of the superposition solution, we turn on the voltage source, v_s, and turn off the independent source, i_s.



This means the current source turns into an open circuit. The remaining circuit has R_1 in parallel with R_3 on the right side and one loop on the left.



We now define the dependent voltage in terms of component values.

$$v_{x1} = (v_s - \alpha v_{x1})\frac{R_2}{R_2 + R_1 \| R_3}$$

or

$$v_{x1}(R_2 + R_1 \| R_3 + \alpha R_2) = v_s R_2$$

or

$$v_{x1} = \frac{v_s R_2}{(1+\alpha)R_2 + R_1 \| R_3}$$

In this circuit, we calculate i_2 from v_x directly:

$$i_{21} = \frac{v_{x1}}{R_2} = \frac{v_s}{(1+\alpha)R_2 + R_1 \| R_3}$$

NOTE: We obtain the same result by replacing the dependent source with a resistor. The current through the dependent source is the same as the current through R_2:

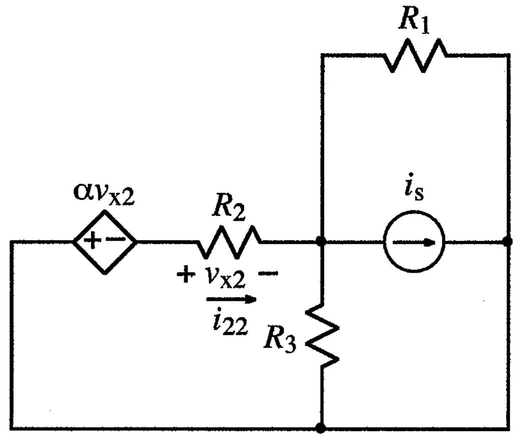
$$i_2 = \frac{v_x}{\mathrm{R}_2}$$

By Ohm's law we can calculate the equivalent resistance for the dependent source:

$$R_{eq} = \frac{v}{i} = \frac{\alpha v_x}{\frac{v_x}{\mathrm{R}_2}} = \alpha R_2$$

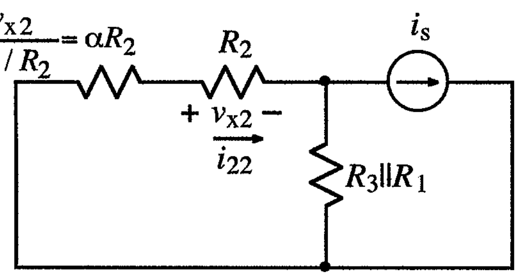
Using this resistance values gives the expression for i_{21} found earlier.

For the second case of the superposition solution, we turn on the current source, i_s, and turn off the voltage source, v_s. The voltage source turns into a wire. We have R_1 in parallel with R_3 in parallel with the branch on the left side, (which is now the dependent source and R_2).



We could use node-voltage to find the voltage at the node in the center of the circuit, but it is easier to replace the dependent source with its equivalent resistance. As before, we find that $R_{eq} = \alpha R_2$.

$$R_{eq} = \frac{\alpha v_{x2}}{v_{x2} / R_2} = \alpha R_2$$



This leaves us with three parallel resistances driven by a current source. The current-divider formula gives the current through R_2:

$$i_{22} = i_s \frac{R_1 \| R_3}{(1+\alpha)R_2 + R_1 \| R_3}$$

We sum the two currents to find the total value of i_2:

$$i_2 = i_{21} + i_{22} = \frac{v_s + i_s \cdot R_1 \| R_3}{(1+\alpha)R_2 + R_1 \| R_3}$$