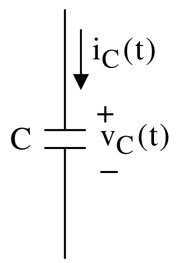
UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ECE 1270 **HOMEWORK #5 Solution** Summer 2010

1. In a-c, the voltage $v_C(t)$ across a 5nF capacitor is listed. Find the current, $i_C(t)$, flowing in the capacitor in each case as a function of time:



(a) $v_C(t) = 8V$
(b) $v_C(t) = 25t\ kV/sec$
(c) $v_C(t) = 5k\cos(2\pi \cdot 20 \cdot t)V$

Use the defining equation for a capacitor in each case:

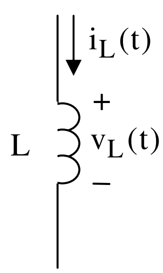
$$i_C = C\frac{dv_C}{dt}$$

(a) $i_C = C\frac{d}{dt}8V = 0A$

(b) $i_C = C\frac{d}{dt}(25{,}000t) = 5nF \cdot 25{,}000V = 125\mu A$

(c) $i_C = C\frac{d}{dt}\left(5k\cos(2\pi\times 20\times t)A\right) = 5n\cdot\left(-5k\sin(2\pi\times 20\times t)20\pi\right) = -500\pi\sin(2\pi\times 20\times t)\mu V$

2. In a-c, the current $i_L(t)$ flowing into a $3\mu H$ inductor is listed. Find the voltage, $v_L(t)$, across the inductor in each case as a function of time.



(a) $i_L(t) = 29nA$
(b) $i_L(t) = 35t\mu A$
(c) $i_L(t) = 2 - 0.2e^{-t/2m\sec}A$

Use the defining equation for a capacitor in each case:

$$v_L = L\frac{di_L}{dt}$$

(a) $v_L = L\frac{di}{dt}29n = L\cdot 0 = 0V$

(b) $v_L = L\frac{d}{dt}\left(35t\mu\right) = 3\mu\cdot 35\mu = 105pV$

(c) $v_L = L\frac{d}{dt}\left(2-0.2e^{-t/2m\sec}\right)V = 3\mu F\cdot\left(-\frac{-0.2}{2m\sec}e^{-t/2m\sec}\right) = 300\mu e^{-t/2m\sec}V$

3. The following equation describes the voltage, v_C, across a capacitor as a function of time. Find the time, t, at which v_C is equal to -6V. Plot $v_C(t)$. You may use Matlab.

$$v_C(t) = 6 - 6\left(1 - e^{-t/10\mu s}\right)V$$

Substitute the value of –6 V for $v_C(t)$ on the left side:

$$-6\text{V} = 6 - 6\left(1 - e^{-t/10\mu s}\right)\text{V}$$

Move constant terms to the left side in order to isolate the exponential:

$$-6\text{V} = 6 - 6 + 6e^{-t/10\mu s}\text{V} \Rightarrow -6\text{V} = 6e^{-t/10\mu s} \Rightarrow -1 = e^{-t/10\mu s}$$

Use the natural log to remove the exponential and solve for t:

$$\ln(-1) = \ln\left(e^{-t/10\mu s}\right) \Rightarrow$$

v_C(t) will never equal -6

Matlab:

```
>> t=[0:1e-6:100e-6];
>> v_C=6-6*(1-exp(-t/10e-6));
>> plot(t,v_C)
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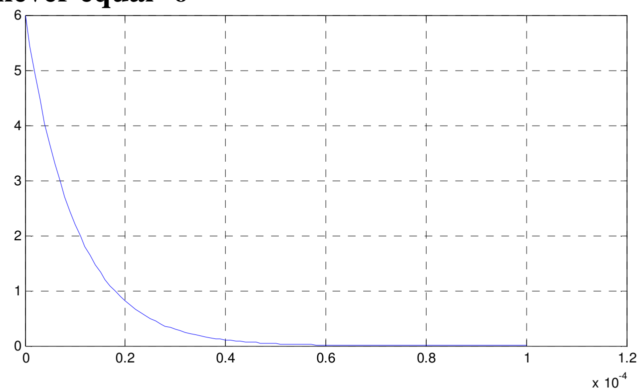


4. The following equation describes the voltage, v_L, across an inductor as a function of time. Find an expression for the current, $i_L(t)$, through the inductor as a function of time. Assume that $i_L(t=0) = 0\text{A}$. Plot $i_L(t)$. You may use Matlab.

$$v_L(t) = 2e^{-t/20ms}\text{V}$$

Use the defining equation for an inductor and solve for *i* in terms of *v* by multiplying both sides by dt:

$$v_L = L\frac{di_L}{dt} \Rightarrow v_L dt = L di_L$$

Second, integrate both sides and use limits that correspond to the variable of integration for each side *and are evaluated at the same points in time* for both sides:

$$\int_0^t v_L dt = \int_{i_L(t=0)}^{i_L(t)} L di_L \Rightarrow \int_0^t v_L dt = L i_L \Big|_{i_L(t=0)}^{i_L(t)} = L\left[i_L(t) - i_L(t=0)\right]$$

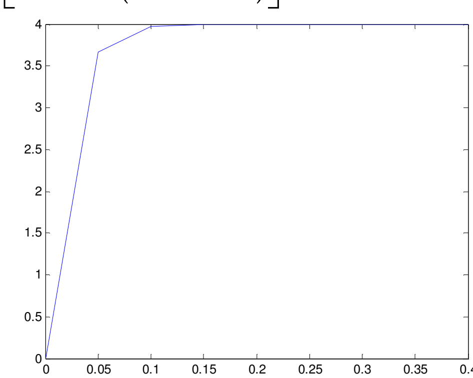
$$i_L(t) = \frac{1}{L}\int_0^t v_L dt + i_L(t=0)$$

The above expression applies to any inductor in any circuit. Now substitute the formula given for $v_L(t)$ and the value given for $i_L(t=0)$ to find $i_L(t)$:

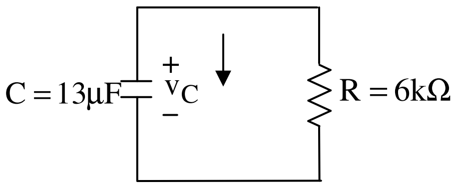
$$i_L(t) = \frac{1}{L}\int_0^t v_L dt + i_L(t=0) = \frac{1}{L}\int_0^t \left[2e^{-t/20ms}V\right]dt + 0A$$

$$i_L(t) = -2\cdot 20m \cdot e^{-t/20ms}\Big|_0^t = \frac{1}{L}\left[-40ms\cdot\left(e^{-t/20ms} - 1\right)\right]V$$

Make an assumption for L to create the graph. (L=10mH):



5. Find the voltage, v_C, on the capacitor in the circuit below as a function of time if the initial condition is $v_C(t=0^+)=2\text{V}$.



The following general form of solution applies to any RC circuit with a single capacitor:

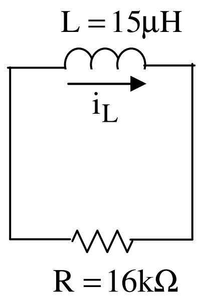
$$v_C(t \geq 0) = v_C(t \to \infty) + \left[v_C(t=0^+) - v_C(t \to \infty) \right] \cdot e^{\frac{-t}{R_{eq} \cdot C}} V$$

[$v_C(t>0)$=*Final Value* +[*Initial Value – Final Value*] $e^{\frac{-t}{\tau}}$]

where the Final Value, $v_C(t \to \infty)$ = 0 because no sources are attached. The equivalent resistance, R_{eq}, is for the circuit after t = 0 (with the C removed) as seen from the terminals where the C is connected.

τ =Req·C=6k·13µ=78msec $\qquad v_c(t) = 0 + \left[2-0\right] e^{-t/78ms} V$

6. Find the current, i_L, through the inductor in the circuit below for t > 0 if $i_L(t=0)=13\text{mA}$.



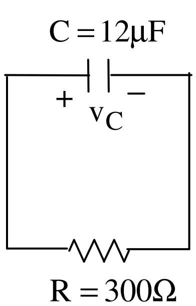
The following general form of solution applies to any RL circuit with a single inductor:

$$i_L(t \geq 0) = i_L(t \to \infty) + \left[i_L(t=0^+) - i_L(t \to \infty) \right] \cdot e^{\frac{-R_{eq}t}{L}} A$$

[$x(t>0)$=*Final Value* +[*Initial Value – Final Value*] $e^{\frac{-t}{\tau}}$]

where the Final Value, $i_L(t \to \infty)$ = 0 because no sources are attached. The equivalent resistance, R_{eq}, is for the circuit after t = 0 (with the L removed) as seen from the terminals where the L is connected. $\tau = \frac{L}{R_{eq}} = \frac{15\mu}{16k} = 0.94n\sec$ $\qquad i_L(t) = 0 + \left[13m - 0\right] e^{-t/0.94ms} A = 13m \cdot e^{-t/0.94ms} A$

7. Find the voltage, v_C, across the capacitor in the circuit below for t > 0 if $v_C(t=0)=5\text{V}$.



The following general form of solution applies to any RC circuit with a single capacitor:

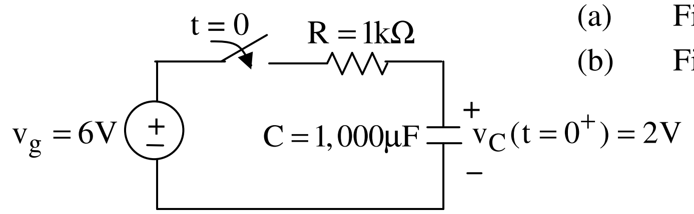
$$v_C(t \geq 0) = v_C(t \to \infty) + \left[v_C(t=0^+) - v_C(t \to \infty) \right] \cdot e^{\frac{-t}{R_{eq} \cdot C}} V$$

where the Final Value, $v_C(t \to \infty)$ = 0 because no sources are attached. The equivalent resistance, R_{eq}, is for the circuit after t = 0 (with the C removed) as seen from the terminals where the C is connected.

τ =Req·C=300·12µ=3.6msec

$$v_c(t) = 0 + \left[5-0\right] e^{-t/3.6ms} V = 5e^{-t/3.6ms} V$$

8. After being open for a long time, the switch closes at t = 0.



(a) Find an expression for $v_C(t)$ for t ≥ 0.

(b) Find the energy stored in the capacitor at time t = 2s.

(a) The following general form of solution applies to any RC circuit with a single capacitor:

$$v_C(t \geq 0) = v_C(t \to \infty) + \left[v_C(t = 0^+) - v_C(t \to \infty) \right] \cdot e^{\frac{-t}{R_{eq} \cdot C}} V$$

The value of v_C(t=0) is given in the problem as 2 V. Note that the C could have any voltage before t = 0 in this circuit if the value were not specified. The voltage would stay on the ideal C indefinitely prior to t = 0. As time approaches infinity, the C will charge to its final value, and current will cease to flow in the C. Thus, the C will become an open circuit. It follows that the current through the R, which is the same as the current through the C, will become zero. By Ohm's law, this in turn means that the voltage drop across the R will become zero, and the voltage across the C will be the same as the source voltage, 6 V.

The equivalent resistance, R_{eq}, is for the circuit after t = 0 (with the C removed) as seen from the terminals where the C is connected.

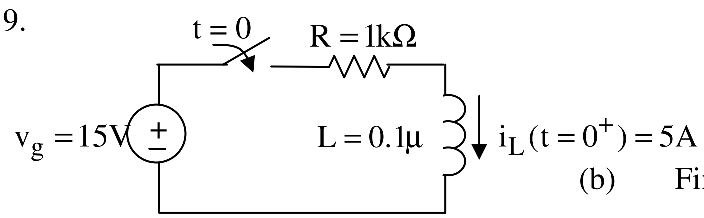
τ =Req·C=1k·1,000μ=1sec $\qquad v_c(t) = 6 + [2 - 6] e^{-t/1s} V = 6 - 4e^{-t/1s} V$

(b) At t=2s: $v_c(2s) = 6 - 4e^{-2s/1s} V = 6 - 4e^{-2} = 5.5V$

The energy in a capacitor is given by the following formula:

$$w_C = \frac{1}{2} C v_C^2 = \frac{1}{2} 1{,}000\mu \cdot (5.5)^2 = 15.1mJ$$

9.



(a) Find an expression for $i_L(t)$ for t ≥ 0. Note: Assume the initial current in the L is created by circuitry not shown in the diagram.

(b) Find the energy stored in the inductor at time t = 1 ms.

(a) The following general form of solution applies to any RL circuit with a single inductor:

$$i_L(t \geq 0) = i_L(t \to \infty) + \left[i_L(t = 0^+) - i_L(t \to \infty) \right] \cdot e^{\frac{-R_{eq} t}{L}} A$$

The value of i_L(t=0) is given in the problem as 5 A, (created by circuitry not shown).

As time approaches infinity, the L current will converge to its final value, and the voltage across the L will cease to change. Thus, diL/dt = 0 and vL = 0, meaning that L will act like a wire. It follows that the current through the L will equal the current through R:

$$i_L(t \to \infty) = \frac{15}{1k} = 15mA$$

The equivalent resistance, R_{eq}, is for the circuit after t = 0 (with the L removed) as seen from the terminals where the L is connected. $\tau = \frac{L}{R_{eq}} = \frac{0.1\mu}{1k} = 0.1n\sec$

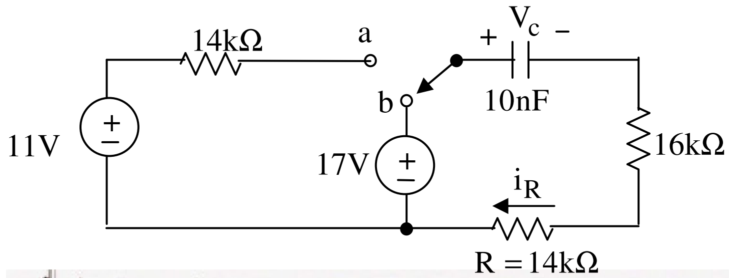
$$i_L(t) = 15m + [5 - 15m] e^{-t/0.1ns} A$$

(a) At t=1ms: $i_L(1m) = 15m + [5 - 15m] e^{-1m/0.1ns} A = 15mA$

The energy in an inductor is given by the following formula:

$$w_L(t = 10m) = \frac{1}{2} L \left[i_L(t = 10m) \right]^2 = \frac{1}{2} 0.1\mu \cdot (15m)^2 = 11.25pJ$$

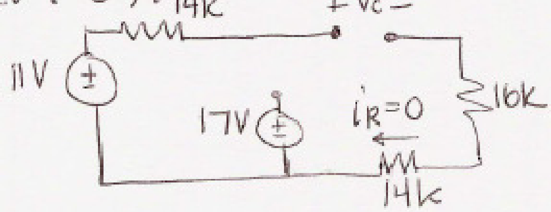
10. The switch has been in a position a for a long time. It is switched to position b at t = 0.



(a) Find an expression for $V_c(t)$ for $t > 0$.

(b) Find the current, i_R, in R as a function of time.

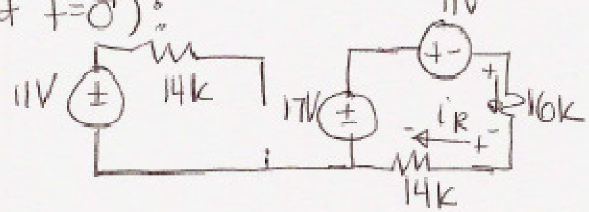
(at $t=0^-$):



$V_c = 11\text{V}$

$i_R = 0$

(at $t=0^+$):

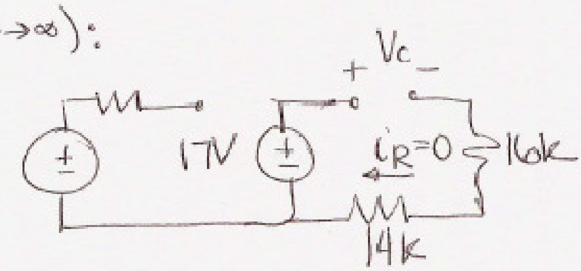


$+i_R(14k) + i_R(16k) + 11 - 17 = 0$

$i_R = \dfrac{6}{30k} = 0.2\text{mA (IC)}$

$V_c(t) = 11\text{V}$

(at $t \to \infty$):



$V_c = 17\text{V}$

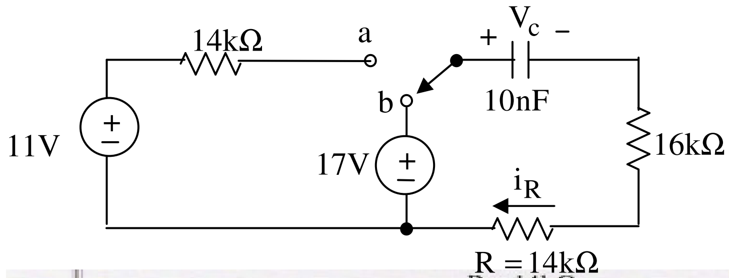
$i_R = 0$

$R_{eq} = 14k + 16k = 30k$

$V_c(t) = 17 + (11-17)e^{-t/30k(10n)} = \boxed{17 - 6e^{-t/300\mu sec}\ \text{V}}$

$i_R(t) = 0 + (0.2m - 0)e^{-t/300\mu sec}\ \text{A} = \boxed{0.2e^{-t/300\mu sec}\ \text{A}}$

10. The switch has been in a position a for a long time. It is switched to position b at t = 0.



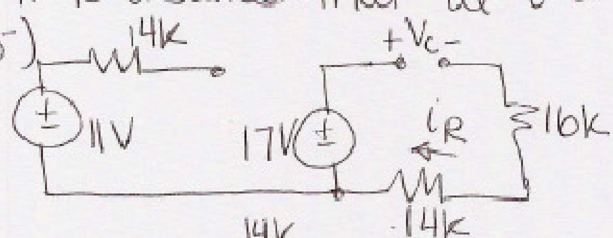
(a) Find an expression for $V_c(t)$ for t > 0.

(b) Find the current, i_R, in R as a function of time.

R = 14kΩ

If it is assumed that at $t = 0^-$ the switch is as shown above:

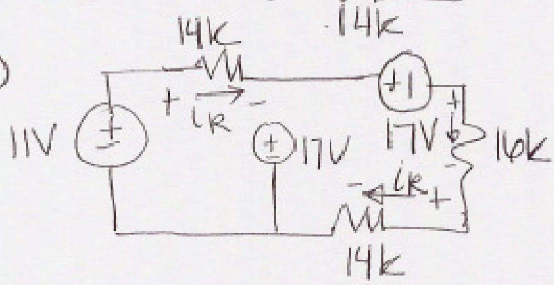
$(t = 0^-)$



$V_c = +17V$

$i_R = 0$

$(t = 0^+)$



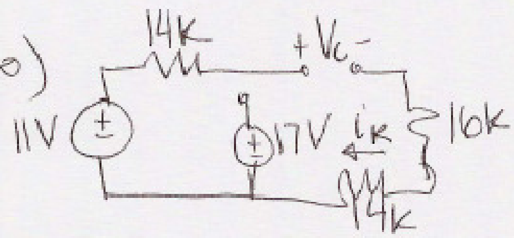
$V_c = +17V$

$+i_R(14k) + i_R(16k) + 17 + i_R(14k) - 11 = 0$

$i_R(44k) = -6$

$i_R = \frac{-6}{44k}$ (I.C.)

$(t \to \infty)$



$V_c = 11V$

$i_R = 0$

$R_{eq} = 14k + 14k + 16k = 44k$

$V_c(t) = 11 + (17 - 11)e^{-t/44k(10n)} = \boxed{11 + 6e^{-t/440\mu sec}\ V}$

$i_R(t) = 0 + \left(-\frac{6}{44k} - 0\right)e^{-t/440\mu sec}$ A

$= \boxed{-\frac{6}{44k}e^{-t/440\mu sec}\ A}$