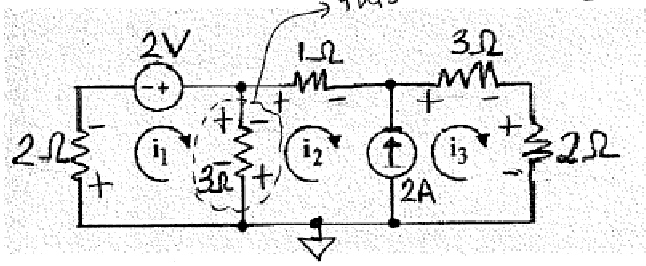
1. Use the mesh-current method to find i_1 and i_2, and i_3.

1. Label polarity

this will have 2 mesh currents through it.



Leftmost loop (through 2Ω → 2V → 3Ω):

① $-2i_1 + 2 - 3i_1 + 3i_2 = 0 \Rightarrow -5i_1 + 3i_2 + 2 = 0$

Loop through outside (2Ω → 2V → 1Ω → 3Ω → 2Ω)

② $-2i_1 + 2 - 1i_2 - 3i_3 - 2i_3 = 0 \Rightarrow -2i_1 - i_2 - 5i_3 + 2 = 0$

Loop through 3Ω → 1Ω → 3Ω → 2Ω:

③ $+3i_1 - 3i_2 - 2i_2 - 3i_3 - 2i_3 = 0 \Rightarrow 3i_1 - 5i_2 - 5i_3 = 0$

Supermesh at 2A source:

④ $2A = (i_3 - i_2)$ & i_3 + because same direction as source's

$i_3 = (2 + i_2)$

④ into ②: $-2i_1 + 2 - i_2 - 5(2 + i_2) = 0 \Rightarrow -2i_1 - 6i_2 - 8 = 0$

⑤ $i_1 = -4 - 3i_2$

⑤ into ①: $-5(-4 - 3i_2) + 3i_2 + 2 = 0 \Rightarrow +22 + 18i_2 = 0 \Rightarrow i_2 = -\frac{22}{18} = \boxed{-\frac{11}{9} = i_2}$

$i_1 = -4 + 3(\cancel{+}\frac{11}{9}) = -\frac{12}{3} + \frac{11}{3} = \boxed{-\frac{1}{3}A = i_1}$

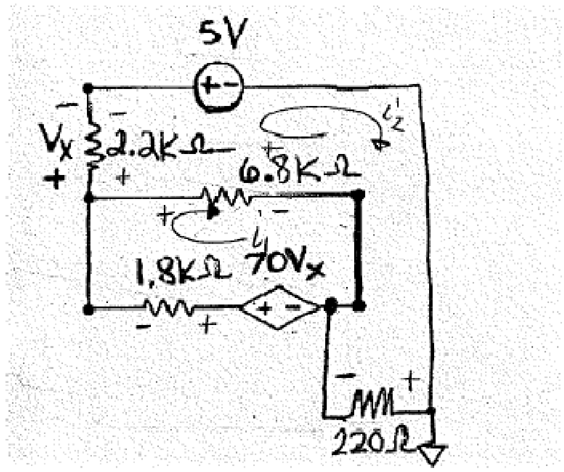
$i_3 = 2 - \frac{11}{9} = \frac{(18 - 11)}{9} = \boxed{\frac{7}{9}A = i_3}$

2. a. Use the mesh-current method to find V_x, V_x must not be in equation.

b. Find power dissipated by the dependent source.

With the labeled mesh currents shown at right:

③ $V_x = 2.2k(i_2)$



Loop through $+70V_x$, 1.8k, 6.8k:

$$① +70V_x - 1.8k\,i_1 - 6.8k\,i_1 + 6.8k\,i_2 = 0 \Rightarrow 70V_x - 8.6k\,i_1 + 6.8k\,i_2 = 0$$

Loop through $+70V_x$, 1.8k, 2.2k, 5V, 220:

$$② +70(2.2k\,i_2) - 1.8k\,i_1 - 2.2k\,i_2 - 5 - 220\,i_2 = 0$$

Loop through 6.8k, 2.2k, 5V, 220:

$$+6.8k\,i_1 - 6.8k\,i_2 - 2.2k\,i_2 - 5 - 220\,i_2 = 0$$

solving ① for i_1: $8.6k\,i_1 = (70(2.2k)+6.8k)\,i_2 = 160{,}800\,i_2$

$\therefore i_1 \doteq 18.7\,i_2$

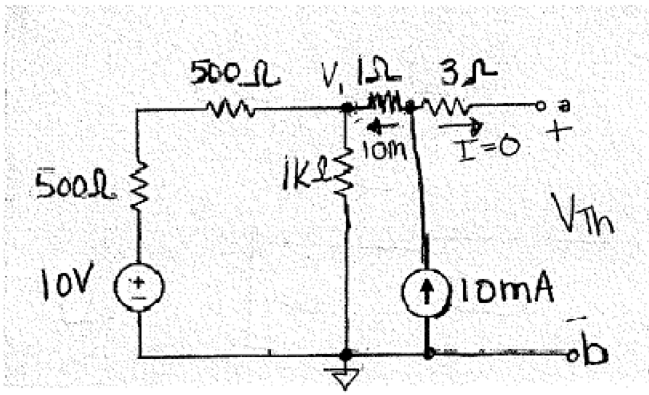
plug into ② $\Rightarrow 70(2.2k)i_2 - 1.8k(18.7)i_2 - 2.2k\,i_2 - 5 - 220\,i_2 = 0$

$i_2(154k - 33{,}660 - 2.2k - 220) = 5 \Rightarrow i_2 = \frac{5}{117{,}920} = 42.4\,\mu A$

$\therefore V_x = 2.2k(42.4\mu) = \boxed{93.3mV}$

power $= 70V_x(i_1) = 70(93.3m)(18.7)(42.4\mu) = \boxed{5.2mW}$

3. Find the Thevenin equivalent circuit at terminals a-b.



① Find V_{th}

Using node-Voltage:

$$\frac{(V_1-10)}{1k}+\frac{V_1}{1k}-10m=0$$

$$V_1\left(\frac{2}{1k}\right)=10m-10m$$

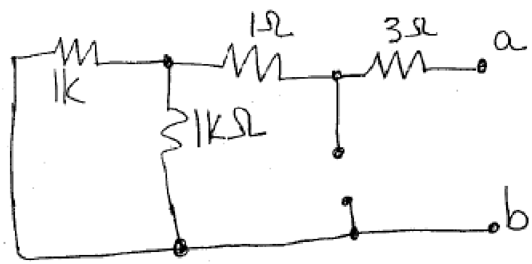
$$\therefore V_1=0$$

V loop from 1k to 1Ω, 3Ω, V_{th}:

$$+0(1k)+10m(1)-0(3)-V_{Th}=0$$

$$\boxed{V_{Th}=10mV}$$

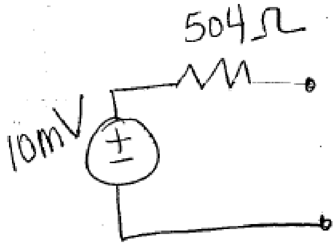
② Redraw circuit for R_{th}: (I=0 looks like open, V=0 short)



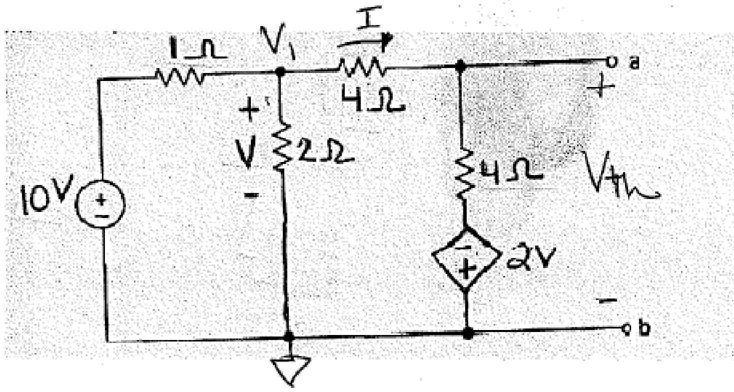
$$\Rightarrow R_{eq}=3+1+(1k\|1k)$$

$$R_{eq}=4+\frac{(1k)(1k)}{2k}=4+500=$$

$$\boxed{R_{th}=504\Omega}$$



4. Find the Thevenin equivalent circuit at terminals a-b.



① Find V_{th}:

·Using node voltage: $\frac{(V_1-10)}{1}+\frac{V_1}{2}+\frac{(V_1-(-2V))}{8}=0$

$V_1\left(1+\frac{1}{2}+\frac{1}{8}\right)=+10-\frac{2(V)}{8}$

writing V in terms of V_1: $V=V_1$

$V_1\left(\frac{8}{8}+\frac{4}{8}+\frac{1}{8}\right)+V_1\frac{2}{8}=+10$

$V_1\left(\frac{15}{8}\right)=+10$

$V_1=\frac{10(8)}{15}=\frac{2(8)}{3}=\boxed{\frac{16}{3}V}$

current through 4Ω:

$I=\left(\frac{V_1+2V}{8}\right)=\frac{3V_1}{8}=\frac{3(16)}{3(8)}=2A$

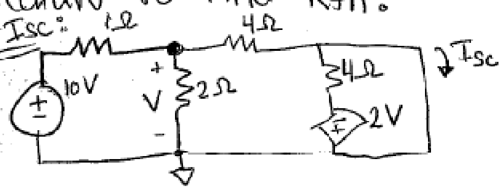
$+V_1-2(4)-V_{th}=0$

$V_{th}=V_1-8$

$V_{th}=\frac{16}{3}-\frac{24}{3}=\boxed{-\frac{8}{3}V}$

② Redraw to find R_{th}:

method for I_{sc}:



$I_{sc}=\frac{V}{4}$

using node voltage: $\frac{(V-10)}{1}+\frac{V}{2}+\frac{V}{4}=0$

$V\left(1+\frac{1}{2}+\frac{1}{4}\right)=10$

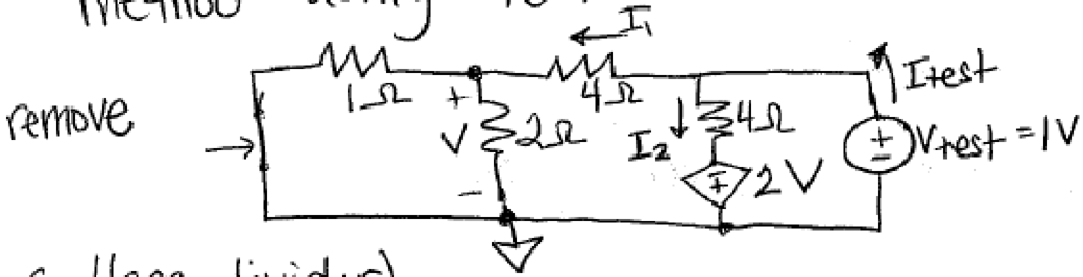
$V\left(\frac{4+2+1}{4}\right)=10 \Rightarrow V=\frac{10}{7}\cdot 4=\frac{40}{7}$

∴ $I_{sc}=\frac{40}{7(4)}=\frac{10}{7}$

$R_{th}=\frac{V_{th}}{I_{sc}}=\frac{-8}{3}\cdot\frac{7}{10}=\boxed{\frac{-28}{15}\Omega}$

② Redraw to find R_{th}:

method using test source ⇒



(voltage-divider):

$$V = \frac{(2\|1)(1)}{(2\|1)+4} = \frac{\frac{2(1)}{3}(1)}{\frac{2}{3}+\frac{12}{3}} = \frac{2}{3}\cdot\frac{3}{14} = \frac{1}{7}V$$

$$I_2 = \frac{1-(-2V)}{4} = \frac{1+2(\frac{1}{7})}{4} = \frac{7+2}{7(4)} = \frac{9}{28}$$

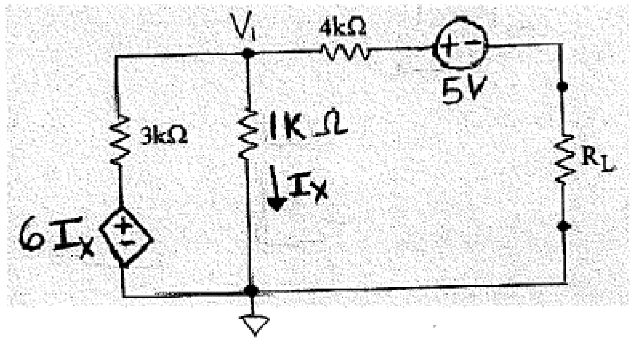
$$I_{test} = I_1 + I_2$$

$$I_1 = \frac{1}{4+\frac{2}{3}} = \frac{1}{\frac{12}{3}+\frac{2}{3}} = \frac{3}{14}$$

$$I_{test} = \frac{3}{14} + \frac{9}{28} = \frac{6+9}{28} = \frac{15}{28}$$

$$R_{th} = \frac{V_{test}}{I_{test}} = \frac{1}{\frac{15}{28}} = \boxed{\frac{28}{15}\Omega}$$

5. Determine the power in the dependent source if R_L =2kΩ



using node voltage:

$$\frac{V_1 - 6I_x}{3k} + \frac{V_1}{1k} + \frac{V_1 - 5}{4k+2k} = 0$$

$$I_x = \frac{V_1}{1k}$$

$$\frac{V_1}{3k} - \frac{6V_1}{3k(1k)} + \frac{V_1}{1k} + \frac{V_1}{6k} - \frac{5}{6k} = 0$$

$$V_1\left(\frac{2k-2(6)+6k+1k}{6k(1k)}\right) = \frac{5}{6k}$$

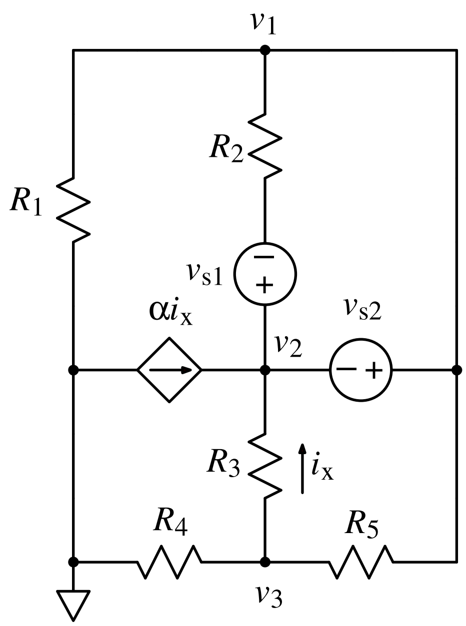
$$V_1\left(\frac{9k-12}{6k(1k)}\right) = \frac{5}{6k(1k)} \Rightarrow \quad V_1 = \frac{5}{6k}\cdot\frac{6k(1k)}{(9k-12)} \approx 556.3\,mV$$

$$I_x = \frac{556.3m}{1k} = .5563mA$$

$$\therefore \text{power} = 6(I_x) * \frac{V_1 - 6I_x}{3k} = 6(.5563mA) * \frac{556.3m - 6(.5563m)}{3k}$$

$$\text{power} = \boxed{0.62\mu W}$$

6.



For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity i_x must not appear in the equations.

sol'n: a) We first define the dependent variable for the dependent source in terms of node voltages.

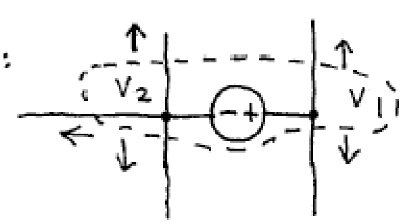
$$i_x = \frac{v_3 - v_2}{R_3}$$

Next, we look for supernodes. We have a supernode between v_1 and v_2. (v_1 is right side of circuit.)

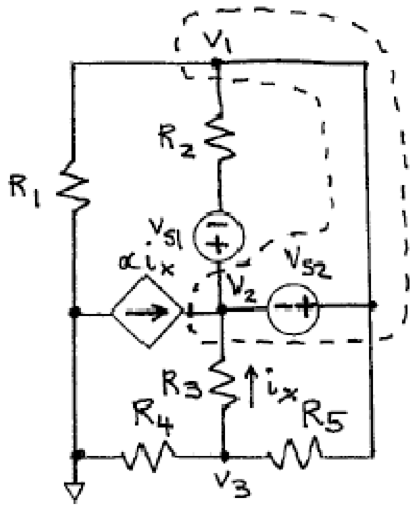
For the supernode, we have a voltage eq'n and a current summation eq'n.

$\boxed{v_1, v_2 \text{ v-eq'n:} \quad v_1 - v_{s2} = v_2}$

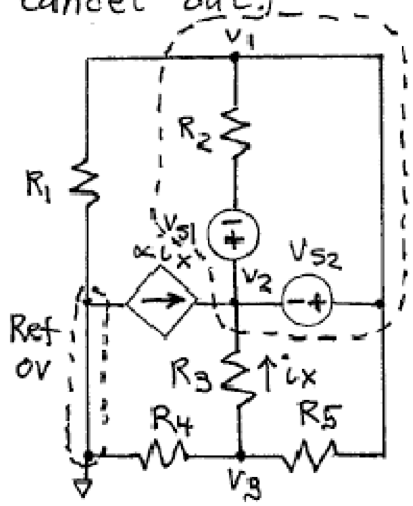
v_1, v_2 i-sum:



At first glance, we might try to use the above dashed bubble, but v_1 includes the node at the top of the circuit.



Since the bubble includes both ends of the vertical branch consisting of R_2 and v_{s1}, we may extend the bubble to include them. (The currents going up and down in this branch will cancel out.)



i-sum eq'n:

$$\frac{v_1}{R_1} - \alpha\left(\frac{v_3 - v_2}{R_3}\right) + \frac{v_2 - v_3}{R_3} + \frac{v_1 - v_3}{R_5} = 0\,\text{A}$$

Finally, we have the i-sum eq'n for v_3

$$\boxed{\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_5} = 0\,\text{A}}$$

7. Make a consistency check on your equations for Problem 6 by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and node voltages for your consistency check, and show that your equations for problem 6 are satisfied for these values. (In other words, plug the values into your equations for problem 1(a) and show that the left side and the right side of each equation are equal.)

For the consistency check, we pick component values that simplify the circuit so we can solve it by inspection.

There are many possible solutions. Here, we consider one example.

Suppose $R_4 = \infty\ \Omega$ (open circuit)

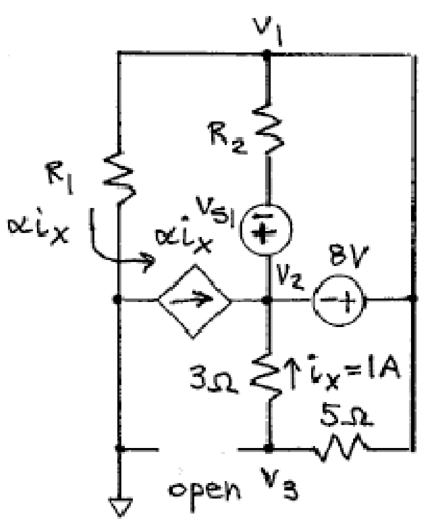
$R_3 = 3\ \Omega$

$R_5 = 5\ \Omega$

$V_{S2} = 8\ V$

These choices give an exact value for i_x:

$$i_x = \frac{8V}{3\Omega + 5\Omega} = 1A$$



We now observe that αi_x flows thru R_1, allowing us to determine v_1, (by Ohm's law).

Suppose $R_1 = 1\,\Omega$
$\alpha = 6$

We have $v_1 = \alpha i_x R_1 = 6 \cdot 1A \cdot 1\,\Omega = 6V$

$v_2 = v_1 - 8V$ (because of 8V src)
or $v_2 = 6V - 8V = -2V$
$v_3 = v_2 + i_x \cdot R_3 = -2V + 1A \cdot 3\,\Omega = 1V$

For completeness, we set $R_2 = 2\,\Omega$
$v_{s1} = +12V$

Now we plug our component and node-voltage values into our eq'ns from part (a) to verify that equality is satisfied.

v_1, v_2 v-eq'n: $6V - 8V \overset{?}{=} -2V$ ✓

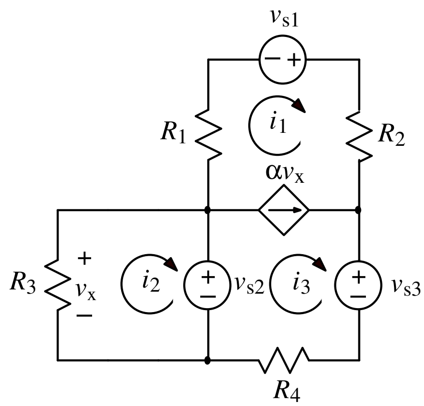
v_1, v_2 i-sum: $\cancel{\dfrac{6V}{1\Omega}} - 6\left(\cancel{\dfrac{\overbrace{1V - -2V}^{3V}}{3\,\Omega}}\right) + \cancel{\dfrac{\overbrace{-2V - 1V}^{-3V}}{3\,\Omega}}$

$+ \cancel{\dfrac{\overbrace{6V - 1V}^{5V}}{5\,\Omega}} \overset{?}{=} 0A$ ✓

v_3 i-sum: $\cancel{\dfrac{1V}{\infty\,\Omega}}^{\,0} + \cancel{\dfrac{\overbrace{1V - -2V}^{3V}}{3\,\Omega}} + \cancel{\dfrac{\overbrace{1V - 6V}^{-5V}}{5\,\Omega}} \overset{?}{=} 0A$ ✓

The eq'ns are all satisfied.

8.



For the circuit shown, write three independent equations for the three mesh currents i1, i2, and i3. The quantity vx must not appear in the equations.

sol'n: We first write the dependent variable for the dependent source in terms of mesh currents.

$$v_x = -i_2 \cdot R_3$$

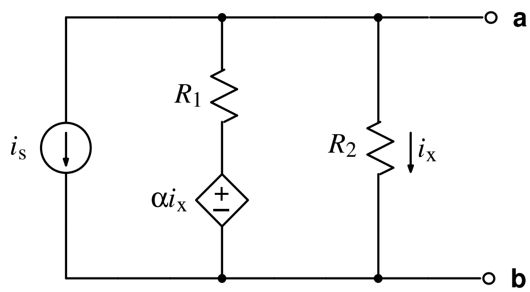
Next, we look for supermeshes. Here, we have a supermesh consisting of the i_1 and i_3 loops.

i_1, i_3 v-loop: (around outside of i_1 and i_3 loops) $v_{s2} - i_1 R_1 + v_{s1} - i_1 R_2 - v_{s3} - i_3 R_4 = 0\text{V}$

i-eqn for i_1, i_3 loops: $-i_1 + i_3 = \alpha(-i_2 \cdot R_3)$

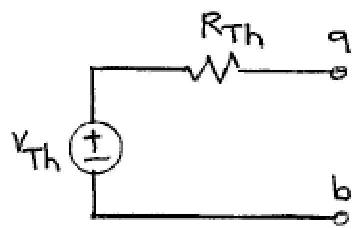
i_2 loop: $-i_2 R_3 - v_{s2} = 0\text{ V}$

9.



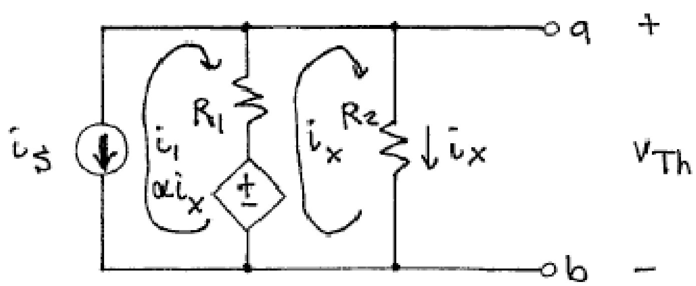
Find the Thevenin equivalent circuit at terminals **a** and **b**. i_X must not appear in your solution. **Note:** $\alpha > 0$.

sol'n: The Thevenin equivalent consists of a voltage source and a resistor.



V_{Th} equals the voltage across a,b (with nothing connected).

The mesh-current method is convenient here.



We have $i_1 = -i_s$ since i_s is on the outside edge of the circuit.

For the right loop, we have the following voltage-drop eq'n:

$$\alpha i_x - i_x R_1 + \underset{\substack{\parallel \\ -i_S}}{i_1} R_1 - i_x R_2 = 0V$$

or $$i_x\left[\alpha - (R_1 + R_2)\right] = i_S R_1$$

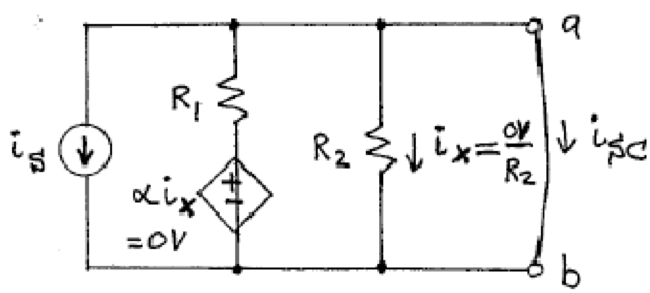
or $$i_x = \frac{i_S R_1}{\alpha - (R_1 + R_2)}$$

By Ohm's law, we obtain v_{Th}:

$$\boxed{v_{Th} = i_x R_2 = \frac{i_S R_1 R_2}{\alpha - (R_1 + R_2)}}$$

To find R_{Th}, we can short **a** to **b**, find the short-circuit current, i_{sc}, and compute R_{Th} by Ohm's law:

$$R_{Th} = \frac{v_{Th}}{i_{sc}}$$



Because R_2 is by-passed by a wire, R_2 has no voltage across it and has no current:

$$i_x = 0\ A$$

Thus, $\alpha i_x = 0V.$

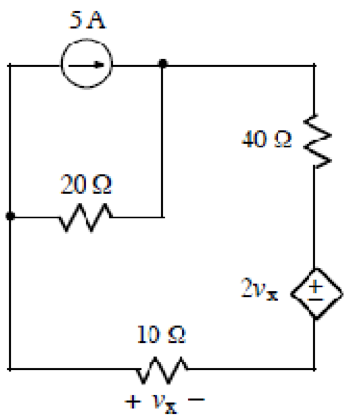
Now we observe that R_1 is also by-passed by the wire from a to b and carries no current.

It follows that $i_{sc} = -i_s$.

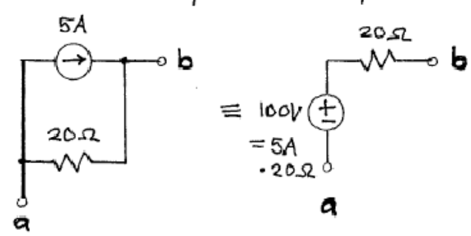
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{\dfrac{i_s R_1 R_2}{\alpha - (R_1 + R_2)}}{-i_s}$$

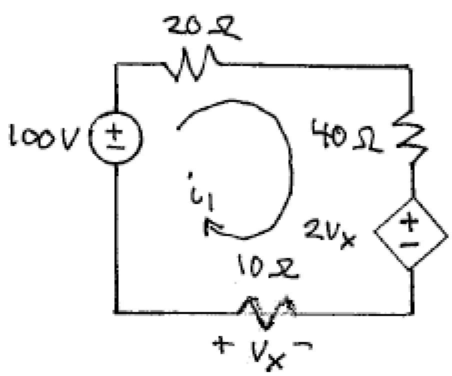
$$\boxed{R_{Th} = \frac{R_1 R_2}{R_1 + R_2 - \alpha}}$$

10. Calculate the power consumed (i.e., dissipated) by the $2v_X$ dependent source. **Note:** If a source supplies power, the power it consumes is negative.



sol'n: We can solve this problem by any method we choose. Node-voltage or mesh-currents could be used, but converting the upper lefthand corner to a Thevenin equivalent yields a simple v-loop.





Now use mesh-current i_1:

$$v_x = -i_1 \cdot 10\,\Omega$$

$$100\,\text{V} - i_1 \cdot 20\,\Omega - i_1 \cdot 40\,\Omega - 2(-i_1 \cdot 10\,\Omega) - i_1 \cdot 10\,\Omega = 0\,\text{V}$$

or $i_1(20\Omega + 40\Omega - 20\Omega + 10\Omega) = 100V$

or $i_1 = \frac{100V}{50\Omega} = 2A$

It follows that $v_x = -2A \cdot 10\Omega = -20V.$

The power for the dependent src is

$$p = 2v_x \cdot i_1 = 2(-20V)\,2A = -80W$$

$$\boxed{p = -80W}$$