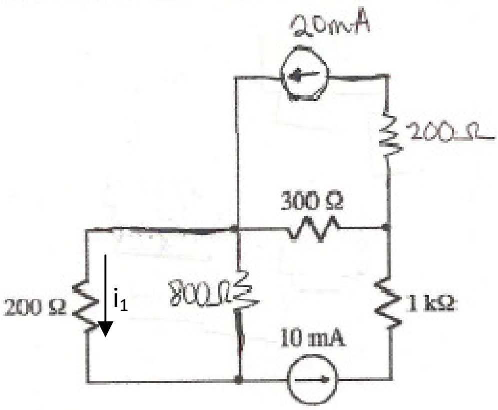
1.



Calculate i_1.

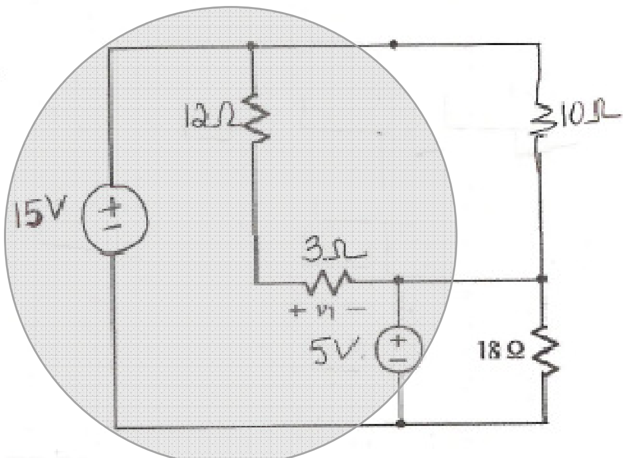
Calculate i_1.

The 10 mA source provides all the current flowing through the 200Ω in parallel with 800Ω. (Note that 10mA flows through the 300ohm from left to right so 10mA will divide through the 200 and 800ohm). Thus, the current divider formula gives the value of *i*1:

$$i_1 = \frac{10m(800)}{(800+200)} = \frac{10m(800)}{1k}$$

$$i_1 = 8mA$$

2.



Calculate v_1.

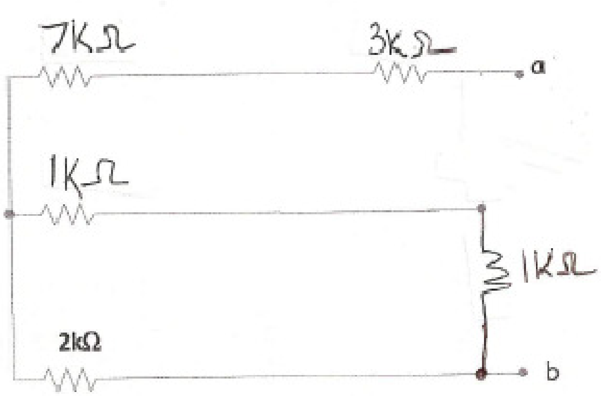
Calculate V_1.

If we follow the wires in the circuit, the 12Ω is in series with the 3Ω, and the 5V source is in series with 15V which is across (highlighted). Thus, the value of *V*1 is given by the voltage-divider formula:

$$V_1 = \frac{10(3)}{12+3} = \frac{30}{15}$$

$$V_1 = 2V$$

3. Find the value of total resistance between terminals **a** and **b**.



Rab=3k+7k+(1k||1k)||2k

$$Rab = 10k + \frac{(1k)(1k)}{1k + 1k}||2k$$

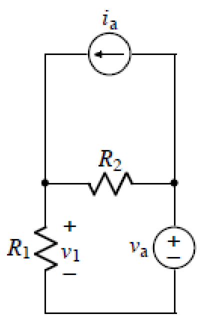
$$Rab = 10k + 2k||2k$$

$$Rab = 10k + \frac{(2k)(2k)}{2k + 2k}$$

$$Rab = 10k + 1k$$

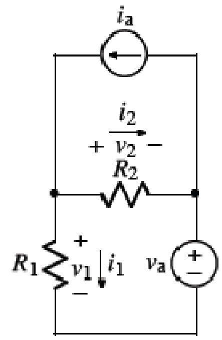
$$Rab = 11{,}000\Omega$$

4.



Derive an expression for v_1. The expression must not contain more than the circuit parameters i_a, v_a, R_1, and R_2.

SOL'N: a) The voltage drops and currents may be measured in one of two directions for each resistor so long as we follow the passive sign convention. One such consistent labeling is shown below.



We observe that no components are in series and carrying the same current. (This is always a necessary thing to check for, however, as we need such equations when there are components in series.)

Looking for voltage-loops, we find that only the lower loop avoids the current source.

$$i_1R_1 - i_2R_2 - v_a = 0 \text{ V}$$

A current summation at the left node gives a second equation for i_1 and i_2:

$$-i_a + i_2 + i_1 = 0 \text{ A}$$

Solving this equation for i_2 in terms of i_1 yields the following result:

$$i_2 = i_a - i_1$$

Substituting for i_2 in the previous equation yields an equation in terms on only i_1:

$$i_1R_1 - (i_a - i_1)R_2 - v_a = 0 \text{ V}$$

or

$$i_1(R_1 + R_2) = v_a + i_aR_2$$

or

$$i_1 = \frac{v_a + i_aR_2}{R_1 + R_2}$$

By Ohm's law, $v_1 = i_1R_1$.

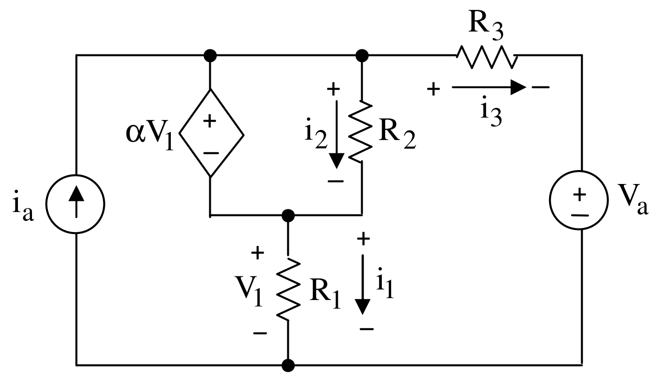
$$i_1 = (v_a + i_aR_2)\frac{R_1}{R_1 + R_2}$$

5. Derive an expression using the circuit in Problem #4 above for the power through R2 resistor. The known values are i_a, V_a, R_1, and R_2.

$i_2 = i_a$-i_1 power = $i_2.i_2.R_2 = (i_a - i_1)^2R_2 = i_a^2R_2 - 2i_ai_1R_2 + i_1^2R_2 =$

power= $= i_a^2R_2 - 2i_a(v_a + i_aR_2)\frac{R_1}{R_1+R_2}R_2 + (v_a + i_aR_2)^2\left(\frac{R_1}{R_1+R_2}\right)^2 R_2$

6.



Derive the expression for V_1 containing not more than circuit parameters α, R_1, R_2, R_3, V_a, and i_a.

The above circuit is labeled with current directions and polarity. Using these labeled currents, there are 3 voltage loops:

$$+V_1 + \alpha V_1 - i_3 R_3 - V_a = 0$$
$$+i_1 R_1 + i_2 R_2 - i_3 R_3 - V_a = 0$$
$$\alpha V_1 - i_2 R_2 = 0$$

Taking a current summation:

$$+i_a - i_1 - i_3 = 0$$
$$i_3 = (i_a - i_1)$$

Taking the first equation from the voltage loops and plugging in the above value for i_3:

$$V_1(\alpha+1) - i_a R_3 + i_1 R_3 - V_a = 0$$

Using Ohm's Law in this equation for V_1:

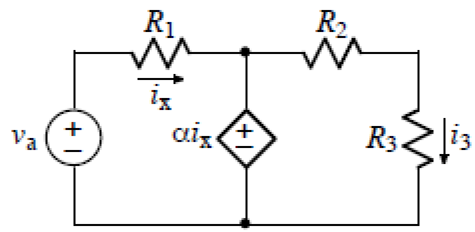
$$V_1 = i_1 R_1$$
$$i_1 R_1(\alpha+1) + i_1 R_3 = i_a R_3 + V_a$$
$$V_1 = i_1 R_1 = \left[\frac{i_a R_3 + V_a}{R_3 + R_1(\alpha+1)}\right] R_1$$

7. Using the circuit shown in Problem #6, derive an expression for the power through R2. The known values are α, i_a, V_a, R_1, R_2 and R_3.

Using the value found in #6 and solving for the Power:

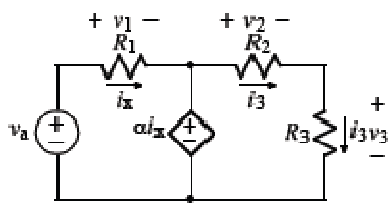
$$P = i_1 \times V_1 = \left[\frac{(i_a R_3 + V_a)}{(R_3 + R_1(\alpha+1))}\right]^2 R_1$$

8.



Derive an expression for i_3. The expression must not contain more than the circuit parameters α, v_a, R_1, R_2, and R_3. **Note:** $\alpha > 0$.

Sol'n: We add to the circuit diagram labels that are consistent with the passive sign convention:



We look for components in series and find that R_2 and R_3 are in series.

Turning to voltage loops, we have valid voltage loops on the left side and right side. The left side yields the following equation:

$$v_a - i_x R_1 - \alpha i_x = 0 \text{ V}$$

We can solve this equation for i_x immediately:

$$i_x = \frac{v_a}{R_1 - \alpha}$$

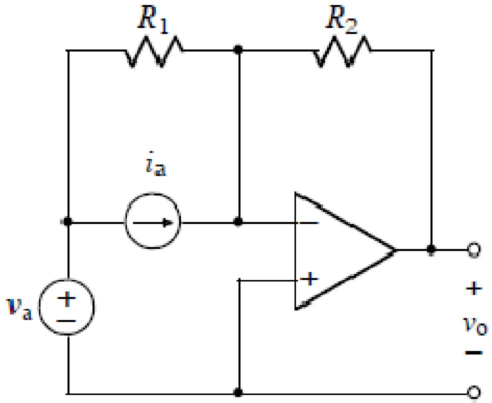
The right side yields the following equation:

$$\alpha i_x - i_3 R_2 - i_3 R_3 = 0 \text{ A}$$

Using the value of i_x from above, we can solve for i_3:

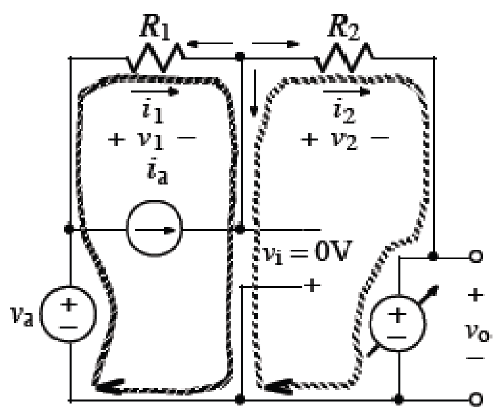
$$i_3 = \frac{\alpha v_a}{(R_1 - \alpha)(R_2 + R_3)}$$

9.



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for v_o in terms of not more than i_a, v_a, R_1, and R_2.

SOL'N: We first remove the op-amp and assume the op-amp output voltage has the value necessary to make the voltage drop across the op-amp inputs equal zero volts. One possible way of labeling the resulting circuit, consistent with the passive sign convention, is shown below.



Looking first for components in series that carry the same current, we find only R_2 and v_o and v_a, which is if little help since we avoid defining a current for a voltage source.

We move on to voltage loops. We must use a loop that passes through $v_i = 0$ V, if possible. On the left, we have a voltage loop that skips v_a to pass through R_1 and then through v_i.

$$v_a - i_1 R_1 + v_i = 0 \text{ V}$$

Note that we use Ohm's law to write v_1 as $i_1 R_1$ and eliminate v_1 immediately. Since $v_i = 0$ V, we may solve for i_1.

$$i_1 = \frac{v_a}{R_1} \qquad (1)$$

On the right side, we have a second voltage loop.

$$-v_i - i_2 R_2 - v_o = 0 \text{ V} \qquad (2)$$

Now we look for a node where we can do a current summation. The top node is the most obvious node where we might have three nonzero currents without having to define a current for a voltage source. (The node on the left side would also work if we observe that i_2 flows in v_a.)

$$-i_1 - i_a + i_2 = 0 \text{ A} \qquad (3)$$

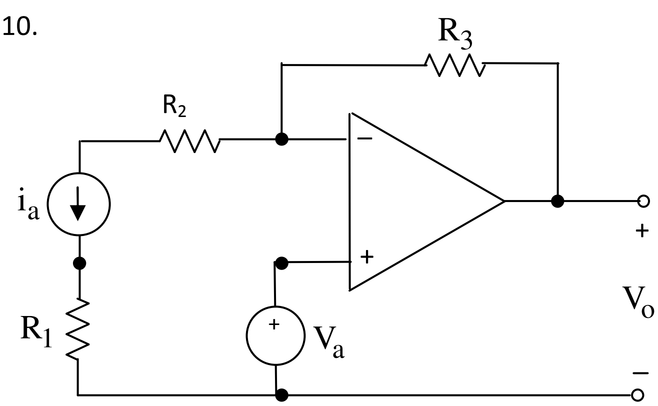
Using the value of i_1 from (1), we can solve (3) for i_2:

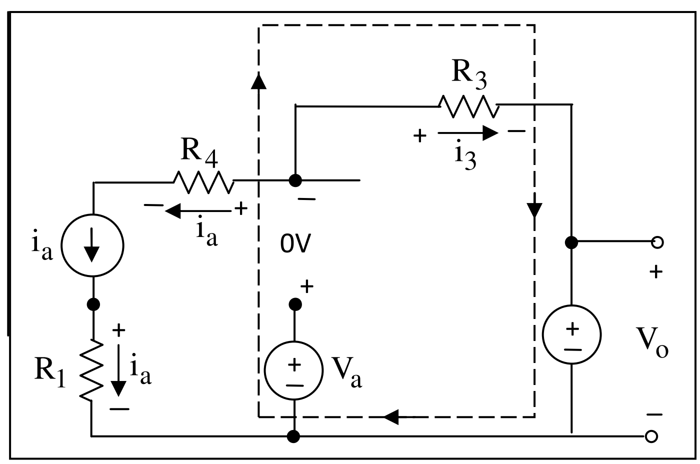
$$i_2 = i_a + \frac{v_a}{R_1}$$

Using this value for i_2 and using $v_o = 0$ V we can solve (2) for v_o:

$$v_o = -i_2 R_2 = -\left(i_a + \frac{v_a}{R_1}\right) R_2$$

10.





The op-amp operates is in the linear mode. Using an appropriate model of the op-amp, derive an expression for V_o in terms of not more than i_a, R_1, R_2, R_3, and V_a.

We first remove the op-amp and assume the op-amp output voltage has the value necessary to make the voltage drop across the op-amp inputs equal zero volts. One possible way of labeling the resulting circuit, consistent with the passive sign convention, is shown above (right).

Looking first for components in series that carry the same current, we see that R_4 and R_3 have equal but opposite currents:

$$i_3 = -i_a$$

Next, we look for voltage loops, making sure we use the 0 V drop across the op-amp inputs at least once. The small voltage loop shown on the diagram above yields the following Equation (using the current through R_3 as i_a:

$$+V_a + i_a R_3 - V_o = 0$$

Solving for Vo:

$$\boxed{V_o = V_a + i_a R_3}$$