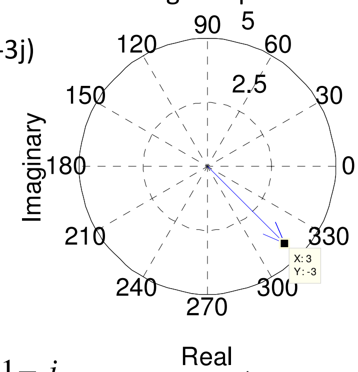
ECE 1270 THE UNIVERSITY OF UTAH

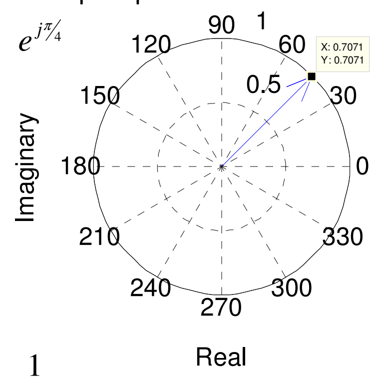
Homework #7 Solution

1. Plot each of the following complex numbers as vector in the complex plane:

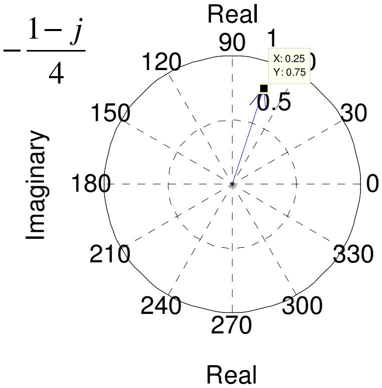
   a. (3-3j)



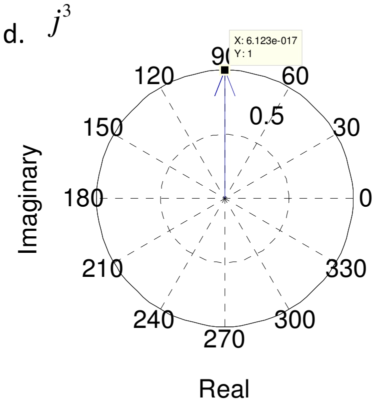
   b. $e^{j\pi/4}$



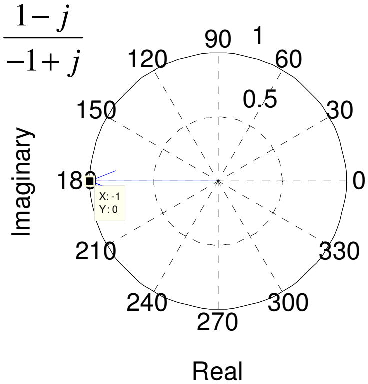
   c. $\dfrac{1+j}{2}-\dfrac{1-j}{4}$



   d. $\dfrac{1}{j^3}$



   e. $\dfrac{1-j}{-1+j}$



2. Give numerical answers to each of the following questions:

   a. Rationalize $\dfrac{-80-j60}{28-j96}$. Express your answer in rectangular form.

$$\frac{-80-j60}{28-j96}=\frac{-80-j60}{28-j96}\cdot\frac{28+j96}{28+j96}=\frac{3520-j9360}{10k}=0.352-j0.936$$

b. Find the polar form of $(1+j)^*\left(\sqrt{1+\frac{\sqrt{3}}{2}}-j\sqrt{1-\frac{\sqrt{3}}{2}}\right)^*$ (Note: The asterisk means conjugate.)

$$(1+j)^*\left(\sqrt{1+\frac{\sqrt{3}}{2}}-j\sqrt{1-\frac{\sqrt{3}}{2}}\right)^*$$

$$=(1-j)\left(\sqrt{1+\frac{\sqrt{3}}{2}}+j\sqrt{1-\frac{\sqrt{3}}{2}}\right)$$

$$=\sqrt{2}\angle-45°\cdot\sqrt{\sqrt{1+\frac{\sqrt{3}}{2}}^{\,2}+\sqrt{1-\frac{\sqrt{3}}{2}}^{\,2}}\angle\tan^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{3}}{2}}}{\sqrt{1+\frac{\sqrt{3}}{2}}}\right)$$

$$=\sqrt{2}\angle-45°\cdot\sqrt{1+\frac{\sqrt{3}}{2}+1-\frac{\sqrt{3}}{2}}\angle\tan^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{3}}{2}}}{\sqrt{1+\frac{\sqrt{3}}{2}}}\right)$$

$$=\sqrt{2}\angle-45°\cdot\sqrt{2}\angle15° = 2\angle-30°$$

c. Find the following phasor: $\mathrm{P}\left[3\sin(25kt-120^o)\right]$.

$$\mathrm{P}[3\sin(25kt-120°)] = -j3\angle-120° = 1\angle-90°\cdot3\angle-120°$$

$$=3\angle-210° = 3\angle150°$$

d. Find the magnitude of $\dfrac{(1-j7)2e^{-j10^o}}{1-e^{j90^o}}$.

d) The magnitude of a product or quotient is the product or quotient of the magnitudes:

$$\left|\frac{(1-j7)2e^{-j10°}}{1-e^{j90°}}\right| = \frac{|1-j7||2||e^{-j10°}|}{|1-e^{j90°}|}$$

$|e^{-j\theta}| = 1$ for any real θ, and $e^{j90°} = j$:

$$\left|\frac{(1-j7)2e^{-j10°}}{1-e^{j90°}}\right| = \frac{\sqrt{1^2+7^2}\cdot2\cdot1}{|1-j|}$$

or

$$\left|\frac{(1-j7)2e^{-j10°}}{1-e^{j90°}}\right| = \frac{\sqrt{50}\cdot2\cdot1}{\sqrt{2}} = 10$$

e. Find the imaginary part of $\frac{1+j\sqrt{3}}{e^{-j30^o}}$.

$$\mathrm{Im}\left[\frac{1+j\sqrt{3}}{e^{-j30^\circ}}\right] = \mathrm{Im}\left[\frac{1+j\sqrt{3}}{\frac{\sqrt{3}}{2}-j\frac{1}{2}}\right] = \mathrm{Im}\left[\frac{1+j\sqrt{3}}{\frac{\sqrt{3}}{2}-j\frac{1}{2}}\cdot\frac{\frac{\sqrt{3}}{2}+j\frac{1}{2}}{\frac{\sqrt{3}}{2}+j\frac{1}{2}}\right]$$

or

$$\mathrm{Im}\left[\frac{1+j\sqrt{3}}{e^{-j30^\circ}}\right] = \mathrm{Im}\left[\frac{\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}+j\left(\frac{3}{2}+\frac{1}{2}\right)}{1}\right]$$

or

$$\mathrm{Im}\left[\frac{1+j\sqrt{3}}{e^{-j30^\circ}}\right] = \mathrm{Im}\left[j2\right] = 2$$

3. a. Write phasors (as both $Ae^{j\phi}$ and $A\angle\phi$) for each of the following signals:

i. v(t)=9cos(2kt+30^o)V => **V=9e^{j30}**

ii. i(t)=2sin(ωt+10^o)mA=2cos(ωt-90^o +10^o)mA => **I=2e^{-j80}**

iii. v(t)=cos(5t+30^o)V+5sin(5t-30^o)V=

V=e^{j30}+5e^{j-120}=cos(30)+jsin(30)+5cos(-120)+jsin(-120)= -1.6340 - 3.8301j

=> **V=4.2e^{j247}**

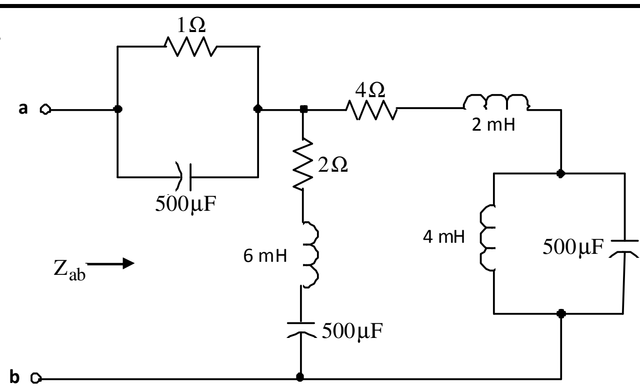
b. Given w=3krad/sec, write inverse phasors for each of the following signals:

i. **I**= $34e^{j20^o}$ A => i(t)=34cos(3kt+20^o)A

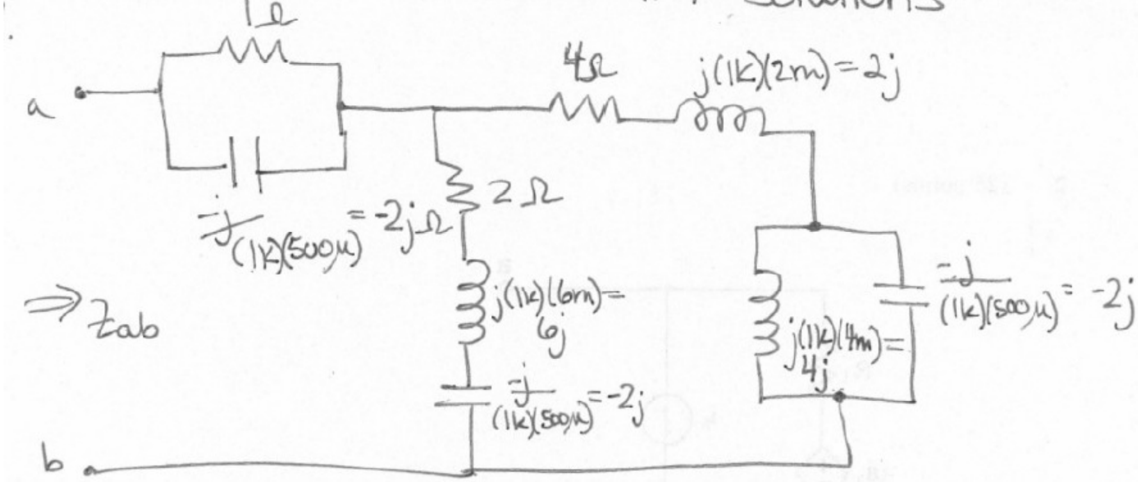
ii. **V**= $-j^3$ V => v(t)=cos(3kt+90^o)V

iii. **I**= $3e^{+\pi-j20^o}$ A => i(t)= $3e^{+\pi}\cos\left(3kt-20^o\right)$ A

4. Given $\omega = 1k$ rad/sec, find Z_{ab}.



Homework #7 Solutions



$$Z_{ab} = (1 \| -2j) + (2+6j-2j) \,\|\, [(4+2j) + (4j \| -2j)]$$

$$Z_{ab} = \frac{1(-2j)}{1-2j} + (2+4j) \,\|\, \left(4+2j + \frac{4j(-2j)}{4j-2j}\right) = \frac{-2j}{1-2j} + (2+4j) \,\|\, \left(4+2j+\overbrace{\frac{8}{2j}}^{-4j}\right)$$

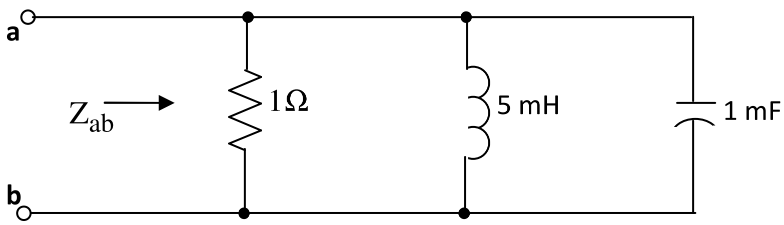
$$Z_{ab} = \frac{-2j(1+2j)}{(1-2j)(1+2j)} + \frac{(2+4j)(4-2j)}{(2+4j+4-2j)} = \frac{-2j+4}{5} + \frac{8-4j+16j-8j^2}{6+2j}$$

$$Z_{ab} = -\frac{2}{5}j + \frac{4}{5} + \frac{16+12j}{6+2j} \cong -\frac{2}{5}j + \frac{4}{5} + \frac{20e^{j37°}}{2\sqrt{10}\, e^{j18°}} = -\frac{2}{5}j + \frac{4}{5} + 3.16e^{j19°}$$

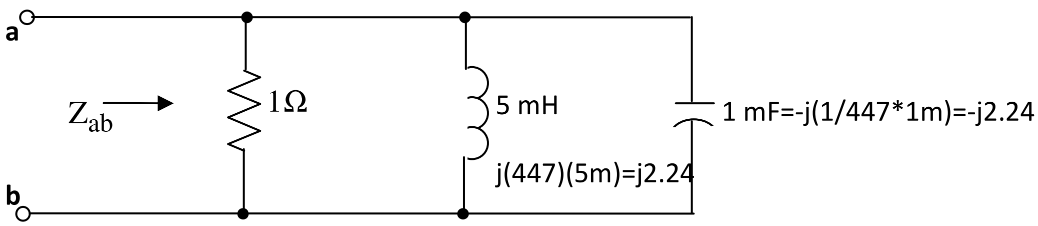
$$Z_{ab} = -\frac{2}{5}j + \frac{4}{5} + 3.16\cos(19°) + 3.16j\sin(19°)$$

$$Z_{ab} \cong -\frac{2}{5}j + \frac{4}{5} + 3 + j = \boxed{(3.8+0.6j)\,\Omega}$$

5.



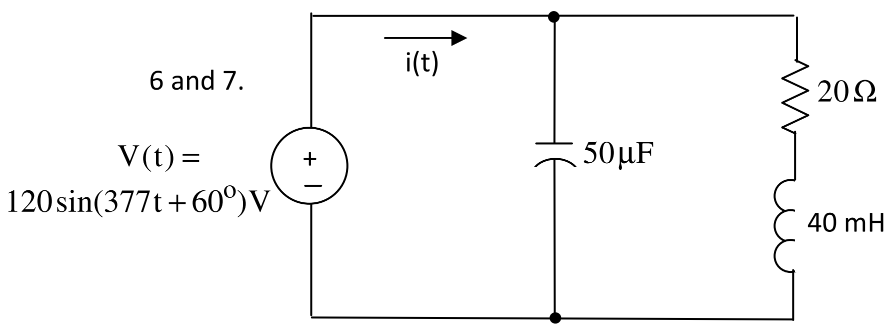
Given $\omega = 447$ rad/sec, find Z_{ab}.



$$Z_{ab} = 1 \,||\, j2.24 \,||\, -j2.24 = \frac{1}{\frac{1}{1} + \frac{1}{j2.24} + \frac{1}{-j2.24}} = \frac{1}{\frac{-j2.24}{-j2.24} + \frac{-1}{-j2.24} + \frac{1}{-j2.24}}$$

$$= \frac{1}{\frac{-j2.24}{-j2.24}} = 1$$

6 and 7.



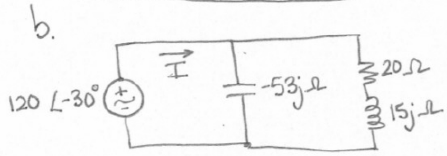
a. Find the phasor value for V(t).

b. Draw the frequency-domain circuit diagram, including the phasor value for V(t) and the impedance values for components.

a. $v(t) = 120\sin(377t + 60^\circ) = 120\cos(377t + 60^\circ - 90^\circ)$

$\boxed{V = 120\angle -30^\circ}$ or $V = 120e^{-j30^\circ}$

b.



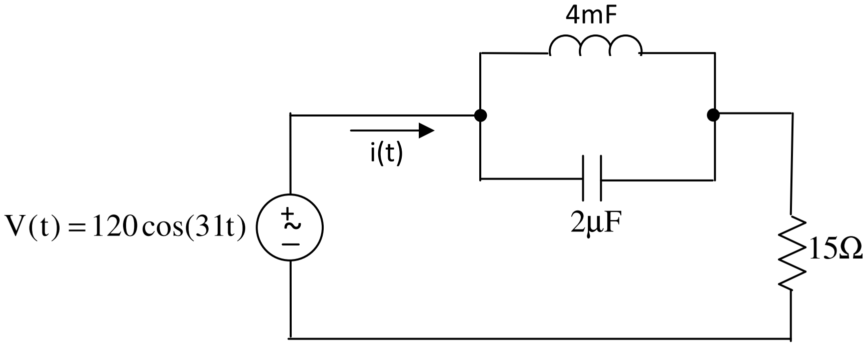
$$I = \frac{120\angle -30^\circ}{\underbrace{-53j \,||\, (20+15j)}_{Z_{eq.}}}$$

$$Z_{eq.} = \frac{-53j(20+15j)}{20-53j+15j} = \frac{-1060j + 795}{(20-38j)} = \frac{1325\angle -53^\circ}{43\angle -62^\circ} = 30.8\angle 9^\circ$$

$$I = \frac{120\angle -30^\circ}{30.8\angle 9^\circ} \cong 4\angle(-30^\circ - 9^\circ) = \boxed{4\angle -39^\circ} \text{ or } 4e^{-j39^\circ} \text{ OR } 4e^{j321^\circ}$$

$i(t) \cong 4\cos(377t - 39^\circ)$ A

8 and 9.



a. Find the phasor value for V(t).

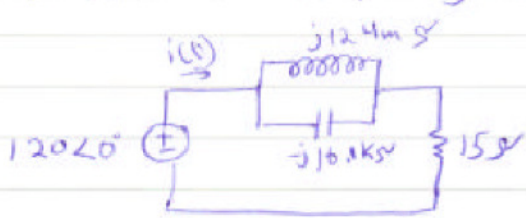
b. Draw the frequency-domain circuit diagram, including the phasor value for V(t) and the impedance values for components.

9. Find the phasor value for i(t) for the circuit in Problem 8.

a) Find the phasor value for V(t)

$V(t) = 120\cos(31t) = 120\angle 0^\circ$ V

b) Draw the frequency-domain circuit diagram



$Z_L = j\,31 \times 4m = j124m\ \Omega$

$Z_C = \frac{-j}{31 \times 2\mu} = -j16.1k\ \Omega$

c) $i(t) = \frac{V(t)}{Z}$

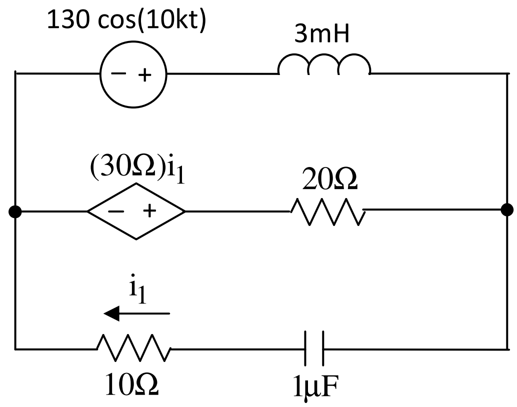
$Z = (Z_L \parallel Z_C) + 15$

$= \frac{-j16.1k \times j124m}{j124m - j16.1k} + 15 = \frac{2k}{-j16.1k} + 15 = j0.124 + 15$

$Z = 15e^{j0.473}$

$I = 120/Z = 8e^{-j0.473}$

10. Find $i_1(t)$.



Take the lower loop

$$(30\Omega)i_1 - 20 i_2 - \frac{-j}{1\mu \times 10k} i_1 - 10 i_1 = 0$$

$$(30\Omega)i_1 - 20 i_2 + j100\, i_1 - 10 i_1 = 0$$

$$20 i_2 = 30 i_1 - 10 i_1 + j100\, i_1$$

$$i_2 = i_1 + j5 i = i_1\, 5.1 \angle \tan^{-1}\left(\frac{5}{1}\right)$$

$$i_2 = i_1\, 5.1 \angle 78.7^\circ$$

$$i = i_1 - i_2$$

$i = \frac{V(t)}{Z}$, $w = 10k$ rad/sec

$$Z = j \times 10k \times 3m + \left((Z_{eq} + 20\Omega) \,//\, \left(\frac{-j}{10k \times 1\mu} + 10\Omega\right)\right)$$

Z_{eq} is impedence of the dependant source

$$Z_{eq} = \frac{V}{i} = \frac{30 i_1}{i_2} = \frac{30 i_1}{i_1\, 5.1 \angle 78.7^\circ} = 5.88 \angle -78.7^\circ$$

or $$\frac{30 i_1}{i_1 + j5 i_1} = \frac{30(1 - j5)}{1 + 25} = 1.15 - j5.77$$

$$Z = j30 + ((1.15 - j5.77 + 20) // (-j100 + 10))$$

$$= j30 + ((21.15 - j5.77) // (-j100 + 10))$$

$$= j30 + \frac{(21.15 - j5.77)(-j100 + 10)}{21.15 - j5.77 - j100 + 10}$$

$$= j30 + \frac{-j2.1k + 211.5 - 577 - j57.7}{31.15 - j105.77}$$

$$= j30 + \frac{-j2.16k - 365.5}{31.15 - j105.77} \times \frac{31.15 + j105.77}{31.15 + j105.77}$$

$$= j30 + \frac{-j67.3k - j38.7k + 228.5k - 11.4k}{12.16k}$$

$$= j30 + 17.85 - j8.72$$

$$= 17.85 + j21.28$$

$$= 27.77 \angle \tan^{-1} \frac{21.28}{17.85}$$

$$= 27.77 \angle 50^\circ$$

$$i = \frac{130 \angle 0^\circ}{27.77 \angle 50^\circ} = 4.68 \angle -50^\circ$$

$$i = i_1 - i_2 = i_1 - i_1(1 + j5) = i_1(1 - 1 - j5)$$

$$\Rightarrow \quad i = -j5\, i_1$$

$$i_1 = \frac{i}{-j5} = \frac{j}{5} i = \frac{1}{5} \angle 90^\circ \, i$$

$$i_1 = 4.68 \angle -50^\circ \times 0.2 \angle 90^\circ = 0.936 \angle 40^\circ \text{ A}$$

i(t) = 0.936 cos(10kt + 40)