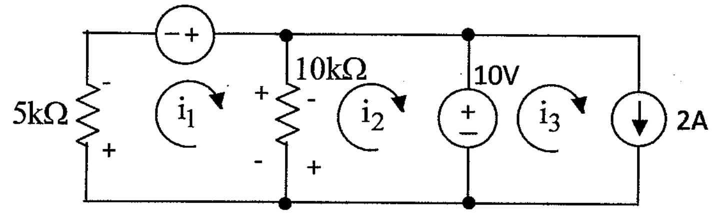
1. Use the mesh-current method to find i_1 and i_2, and i_3.



Procedure for solving for mesh-currents:

1. Label every mesh with a mesh current variable.
2. Label polarity on every R. If R is between 2 meshes, there will be polarities labeled for two mesh currents.
3. If an outside branch contains a current source (dependent or independent), the mesh current is equal to that current source value (+ if they are in the same direction or – if opposite directions).
4. If the circuit contains a dependent source, the dependent variable needs to be written in terms of the mesh currents.
5. If a current source is located between two meshes, a supermesh equation is written. The supermesh equation is the current source value equal to the two mesh currents going through it. (+ if the same direction, - if opposite directions).
6. A voltage loop is taken through pathways without current sources in them. The voltage loop equation will contain two currents going through a resistor located between 2 meshes.
7. Simultaneous equations are solved to obtain the mesh current values. These mesh current values can be used to determine any other unknown variable in the circuit.

Step 1 and 2 are shown in the figure above:

Step 3: $i_3 = 2A$

Step 4: skip

Step 5: skip

Step 6(three voltage loops possible):

$$-i_1(5k) + 20 - i_1(10k) + i_2(10k) = 0$$
$$+i_1(10k) - i_2(10k) - 10 = 0$$
$$-i_1(5k) + 20 - 10 = 0$$

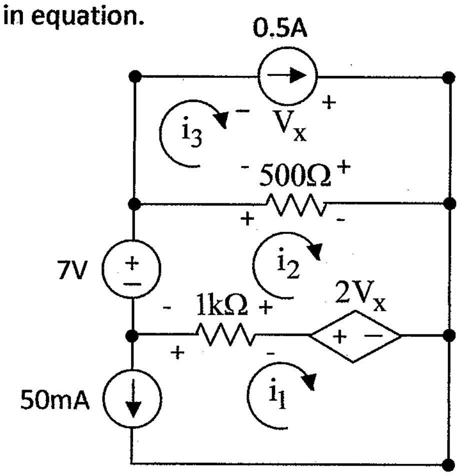
The last equation only has i_1 as an unknown, solving this gives:

$$-i_1(5k) + 20 - 10 = 0$$
$$i_1(5k) = 10 \quad \Rightarrow \quad i_1 = 2mA$$

Using the second equation and this known value yields:

$$+i_2(10k) = +i_1(10k) - 10$$
$$+i_2 = +i_1 - 1m$$
$$+i_2 = 2m - 1m = 1mA$$
$$i_1 = 2mA \quad i_2 = 1mA \quad i_3 = 2A$$

2. a. Use the mesh-current method to find V_X, V_X must not be in equation.
   b. Find power dissipated by the dependent source.



Step 1 and 2 are shown in the figure at the right:

Step 3: $i_1 = -50mA$, $i_3 = +0.5A$

Step 4: Vx is the voltage across the current source and the 500 ohm :

$$V_x = (+i_3 - i_2)500 = (+0.5 - i_2)500 = +250 - i_2 500$$

Step 5: No supermesh

Step 6(only one voltage loop possible):

$$+2V_x - i_2 1k + i_1 1k + 7 + V_x = 0$$

$$3(+250 - i_2 500) - i_2 1k + 50m(1k) + 7 = 0$$

$$i_2 2.5k = 807$$

$$i_2 = 282.8mA$$

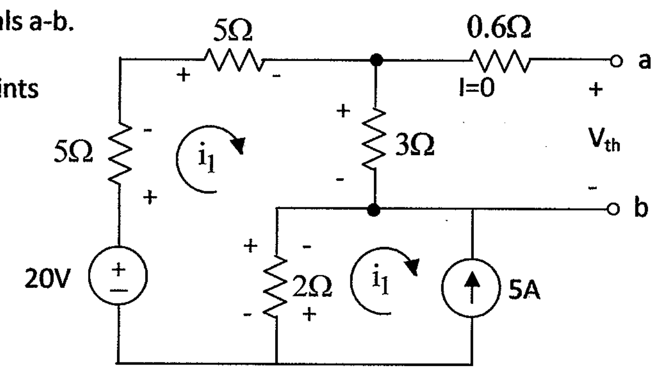
Plugging this value into the equation for Vx above gives:

$$V_x = +250 - (282.8m)500 = 108.6V$$

Power in the dependent source:

$$P = I * V = (i_1 - i_2)2(V_x) = (-282.8m + (-50m))2(108.6) = -72.3W$$

3. Find the Thevenin equivalent circuit at terminals a-b.



Find V_{th}. This is the open circuit voltage between points a-b: (Using mesh currents)

$$i_1 = -5A$$

$$+20 - i_1 5 - i_1 5 - i_1 3 - i_1 2 + i_2 2 = 0$$

$$20 - i_1 15 + i_2 2 = 0$$

$$20 - i_1 15 + (-5)2 = 0$$

$$i_1 = \frac{2}{3}$$

An equation for Vth is needed:

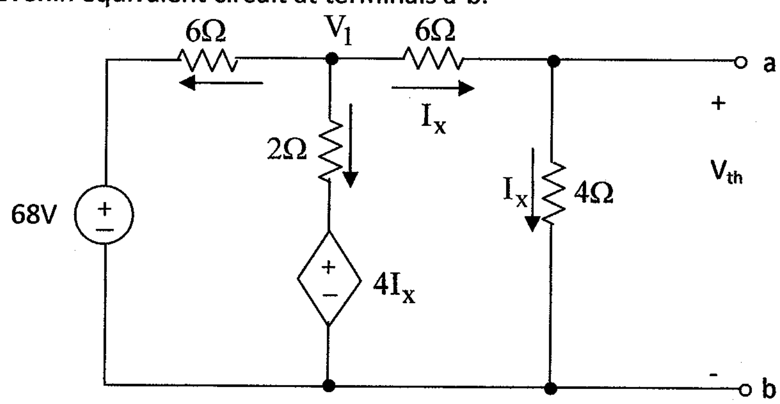
$$V_{th} - 0 - i_1(3) = 0$$

$$V_{th} = \frac{2}{3}(3) = 2V$$

Rth is found by removing all the independent sources (20V becomes a wire, 5A becomes an open):

$$R_{th} = 0.6 + 3 \,||\, (10+2) = 0.6 + \frac{1}{\frac{1}{3} + \frac{1}{12}} = 3\Omega$$

4. Find the Thevenin equivalent circuit at terminals a-b.



Using Node-Voltage Method – Procedure:

1. a. Place a reference point (0V).
   b. Label every essential node with a node-voltage.
2. Place all currents through every R.
3. If the circuit contains a dependent source, the dependent variable needs to be written in terms of the node voltages.
5. If only a single voltage source is located in a branch, a supernode equation is written. The supernode equation is the voltage source value equal to the two nodes on both sides of it. The voltage difference is taken as + at the positive side and – at the negative sign.
6. A current summation is taken at all node voltage variables. Each current term is a positive on the + side of the labeled current and – on the negative side of the labeled current.
7. Simultaneous equations are solved to obtain the node voltage values. These node voltage values can be used to determine any other unknown variable in the circuit.

In this case, the reference point is placed at b. The dependent variable is written in terms of V_1.

$$I_x = \frac{V_1}{(6+4)} = \frac{V_1}{10}$$

There are no supernodes for this circuit, so a current summation is written:

$$\frac{(V_1 - 68)}{6} + \frac{\left(V_1 - 4\left(\frac{V_1}{10}\right)\right)}{2} + \frac{(V_1)}{10} = 0$$

$$V_1\left(\frac{1}{6} - \frac{1}{5} + \frac{1}{2} + \frac{1}{10}\right) = \frac{(68)}{6}$$

$$V_1 = \frac{(68)}{6} \cdot \left(\frac{30}{17}\right) = 20V$$

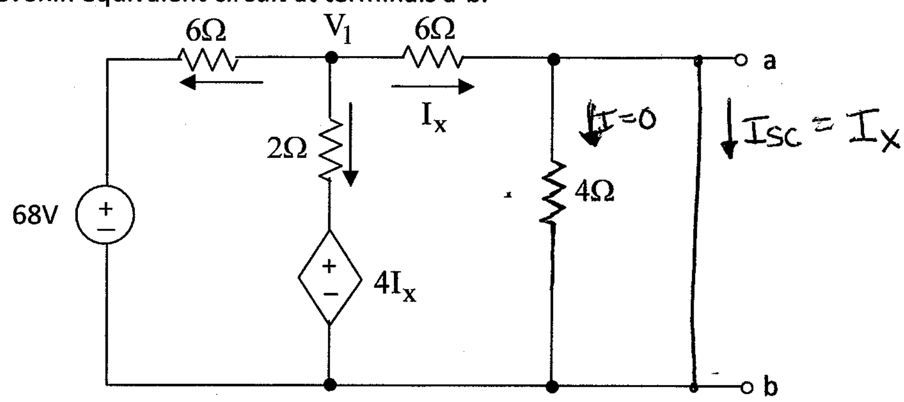
Using this solution for V_1:

$$I_x = \frac{V_1}{10} = \frac{20}{10} = 2A$$

Using ohm's law:

$$V_{th} = I_x(4) = 2(4) = \boxed{8V}$$

4. Find the Thevenin equivalent circuit at terminals a-b.



Solving using node-voltage: $\cdot\ I_x = \frac{V_1}{6}$

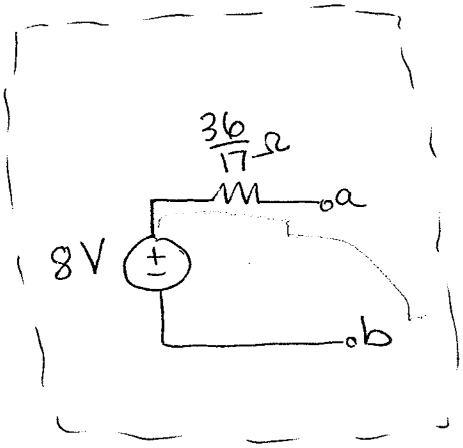
$$\frac{(V_1-68)}{6} + \frac{V_1 - 4\left(\frac{V_1}{6}\right)}{2} + \frac{V_1}{6} = 0$$

$$V_1\left(\frac{1}{6} + \frac{1}{2} - \frac{2}{6} + \frac{1}{6}\right) = \frac{68}{6}$$

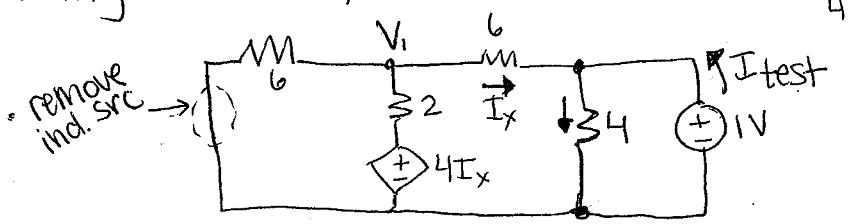
$$V_1 = \frac{68}{6} \cdot \frac{6}{3} = \frac{68}{3}$$

$$I_{sc} = \frac{V_1}{6} = \frac{68}{3(6)} = \frac{68}{18}$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{18}{68} \cdot (8) = \frac{144}{68} = \boxed{\frac{36}{17}}$$



Using a test source: (I used a vsrc to set a known current through 4Ω resistor)



$$-I_x + \frac{1}{4} - I_{test} = 0$$

$$I_{test} = -I_x + \frac{1}{4}$$

where $I_x = \frac{V_1 - 1}{6}$

$$\frac{(V_1)}{6} + \frac{V_1 - 4\left(\frac{V_1}{6} - \frac{1}{6}\right)}{2} + \frac{(V_1-1)}{6} = 0$$

$$V_1\left(\frac{1}{6} + \frac{1}{2} - \frac{2}{6} + \frac{1}{6}\right) = -\frac{2}{6} + \frac{1}{6} = -\frac{1}{6}$$

$$V_1 = -\frac{1}{6} \cdot \frac{6}{3} = -\frac{1}{3}$$

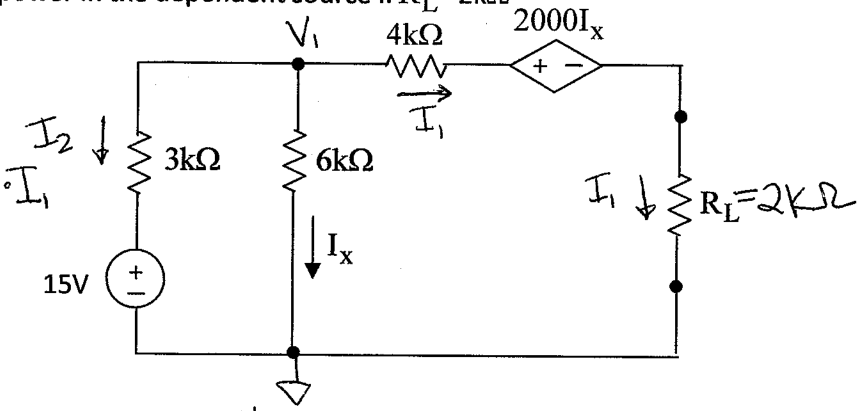
$$I_x = -\frac{1}{3(6)} - \frac{1}{6} = -\frac{1}{18} - \frac{3}{18} = \frac{-4}{18}$$

$$\therefore\ I_{test} = +\frac{4}{18} + \frac{1}{4} = \frac{16}{72} + \frac{18}{72} = \frac{34}{72}$$

$$R_{th} = \frac{V_{test} = 1V}{I_{test}} = \frac{72}{34} = \boxed{\frac{36}{17}\Omega}$$

(same)

5. Determine the power in the dependent source if $R_L = 2k\Omega$



power $= (2000I_x)\cdot I_1$

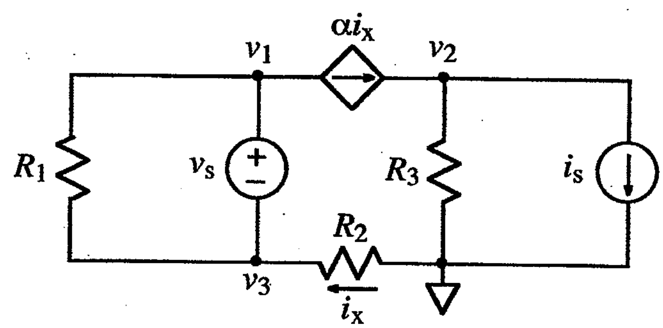
$$\frac{(V_1 - 15)}{3k} + \frac{V_1}{6k} + \frac{V_1 - 2000\left(\frac{V_1}{6k}\right)}{4k + 2k} = 0$$

$$V_1\left(\frac{2}{6k} + \frac{1}{6k} + \frac{1}{6k} - \frac{2000}{6k(6k)}\right) = \frac{15}{3k}$$

$$V_1\left(\cancel{\frac{12k + 6k + 6k - 2000}{6k(6k)}}\right) = \frac{15}{3k}\cdot\frac{6k(6k)}{22k} = \frac{30(6k)}{22k} = 8.2V$$

$$I_1 = \frac{V_1 - 2000\left(\frac{V_1}{6k}\right)}{6k} = .91m$$

$$\therefore \text{ power} = 2000\left(\frac{8.2}{6k}\right)\cdot(.91m) = \boxed{+2.5mW}$$



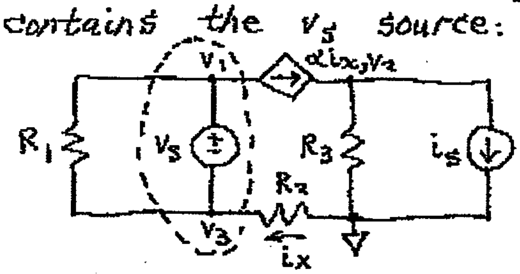
6. For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity i_x must not appear in the equations.

7. Make a consistency check on your equations for part 1(a) by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and node voltages for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a) and show that the left side and the right side of each equation are equal.)

6. sol'n: Define i_x in terms of node V's:

$$i_x = \frac{0V - V_3}{R_2} = -\frac{V_3}{R_2}$$

Now we write three eq'ns for the node V's. We have a supernode for V_1 and V_3, so we sum the currents out of a bubble around V_1 and V_3 that contains the V_s source:



The sum of currents out of the bubble has two terms that cancel out:

$$\frac{v_1 - \cancel{v_3}}{\cancel{R_1}} + \alpha\overbrace{\left(\frac{-v_3}{R_2}\right)}^{i_x} + \frac{v_3 - \cancel{v_1}}{\cancel{R_1}} + \frac{v_3}{R_2} = 0\,A$$

or

$$v_3\left(\frac{1}{R_2} - \frac{\alpha}{R_2}\right) = 0\,A \qquad (1)$$

We also get a voltage eq'n for the supernode:

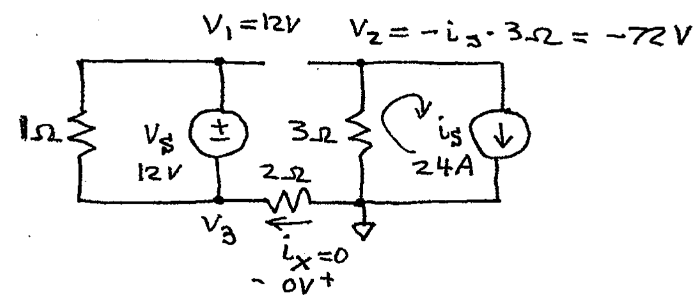
$$v_1 - v_3 = v_s \qquad (2)$$

Our third eq'n comes from node v_2:

$$-\alpha\left(\frac{-v_3}{R_2}\right) + \frac{v_2}{R_3} + i_s = 0\,A \qquad (3)$$

7. Many different consistency checks are possible. One example is given here.

Let $\alpha = 0$, $v_s = 12V$, $i_s = 24A$
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$.



We must have $i_x = 0$ or we will accumulate charge on the left side of the circuit. Thus, $v_3 = 0V$.

Since the v_s source connects v_3 to v_1, we have $v_1 = v_3 + v_s = 0V + 12V = 12V$.

On the right side, i_s flows around the loop, giving $v_2 = -i_s \cdot 3\Omega = -24A \cdot 3\Omega = -72V$.

$$v_1 = 12V, \quad v_2 = -72V, \quad v_3 = 0V$$

Now we plug these values into our 3 eq'ns from part (a):

$$0V\left(\frac{1}{2\Omega} - \frac{0}{2\Omega}\right) \stackrel{?}{=} 0A \qquad (1)$$

or $$0A \stackrel{?}{=} 0A \checkmark$$

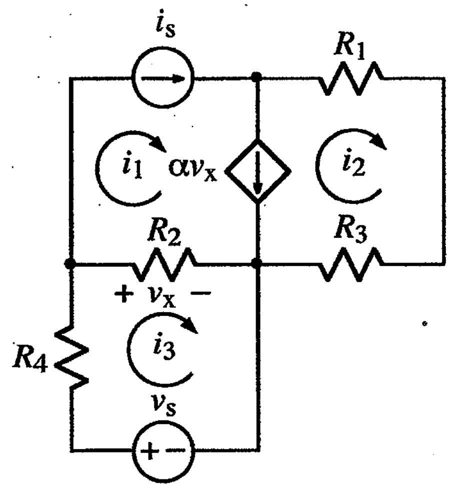
$$12V - 0V \stackrel{?}{=} 12V \checkmark \qquad (2)$$

$$-0\left(-\frac{12V}{2\Omega}\right) + \frac{-72V}{3\Omega} + 24A \stackrel{?}{=} 0A$$

or $$-24A + 24A \stackrel{?}{=} 0A \checkmark \qquad (3)$$

The eq'ns are all satisfied, completing the consistency check.

8.



For the circuit shown, write three independent equations for the three mesh currents i_1, i_2, and i_3. The quantity v_x must not appear in the equations.

soln: c) The loops for i_1 and i_2 share a current source, αv_x, meaning they form a super-mesh. For the voltage loop around the outside of the i_1 and i_2 loops, we have the following problem: the i_1 loop includes current source i_s. Thus, we must abandon the outer voltage loop.

Instead, we have $\boxed{i_1 = i_s}$, since src i_s is on the outside edge of the circuit. We need two eq'ns for the two loops, and the other eq'n is the usual current source eq'n for the αv_x source in terms of i_1 and i_2:

$$\alpha v_x = i_1 - i_2$$

We must write v_x in terms of mesh currents, however:

$$v_x = (i_3 - i_1) R_2$$

Using this expression for v_x, we obtain a second equation for the super-mesh:

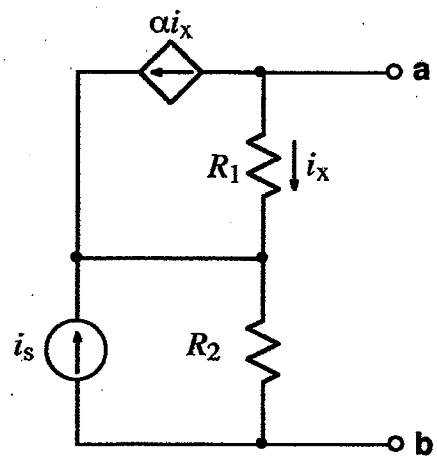
$$\boxed{(i_3 - i_1) R_2 = i_1 - i_2}$$

The voltage loop for i_3 gives the third and final equation:

$$\boxed{-i_3 R_4 - i_3 R_2 + i_1 R_2 + V_S = 0V}$$

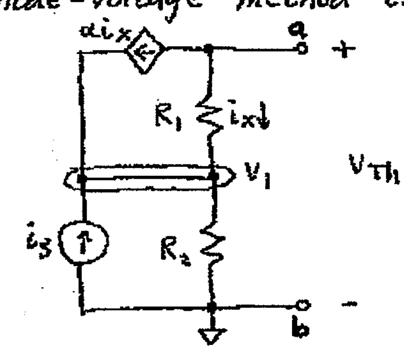
Note: i_1 may be replaced by i_S in the preceding two eq'ns.

9.



Find the Thevenin equivalent circuit at terminals **a** and **b**. i_x **must not appear in your solution. Note:** $\alpha > 0$.

sol'n: $v_{Th} = v_{a,b}$ with nothing connected across terminals a and b. We can use the node-voltage method to find v_{Th}.



We have one node: v_1. After defining i_x in terms of v_1, we sum the currents out of the v_1 node. For i_x, however, the only eq'n we can write is the following:

$$i_x = -\alpha i_x \quad \text{where } \alpha > 0$$

or

$$i_x(1+\alpha) = 0A$$

or

$$i_x = 0A$$

So no current flows in either R_1 or the dependent source. Thus, our node-voltage eq'n for v_1 is as follows:

$$-i_s + \frac{v_1}{R_2} = 0\text{ A}$$

or

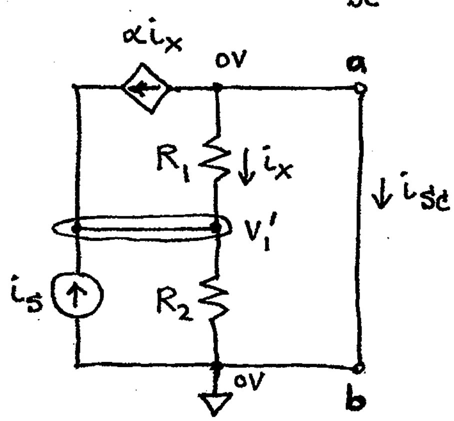
$$v_1 = i_s R_2$$

Since $i_x = 0$, the voltage drop across R_1 is zero. It follows that

$$\boxed{v_{Th} = v_1 = i_s R_2}$$

One way to find R_{Th} is to short **a** to **b**, find the current, i_{sc}, flowing in the short, and compute

$$R_{Th} = \frac{v_{Th}}{i_{sc}} .$$



We use the node-voltage method again.

i_x defined in terms of v_1' is

$$i_x = -\frac{v_1'}{R_1}$$

The summation of currents out of the v_1' node is as follows:

$$-i_s + \frac{v_1'}{R_2} - \alpha\left(\frac{-v_1'}{R_1}\right) - -\frac{v_1'}{R_1} = 0A$$

or

$$v_1'\left(\frac{1+\alpha}{R_1} + \frac{1}{R_2}\right) = i_s$$

or

$$v_1' = \frac{i_s}{\frac{1+\alpha}{R_1} + \frac{1}{R_2}} \cdot \frac{R_1R_2}{R_1R_2}$$

or

$$v_1' = \frac{i_s R_1 R_2}{(1+\alpha)R_2 + R_1}$$

and

$$i_x = -\frac{v_1'}{R_1} = -\frac{i_s R_2}{(1+\alpha)R_2 + R_1}$$

We find i_{sc} by summing currents out of the top node.

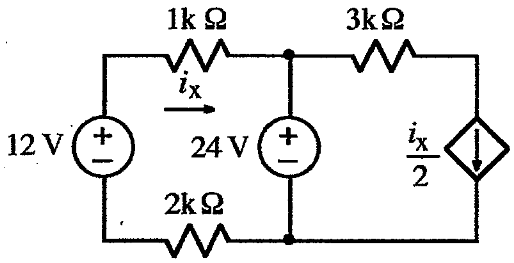
$$\alpha i_x + i_x + i_{sc} = 0A$$

or

$$i_{sc} = -(1+\alpha)i_x = \frac{(1+\alpha)i_s R_2}{(1+\alpha)R_2 + R_1}$$

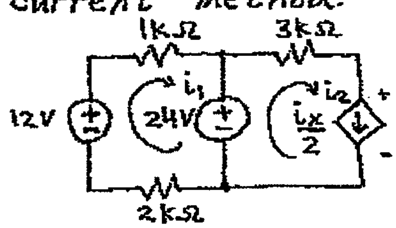
Thus, $$\boxed{R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{\cancel{i_s R_2}}{\frac{(1+\alpha)\cancel{i_s R_2}}{(1+\alpha)R_2 + R_1}} = \frac{(1+\alpha)R_2 + R_1}{1+\alpha}}$$

10.



Calculate the power consumed (i.e., dissipated) by the $i_x/2$ dependent source. **Note:** If a source supplies power, the power it consumes is negative.

soln: Any method of soln is acceptable. Here, the soln employs the mesh-current method.



$i_x = i_1$ so we have $\frac{i_1}{2}$ dependent src

i_1 loop: $12V - i_1 \cdot 1k\Omega - 24V - i_1 \cdot 2k\Omega = 0V$

Solving for i_1, we have

$$i_1 = \frac{24V - 12V}{1k\Omega + 2k\Omega} = -\frac{12V}{3k\Omega} = -4\text{ mA}$$

We have $i_x = i_1$, so the loop current i_2 is $i_x/2 = i_1/2$. (Note that i_2 is the same as the current in the dependent src, which is on the outside edge of the circuit.)

$$i_2 = \frac{i_x}{2} = \frac{i_1}{2} = -2\text{ mA}$$

To find the voltage drop across the dependent source, we use a voltage loop on the right side.