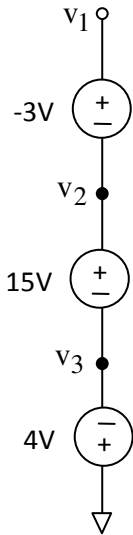


1.



Find the absolute voltages at all the labeled nodes in the above circuit. Hint: Start at the bottom and work upwards.

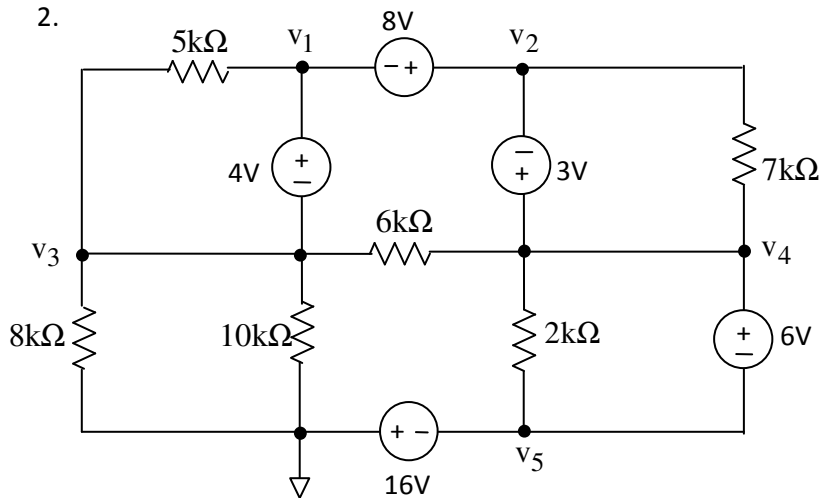
Sol: Starting at the reference, which is 0 V, we work our way upwards. If we enter a source at the - sign, we add the voltage of the source. Otherwise, we subtract the voltage of the source.

$$V_3 = -4V$$

$$V_2 = +V_3 + 15V = -4 + 15 = 11V$$

$$V_1 = -4 + 15 + (-3) = +8V$$

2.



3. Using Ohm's law and the node voltages found in Problem 2, find the currents for all the resistors in Problem 2.

Currents may be measured in one of two directions for each resistor. Here, they will all be measured with the arrow pointing down or to the right. The resistor currents are found by taking the difference of the node voltage on each end of the resistor and dividing by the resistance.

$$\text{Across } 8k\Omega \text{ resistor: } \frac{v_3 - 0}{8k} = \frac{-25}{8k} = -3.125mA$$

$$\text{Across } 10k\Omega \text{ resistor: } \frac{v_3 - 0}{10k} = \frac{-25}{10k} = -2.5mA$$

$$\text{Across } 5k\Omega \text{ resistor: } \frac{v_3 - v_1}{5k} = \frac{-25 - (-21)}{5k} = -0.8mA$$

Find the absolute voltages at all the labeled nodes in the above circuit. Hint: This may be done by inspection.

Sol: The node voltages are found by starting at the reference, (0 V), and stepping from node to node via voltage sources. If we enter a voltage source at the - sign and exit at the + sign, then we add the voltage of the source.

**NOTE:** Nodes connected by wires are really the same node and have the same voltage.

Starting from the reference and working towards the right side, we get the following successive voltages:

$$V_5 = -16V$$

$$V_4 = -16 + 6 = -10V$$

$$V_2 = -16 + 6 - 3 = -13V$$

$$V_1 = -16 + 6 - 3 - 8 = -21V$$

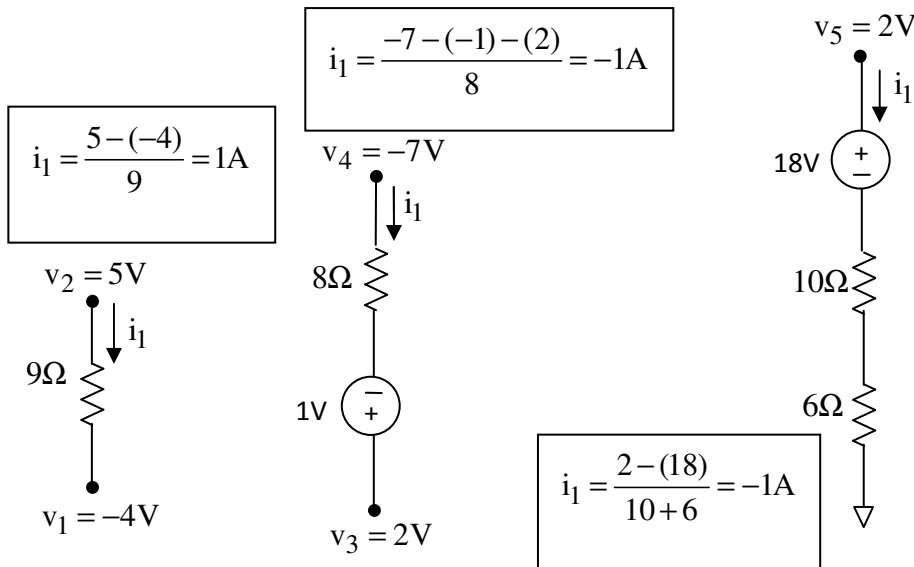
$$V_3 = -16 + 6 - 3 - 8 - 4 = -25V$$

$$\text{Across } 6k\Omega \text{ resistor: } \frac{v_3 - v_4}{6k} = \frac{-25 - (-10)}{6k} = -2.5mA$$

$$\text{Across } 2k\Omega \text{ resistor: } \frac{v_4 - v_5}{2k} = \frac{-10 - (-16)}{2k} = +3mA$$

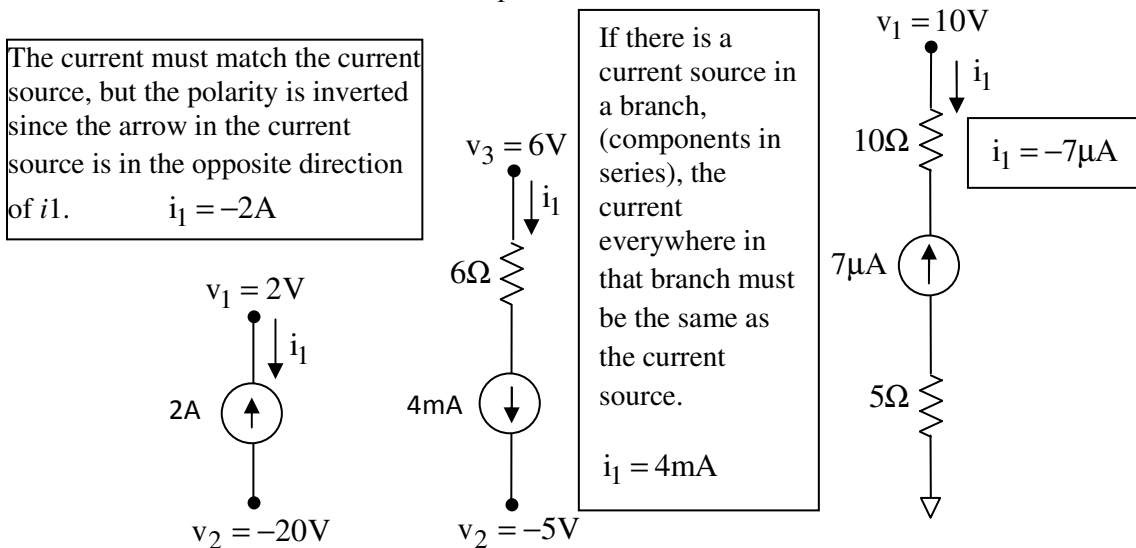
$$\text{Across } 7k\Omega \text{ resistor: } \frac{v_2 - v_4}{7k} = \frac{-13 - (-10)}{7k} = -0.43mA$$

4. Find the value of current,  $i_1$ , for each of the above circuits.

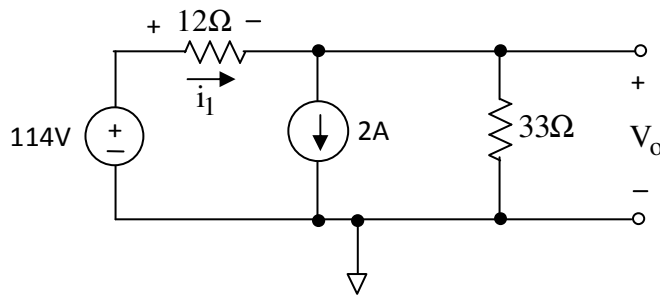


To determine the signs, if we were to mark a + and - for a voltage measurement across the  $9\Omega$  resistor that is in the passive sign direction for  $i_1$ , then we add  $v_2$  (which is next to the + sign) and subtract  $v_1$  (which is next to the - sign).

5. Find the value of current,  $i_1$ , for each of the above circuits.



6.



Use the node-voltage method to find  $i_1$  and  $V_o$ .

Set the reference point at the bottom node and  $V_o$  as the top node. We set the sum of currents out of  $v_1$  equal to zero, group terms for  $V_o$  and then solve:

$$\frac{V_o - 114}{12} + 2 + \frac{V_o - 0}{33} = 0$$

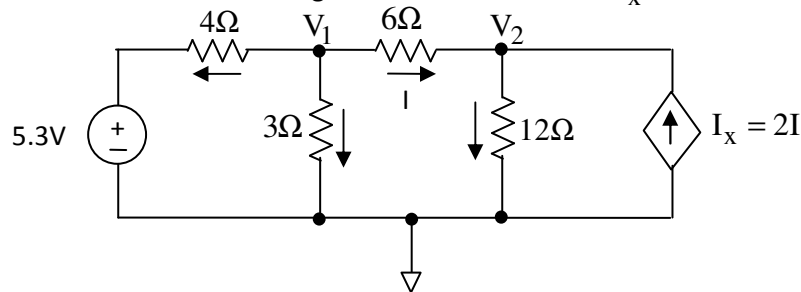
$$V_o \left( \frac{1}{12} + \frac{1}{33} \right) = \frac{114}{12} - 2$$

$$V_o \left( \frac{33}{12(33)} + \frac{12}{(12)33} \right) = \frac{114}{12} - \frac{24}{12}$$

$$V_o \left( \frac{45}{396} \right) = \frac{90}{12}$$

$$V_o = \frac{90}{12} \cdot \frac{396}{45} = \frac{35640}{540} = 66V$$

$$i_1 = \frac{114 - V_o}{12} = \frac{114 - 66}{12} = 4A$$

7. Use the node-voltage method to determine  $I_x$ .

Using the bottom node as the reference node, two equations stated below are determined by setting the sum of currents (directions as shown) out each node equal to zero.

$$\frac{V_1 - 5.3}{4} + \frac{V_1}{3} + \frac{V_1 - V_2}{6} = 0 \quad (V_1 \text{ node})$$

$$-\left( \frac{V_1 - V_2}{6} \right) + \frac{V_2}{12} - 2 \left( \frac{V_1 - V_2}{6} \right) = 0 \quad (V_2 \text{ node})$$

Solving the first equation for  $V_1$ :

$$V_1 \left( \frac{1}{4} + \frac{1}{3} + \frac{1}{6} \right) = \frac{5.3}{4} + \frac{V_2}{6}$$

$$V_1 \left( \frac{3}{12} + \frac{4}{12} + \frac{2}{12} \right) = \frac{5.3}{4} + \frac{V_2}{6}$$

$$V_1 \left( \frac{9}{12} \right) = \left( \frac{5.3}{4} + \frac{V_2}{6} \right) \cdot \left( \frac{12}{9} \right) = \left( \frac{5.3}{3} + \frac{2V_2}{9} \right)$$

Plugging this variable into the second equation and solving for  $V_2$ :

$$-\left( \frac{V_1 - V_2}{6} \right) + \frac{V_2}{12} - 2 \left( \frac{V_1 - V_2}{6} \right) = 0$$

$$V_1 \left( \frac{-1}{6} + \frac{-2}{6} \right) + V_2 \left( \frac{1}{6} + \frac{1}{12} + \frac{2}{6} \right) = 0$$

$$\left( \frac{5.3}{3} + \frac{2V_2}{9} \right) \cdot \left( \frac{-3}{6} \right) + V_2 \left( \frac{7}{12} \right) = 0$$

$$\left( \frac{-5.3}{6} + \frac{-V_2}{9} \right) + V_2 \left( \frac{7}{12} \right) = 0$$

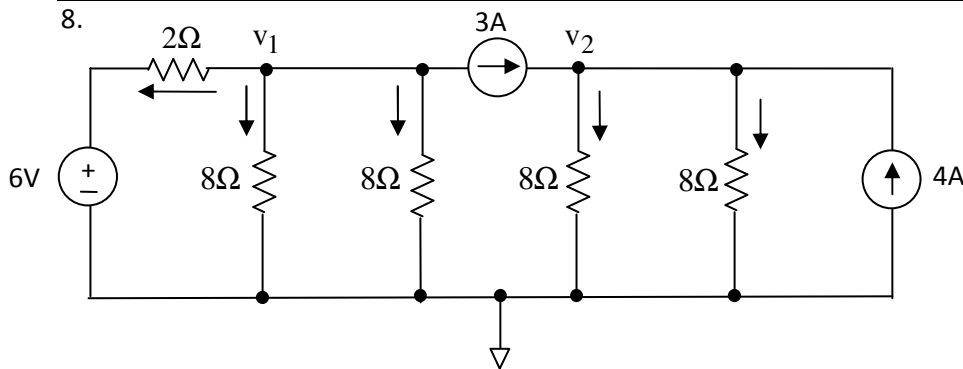
$$V_2 \left( \frac{7}{12} + \frac{-1}{9} \right) = \left( \frac{5.3}{6} \right) \cdot \left( \frac{-9(12)}{7(9) - 12} \right) = 1.9V$$

$$V_1 = \left( \frac{5.3(3)}{9} + \frac{2(1.87)}{9} \right) = 2.2V$$

Once  $V_1$  and  $V_2$  are solved,  $I_x = 2I$  can be found:

$$I = \left( \frac{V_1 - V_2}{6} \right) = \left( \frac{2.2 - 1.9}{6} \right) = 50mA$$

$$I_x = 2I = 100mA$$



- a. Use the node-voltage method to find  $v_1$  and  $v_2$ .
- b. Determine the amount of power supplied by the voltage source.

(a) We derive two equations by setting the sum of currents out each node equal to zero.

$$\left(\frac{v_1 - 6}{2}\right) + \left(\frac{v_1}{8}\right) + \left(\frac{v_1}{8}\right) + 3 = 0 \Rightarrow v_1 \left(\frac{4}{8} + \frac{1}{8} + \frac{1}{8}\right) = -3 + 3$$

$$-3 + \left(\frac{v_2}{8}\right) + \left(\frac{v_2}{8}\right) - 4 = 0 \Rightarrow v_2 \left(\frac{1}{8} + \frac{1}{8}\right) = +7$$

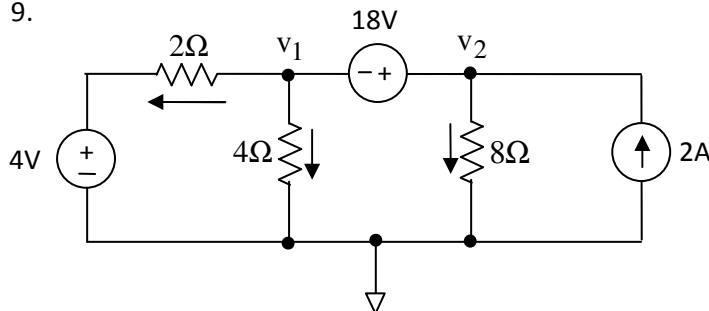
Solving the first equation for :

$$v_1 \left(\frac{6}{8}\right) = 0 \Rightarrow v_1 = 0$$

$$v_2 \left(\frac{2}{8}\right) = +7 \Rightarrow v_2 = 7 \left(\frac{8}{2}\right) = 28V$$

(b) Power =  $I \cdot V = I \cdot 6 = \left(\frac{v_1 - 6}{2}\right) \cdot 6 = \left(\frac{0 - 6}{2}\right) \cdot 6 = -18W$  (generating)

9.



Use the node-voltage method to find  $v_1$  and  $v_2$ .

$$v_1 - v_2 = -18 \Rightarrow v_1 = +v_2 - 18$$

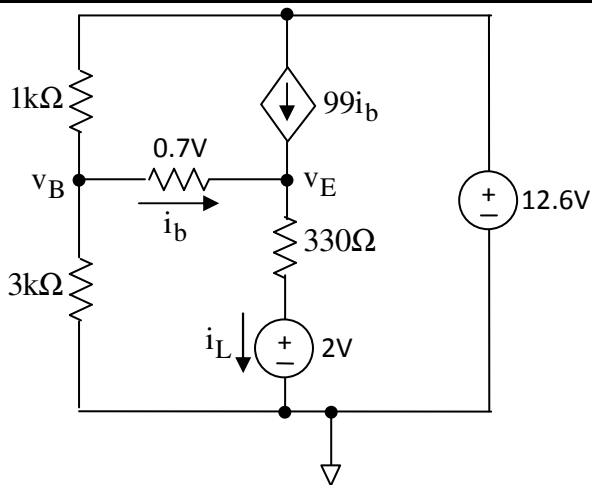
$$\left(\frac{v_1 - 4}{2}\right) + \left(\frac{v_1}{4}\right) + \left(\frac{v_2}{8}\right) - 2 = 0 \Rightarrow v_1 \left(\frac{1}{2} + \frac{1}{4}\right) + \left(\frac{v_2}{8}\right) = +2 + 2$$

Using the first eq. and putting it into the 2<sup>nd</sup> eq.:

$$(v_2 - 18) \cdot \left(\frac{1}{2} + \frac{1}{4}\right) + \left(\frac{v_2}{8}\right) = +2 + 2 \Rightarrow v_2 \left(\frac{7}{8}\right) = +\frac{54}{4} + \frac{16}{4}$$

$$v_2 = +\frac{70}{4} \left(\frac{8}{7}\right) = 20 \Rightarrow v_1 = v_2 - 18 = 20 - 18 = +2$$

10.



Use the node-voltage method to find  $v_B$  and  $v_E$ . Then find  $i_L$ .

Use the bottom node as the reference node.

Sol'n: Write current summations for  $v_B$  and  $v_E$  nodes. The second equation is the supernode equation. The first equation is due to a supernode between  $v_B$  and  $v_E$

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + \frac{V_E - 2}{330} - 99i_b = 0$$

$$V_B - V_E = 0.7$$

The first equation still has  $i_b$  in it. This needs to be removed by writing  $i_b$  in terms of  $v_B$  and  $v_E$ . This can be done by a current summation for the  $v_B$  node:

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + i_b = 0$$

$$i_b = \frac{-V_B}{3k} + \frac{-V_B}{1k} + \frac{12.6}{1k}$$

Plugging this into the first equation above to remove  $i_b$  from it:

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + \frac{V_E - 2}{330} - 99 \left( \frac{-V_B}{3k} + \frac{-V_B}{1k} + \frac{12.6}{1k} \right) = 0$$

$$V_B - V_E = 0.7$$

$$V_E = V_B - 0.7$$

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + \frac{(V_B - 0.7) - 2}{330} - 99 \left( \frac{-V_B}{3k} + \frac{-V_B}{1k} + \frac{12.6}{1k} \right) = 0$$

$$V_B \left( \frac{1}{3k} + \frac{1}{1k} + \frac{1}{330} + \frac{99}{3k} + \frac{99}{1k} \right) = \frac{12.6}{1k} + \frac{2.7}{330} + \frac{99(12.6)}{1k}$$

$$V_B \left( \frac{100(330)}{3k(330)} + \frac{100(330)(3)}{3k(330)} \right) = \frac{(100)12.6(330)}{1k(330)}$$

$$V_B = 9.3$$

$$V_E = V_B - 0.7 = 9.3 - 0.7 = 8.6V$$

$$i_L = \frac{V_E - 2}{330} = \frac{8.6 - 2}{330} = 20mA$$