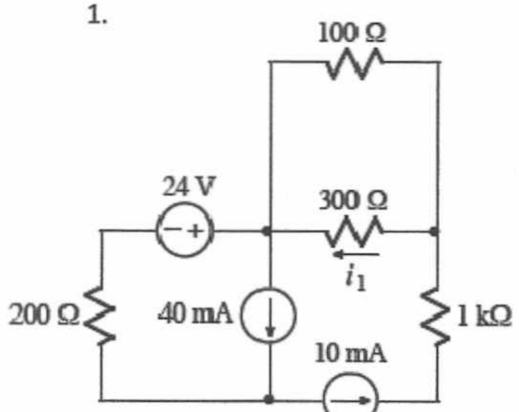
1.

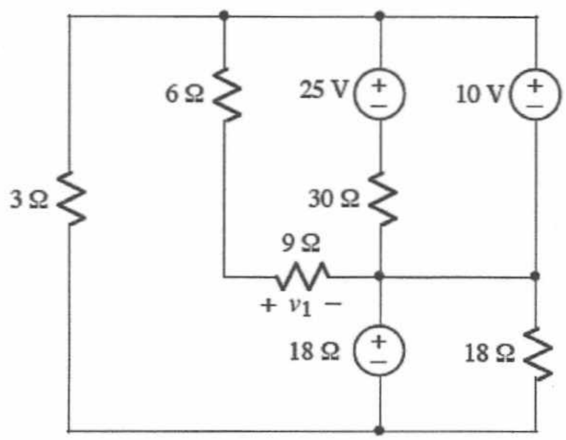


Calculate i_1.

SOL'N: The 10 mA source provides all the current flowing through the 100 Ω in parallel with 300 Ω. Thus, the current divider formula gives the value of i_1:

$$i_1 = 10\text{ mA}\frac{100\ \Omega}{100\ \Omega + 300\ \Omega} = 10\text{ mA}\frac{1}{4} = 2.5\text{ mA}$$

2.

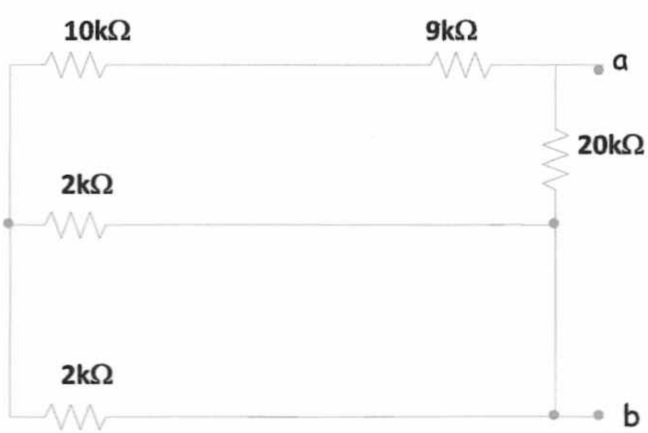


Calculate v_1.

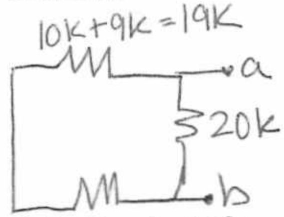
SOL'N: If we follow the wires in the circuit, the 6 Ω is in series with the 9 Ω, and the 10 V source is directly across them. Thus, the value of v_1 is given by the voltage-divider formula:

$$v_1 = 10\text{ V}\cdot\frac{9\ \Omega}{6\ \Omega + 9\ \Omega} = 10\text{ V}\cdot\frac{9\ \Omega}{15\ \Omega} = 10\text{ V}\cdot\frac{3}{5} = 6\text{ V}$$

3. Find the value of total resistance between terminals **a** and **b**.

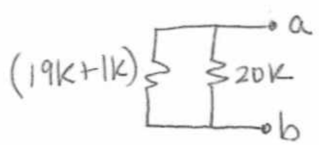


$10k + 9k = 19K$



$2k \| 2k = \frac{2k(2k)}{2k+2k} = 1K$

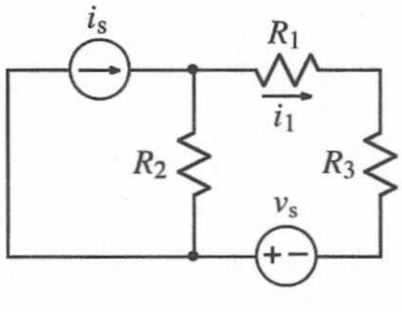
$\boxed{R_{eq} = 10k}$



$R_{eq} = 20k \| 20k$

$R_{eq} = \frac{20k(20k)}{20k+20k}$

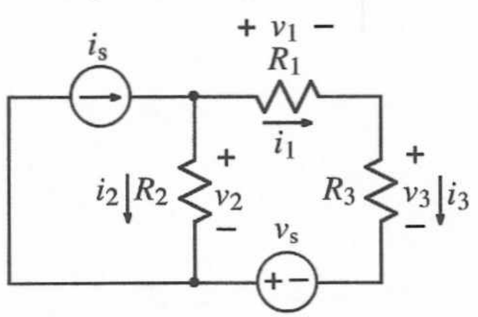
Ex:



4. Derive an expression for i_1. The expression must not contain more than the circuit parameters i_s, v_s, R_1, R_2, and R_3.

5. Make at least one consistency check (other than a units check) on your expression. In other words, choose component values that make it possible to solve the circuit by inspection, and verify that your answer to (a) gives that answer. Specify your consistency check by listing a numerical value for every source and resistor.

4. **SOL'N:** a) The voltage drops and currents may be measured in one of two directions for each resistor so long as we follow the passive sign convention. One such labeling is shown below.



We observe that R_3 is in series with R_1 and carries the same current.

$$i_3 = i_1$$

Looking for voltage-loops, we find that only the smaller loop on the right avoids the current source.

$$i_2R_2 - i_1R_1 - i_1R_3 + v_s = 0 \text{ V}$$

4. (cont) or

$$i_2 R_2 - i_1(R_1 + R_3) + v_s = 0 \text{ V}$$

A current summation at the top-center node gives a second equation for i_1 and i_2:

$$-i_s + i_2 + i_1 = 0 \text{ A}$$

Solving this equation for i_2 in terms of i_1 yields the following result:

$$i_2 = i_s - i_1$$

Substituting for i_2 in the previous equation yields an equation in terms on only i_1:

$$(i_s - i_1)R_2 - i_1(R_1 + R_3) + v_s = 0 \text{ V}$$

or

$$-i_1(R_2 + R_1 + R_3) = -v_s - i_s R_2$$

or

$$i_1 = \frac{v_s + i_s R_2}{R_1 + R_2 + R_3}$$

5. A variety of consistency checks are possible. One possible check is to let R_1 become an open circuit:

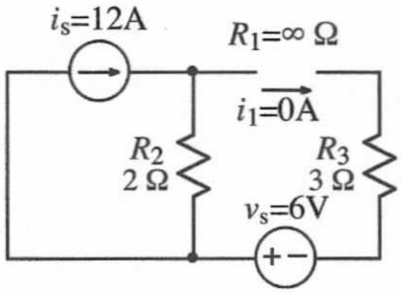
$$R_1 = \infty\ \Omega$$

In that case, no current can flow in R_1, meaning $i_1 = 0$ by inspection.

We may assign other values to the remaining components to see if the answer to (a) gives the correct answer, (i.e., $i_1 = 0$). One choice of component values is shown below.

NOTE: Picking numerical values for components other than R_1 is optional, since it is easy to see what value the answer in (a) will give when $R_1 = \infty\ \Omega$.

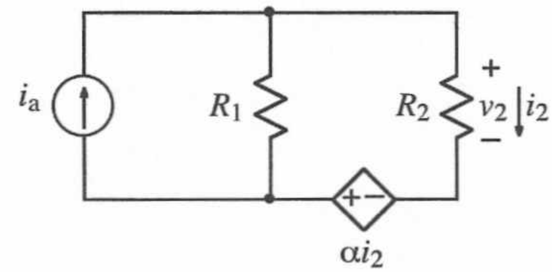
5(cont)



The answer to (a) gives the following result when the chosen numerical values for the circuit are used:

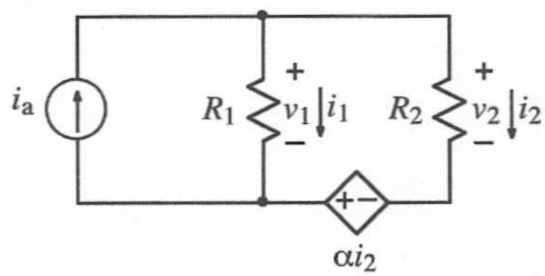
$$i_1 = \frac{6\text{ V} + 12\text{ A}\cdot 2\Omega}{\infty + 2\ \Omega + 3\ \Omega} = \frac{\text{constant}}{\infty}\text{A} = 0\text{ A}$$

Ex: 6.



Derive an expression for v_2. The expression must not contain more than the circuit parameters α, i_a, R_1, and R_2. **Note:** $\alpha < 0$.

SOL'N: We add to the circuit diagram a label that is consistent with the passive sign convention:



We look for components in series and find that only R_2 and the dependent source are in series. This may prove useful later on, but gives us the current for a voltage source, which we may ignore when solving for v_2.

We have one voltage loop on the right side that avoids the current source:

$$i_1 R_1 - i_2 R_2 + \alpha i_2 = 0 \text{ V}$$

or

$$i_1 R_1 - i_2 (R_2 - \alpha) = 0 \text{ V}$$

A current summation at the top-center node gives the following equation:

$$-i_a + i_1 + i_2 = 0 \text{ A}$$

Solving this equation for i_1 in terms of i_2 we have an expression we can substitute into the voltage-loop equation to solve for i_2:

6. (cont)

$$i_1 = i_a - i_2$$

Now for the substitution:

$$(i_a - i_2)R_1 - i_2(R_2 - \alpha) = 0 \text{ V}$$

or

$$i_a R_1 - i_2(R_1 + R_2 - \alpha) = 0 \text{ V}$$

or

$$i_2 = \frac{i_a R_1}{R_1 + R_2 - \alpha}$$

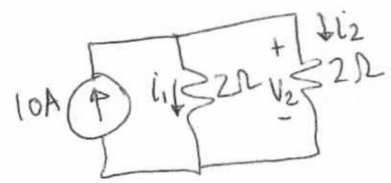
We use Ohm's law to find v_2:

$$v_2 = i_2 R_2 = \frac{i_a R_1 R_2}{R_1 + R_2 - \alpha}$$

7. perform a consistency check for derived equation from 6.

Let $\alpha = 0$, $R_1 = 2\,\Omega$, $R_2 = 2\,\Omega$ and $i_a = 10\text{A}$

New circuit:



Current divider:

$$i_2 = \frac{10(2)}{4} = 5\text{A}$$

$$V_2 = i_2 \cdot (2) = \underline{\underline{10V}}$$ ✓

From eq. above in 6:

$$V_2 = \frac{i_a R_1 R_2}{R_1 + R_2 - \alpha} = \frac{10(2)(2)}{2+2-0} = \underline{\underline{10V}}$$ same ✓

sol'n

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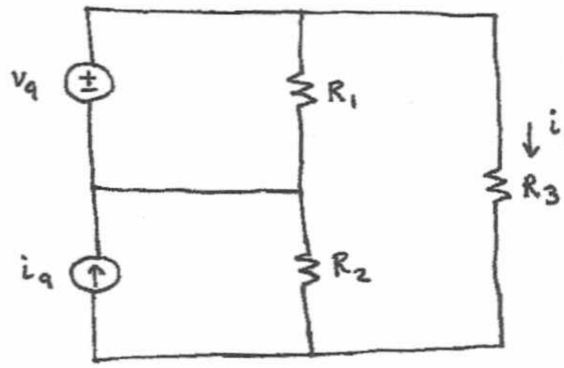
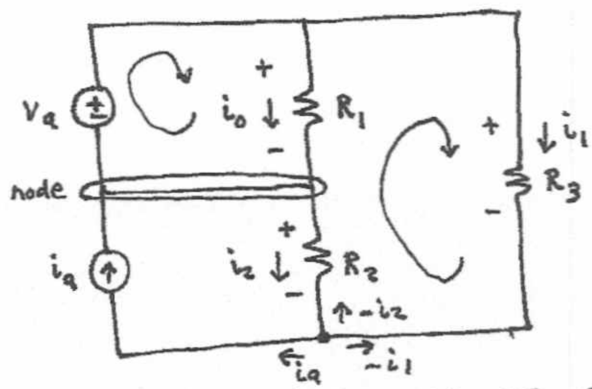
8.



Derive expression for i_1. Expression must contain no other parameters than v_a, i_a, R_1, R_2, R_3.

sol'n: Use Kirchhoff's Laws. Label resistors.



We'll use Ohm's Law as we go by writing voltages for R's as $v = iR$.

Voltage loops:

Upper left loop $+v_a - i_0 R_1 = 0V$

We can solve this. $i_0 = \dfrac{v_a}{R_1}$

Right side loop $+i_2 R_2 + i_0 R_1 - i_1 R_3 = 0V$ (1)

These are the only v-loops that are allowed, (i.e., no v needed for i src), and are not redundant, (i.e., are not bigger loops equivalent in content to several smaller loops).

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soln: 8 cont.

current sums at nodes:

The only node where we can sum currents (without having to define a current for a voltage source) is the bottom node.

Bottom node $\quad i_a - i_2 - i_1 = 0A \qquad (2)$

Now we have the last two eqns in two unknowns that we can solve for i_1.

Solve eqn (1) for i_2 so we can eliminate it.

$$i_2 = \frac{i_1 R_3 - i_0 R_1}{R_2} = \frac{i_1 R_3 - \frac{v_a}{R_1} \cdot R_1}{R_2}$$

$$\text{or} \quad i_2 = \frac{i_1 R_3 - v_a}{R_2}$$

Substitute this into eqn (2); solve for i_1.

$$i_1 = i_a - i_2 = i_a - \frac{i_1 R_3 - v_a}{R_2}$$

$$i_1 + i_1 \frac{R_3}{R_2} = i_a + \frac{v_a}{R_2}$$

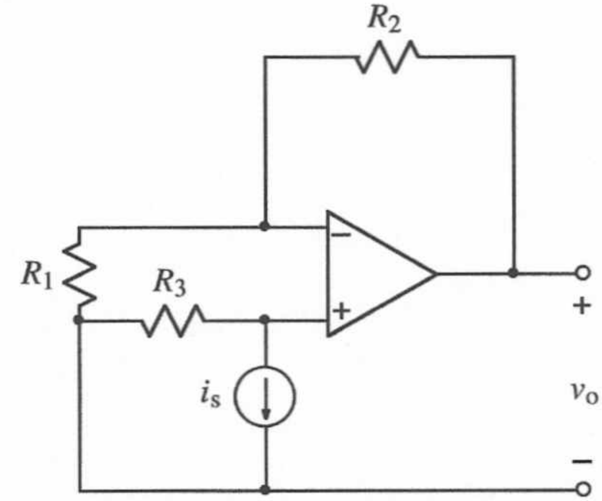
$$i_1 \left(1 + \frac{R_3}{R_2}\right) = i_a + \frac{v_a}{R_2}$$

$$i_1 \frac{R_2 + R_3}{R_2} = i_a + \frac{v_a}{R_2}$$

$$i_1 = \left(i_a + \frac{v_a}{R_2}\right) \frac{R_2}{R_2 + R_3}$$

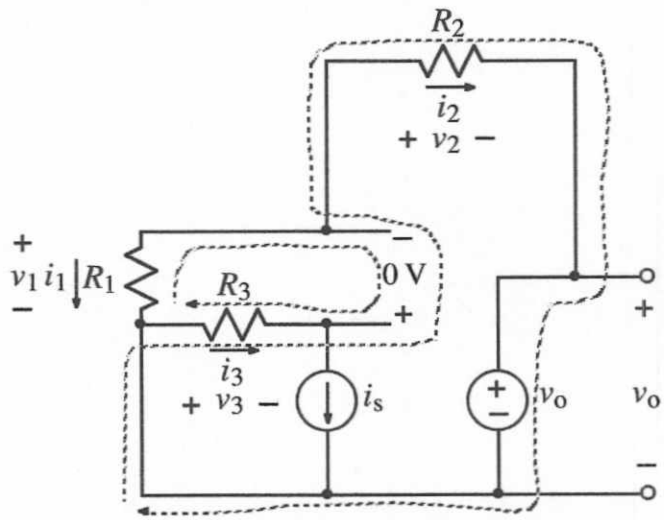
$$\boxed{i_1 = \frac{i_a R_2 + v_a}{R_2 + R_3}}$$

Ex: 9.



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for v_o in terms of not more than i_s, R_1, R_2, and R_3.

SOL'N: We first remove the op-amp and assume the op-amp output voltage has the value necessary to make the voltage drop across the op-amp inputs equal zero volts. One possible way of labeling the resulting circuit, consistent with the passive sign convention, is shown below.



Looking first for components in series that carry the same current, we see that R_1 and R_2 have equal but opposite currents:

9. (cont.)

$$i_2 = -i_1$$

We also have R_3 in series with current source i_s. By Ohm's law, the voltage across R_3 is $i_s R_3$:

$$v_3 = i_s R_3$$

Next, we look for voltage loops, making sure we use the 0 V drop across the op-amp inputs at least once.

The small voltage loop shown on the diagram above yields the following equation:

$$i_1 R_1 + 0 \text{ V} + i_s R_3 = 0 \text{ V}$$

Solving for i_1, we have the following result:

$$i_1 = -\frac{i_s R_3}{R_1}$$

The large voltage loop shown on the diagram above yields the following equation:

$$-i_s R_3 - 0 \text{ V} - i_2 R_2 - v_o = 0 \text{ V}$$

or, substituting for i_2, we have an equation for v_o in terms of i_1:

$$-i_s R_3 - 0 \text{ V} + i_1 R_2 - v_o = 0 \text{ V}$$

or

$$v_o = -i_s R_3 + i_1 R_2$$

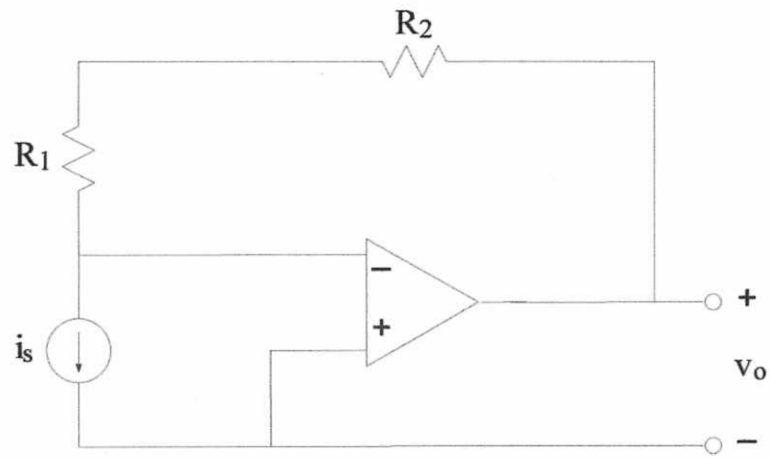
Substituting for i_1 yields the following final answer:

$$v_o = -i_s R_3 - \frac{i_s R_3}{R_1} R_2$$

or

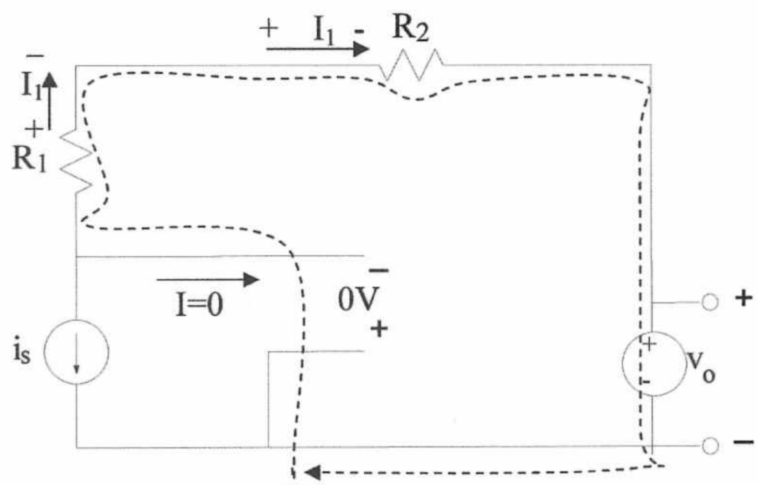
$$v_o = -i_s R_3 \left(1 + \frac{R_2}{R_1}\right)$$

10.



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for v_O in terms of not more than v_S, i_S, R_1, and R_2.

Removing the op-amp, putting Vo source on output, and putting 0v between input terminals results in the following:



Taking a loop as shown in the above figure: $-0-I_1R_1-I_1R_2-Vo=0 \Rightarrow -I_1(R_1+R_2)=Vo$

Current summation results in : $-i_s+I_1+0=0 \Rightarrow +i_s+=I_1$

Therefore from the v-loop equation: $\mathbf{Vo=-i_s(R_1+R_2)}$