1. **Solve the following simultaneous equations for i_1, i_2, and i_3:**

Eq. 1 $$2(i_1+i_2)-10+(3i_2-i_1-4i_3)=0$$

Eq. 2 $$-3(i_1+i_2)+2(i_1+3i_3)=0$$

Eq. 3 $$i_1-5-i_2=0$$

Solving Eq. 3 for i_1=> $$i_1 = 5 + i_2$$

Plugging this result into Eq. 2=> $$-3(5+i_2)-3i_2+2(5+i_2)+6i_3=0$$

$$-15-3i_2-3i_2+10+2i_2+6i_3=0$$

$$-5-4i_2+6i_3=0$$

{Solving for i_3=>} $$i_3=\frac{+5+4i_2}{6}$$

Plugging this result into Eq. 1=> $$2(5+i_2)+2i_2-10+3i_2-5-i_2-4\left(\frac{+5+4i_2}{6}\right)=0$$

$$+6\left(\frac{6}{6}\right)i_2-5\left(\frac{6}{6}\right)-\left(\frac{+20+16i_2}{6}\right)=0$$

$$\left(\frac{20}{6}\right)i_2-\left(\frac{+50}{6}\right)=0$$

$$i_2=\left(\frac{50}{6}\right)\left(\frac{6}{20}\right)=\boxed{2.5A}$$

$$i_3=\frac{+5+4(2.5)}{6}=\boxed{2.5A}$$

$$i_3=5+2.5=\boxed{7.5A}$$

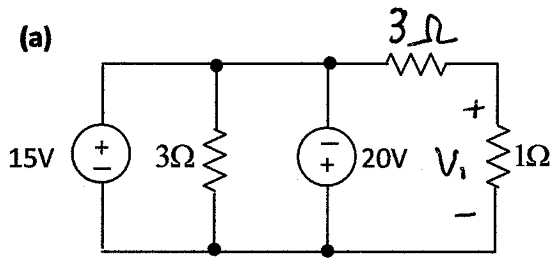
2. **Perform the following calculations. Write the answers with appropriate prefixes (such as μ, m, k etc.) for engineering untis:**

a) $P = 7.2\text{ mA}\times 6\text{ kV}$ (Note: V*A=W) = $7.2*10^{-3}x6*10^{3}=\boxed{43.2W}$

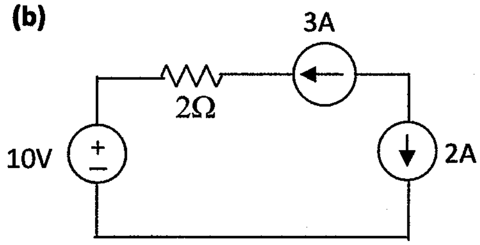
b) $R = 3.3\text{ k}\Omega + 1.6\text{ }\mu\Omega$ =3300+0.0000016=$\boxed{3{,}300.0000016\Omega}$

3. Determine whether each of the following circuits is valid or invalid.

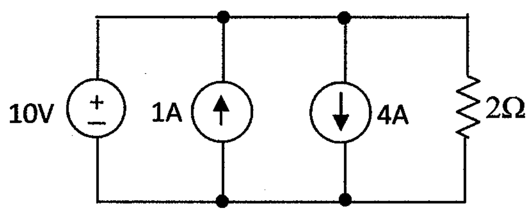
(a)



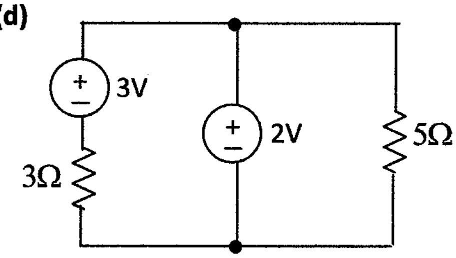
(b)



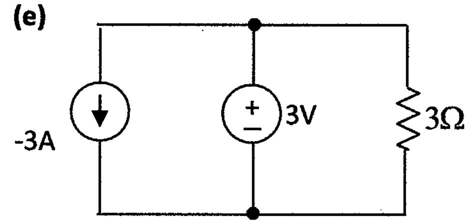
(c)



(d)



(e)



(a) This circuit is INVALID. By Kirchhoff's laws, components in parallel must have the same voltage drop across them. Here, the two voltage sources disagree across the 3 Ω resistor.

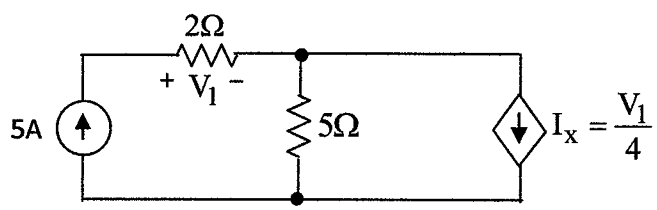
(b) This circuit is INVALID. By Kirchhoff's laws components in series must carry the same current. The current sources disagree on the current and will try to force the current to have two different values: 3 A and 2 A.

(c) This circuit is VALID. Current sources in parallel sum. The voltage across both current sources and the 2Ω is 10V. Here, the current flowing through the 2 Ω is 10/2=5A. Summing the current will result in –I_10V-1+4+5=0 => I+10V=8A flowing upward through the 10V voltage source. Note that current sources will produce whatever voltage is necessary to force a specified current flow in a circuit.

(d) This circuit is VALID. By Kirchhoff's laws, components in parallel must have the same voltage drop across them. Here, this is possible.

(e) This circuit is VALID. The 3Ω has 3/3=1A flowing downward through it. This means +(-3)-I_3V+1=0 gives I_3V=-2A flowing through the voltage source (2A is flowing downward through the voltage source.)

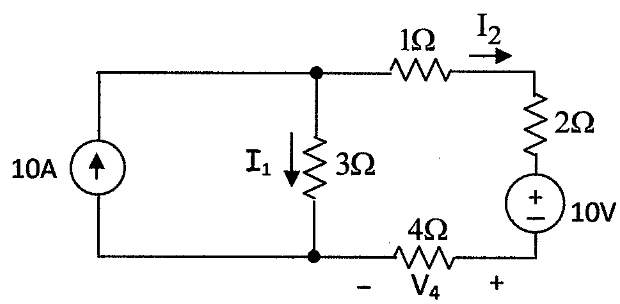
4. **Find I_x in the circuit below.**



V_1=5A*2Ω=10V

Ix=V1/4=10/4=$\boxed{2.5A}$

5. **Find I_2 in the circuit below if V_4=8V.**

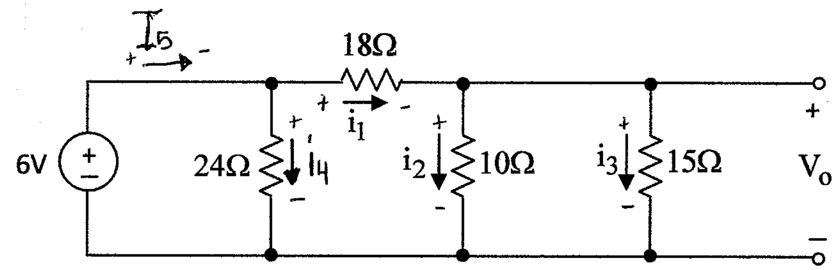


V4=8V=I_2*4Ω=> I_2=8/4Ω=$\boxed{2A}$

Using a current summation at the top node: $-10+I_1+I_2=0 \Rightarrow I_1=10-2=$**8A**

9. (a) Find i_1, i_2, i_3, and v_o.

(b) Find the power dissipated in the 24Ω resistor and the power supply.



KVL: $+6 - i_4(24) = 0 \Rightarrow i_4 = \frac{6}{24} = 250mA$

$+6 - i_1(18) - i_2(10) = 0$

① $+6 - i_1(18) - i_3(15) = 0$

$+i_2(10) - i_3(15) = 0$

$+i_2(10) - V_o = 0$

$+i_3(15) - V_o = 0$

$+i_4(24) - i_1(18) - i_2(10) = 0$

$+i_4(24) - i_1(18) - i_3(15) = 0$

∴ $V_o = i_2(10)$ or $V_o = i_3(15)$

$i_2(10) = i_3(15)$

$i_2 = i_3\left(\frac{3}{2}\right)$

Using ① and ②:

$+6 - i_3\left(\frac{5}{2}\right)18 - i_3(15) = 0$

$+6 = \left[\frac{18(5)+30}{2}\right] i_3$

$\frac{12}{120} = i_3 = 100mA$

$i_1 = \frac{5}{2}(100m) = 250mA$

$i_2 = \frac{3}{2}(100m) = 150mA$

$V_o = i_2(10) = 1.5V$

KCL:

$-i_1 + i_2 + i_3 = 0$

$-i_1 + i_3\left(\frac{3}{2}\right) + i_3\left(\frac{2}{2}\right) = 0$

② $i_1 = \frac{5}{2} i_3$

(b) 24Ω power:

$P = I \cdot V = I^2 R$

$I_{24\Omega} = i_4 = 250mA$

∴ $P_{24\Omega} = (250m)^2 \cdot 24$

$\boxed{P_{24\Omega} = 1.5W}$

power supply power:

$\pm$ 6V I_5 + $\quad P = (-) I \cdot V$

signs are opposite

$-I_5 + i_4 + i_1 = 0$

$I_5 = 250m + 250m = 500m$

$P_{6V} = -500m(6) = \boxed{-3W}$

sol'n

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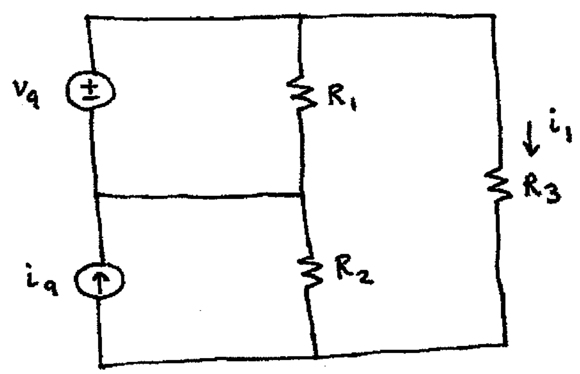
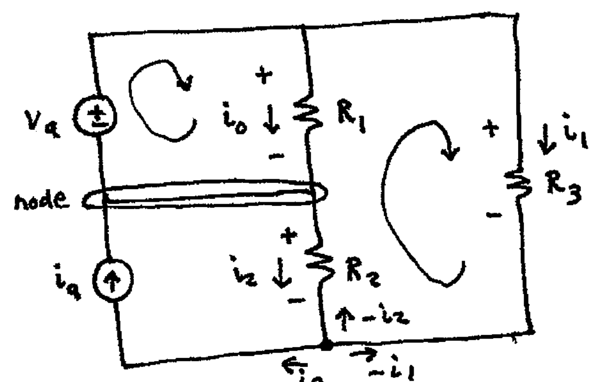
7.



Derive expression for i_1. Expression must contain no other parameters than v_a, i_a, R_1, R_2, R_3.

sol'n: Use Kirchhoff's Laws. Label resistors.



We'll use Ohm's Law as we go by writing voltages for R's as $v = iR$.

Voltage loops:

Upper left loop $+v_a - i_o R_1 = 0V$

We can solve this. $i_o = \dfrac{v_a}{R_1}$

Right side loop $+i_2 R_2 + i_o R_1 - i_1 R_3 = 0V$ (1)

These are the only v-loops that are allowed, (i.e., no v needed for i src), and are not redundant, (i.e., are not bigger loops equivalent in content to several smaller loops).

soln

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Soln: cont.

current sums at nodes:

The only node where we can sum currents (without having to define a current for a voltage source) is the bottom node.

Bottom node $\quad i_q - i_2 - i_1 = 0\,A \qquad (2)$

Now we have the last two eqns in two unknowns that we can solve for i_1.

Solve eqn (1) for i_2 so we can eliminate it.

$$i_2 = \frac{i_1 R_3 - i_0 R_1}{R_2} = \frac{i_1 R_3 - \frac{v_q}{R_1} \cdot R_1}{R_2}$$

$$\text{or} \quad i_2 = \frac{i_1 R_3 - v_q}{R_2}$$

Substitute this into eqn (2); solve for i_1.

$$i_1 = i_q - i_2 = i_q - \frac{i_1 R_3 - v_q}{R_2}$$

$$i_1 + i_1 \frac{R_3}{R_2} = i_q + \frac{v_q}{R_2}$$

$$i_1 \left(1 + \frac{R_3}{R_2}\right) = i_q + \frac{v_q}{R_2}$$

$$i_1 \frac{R_2 + R_3}{R_2} = i_q + \frac{v_q}{R_2}$$

$$i_1 = \left(i_q + \frac{v_q}{R_2}\right) \frac{R_2}{R_2 + R_3}$$

$$\boxed{i_1 = \frac{i_q R_2 + v_q}{R_2 + R_3}}$$

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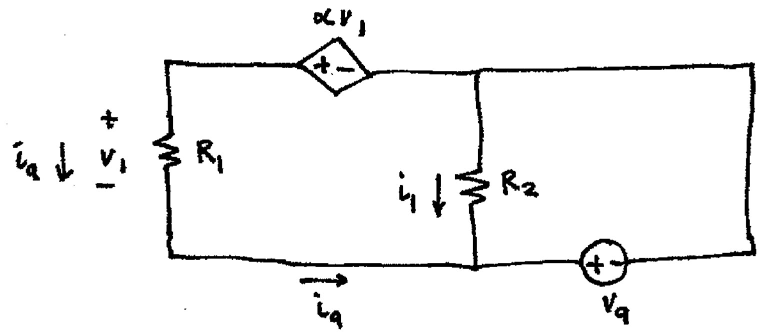
sol'n

8.



a. Derive expression for i_a containing not more than circuit parameters α, v_a, R_1, R_2.

sol'n: a) Use Kirchhoff's Laws. We can save effort by observing that v_a is across two circuits that we may solve separately: R_2 is the first circuit, and R_1 and αv_1 src are the other circuit.

We may thus ignore R_2, and we may use an outer v-loop.

(We use $i_a R_1$ to replace v_1 everywhere.)

$$+ i_a R_1 - \alpha(\underbrace{i_a R_1}_{v_1}) + v_a = 0V$$

or $i_a (R_1 - \alpha R_1) = -v_a$

or $\boxed{i_a = \dfrac{-v_a}{(1-\alpha) R_1}}$

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sol'n

sol'n: 8. (Example) A consistency check means we pick values for components that make the circuit so simple that we can solve it by inspection. We then check our answer to (a) against the simple sol'n.

Many checks are possible. This is only one possible check.

Let $\alpha = 0$ so dependent v-src becomes 0V (or wire).

Then $-v_a$ is across R_1. Thus, $i_a = \frac{-v_a}{R_1}$.

We let $R_1 = 1\Omega$, $R_2 = 2\Omega$, $v_a = 12V$.

Then $i_a = \frac{-12V}{1\Omega} = -12A$.

Now we see if our formula from (a) gives this answer.

$$i_a = \frac{-v_a}{(1-\alpha)R_1} = \frac{-12V}{(1-0)\,1\Omega} = -12A \checkmark$$

Agrees with answer from simplified circuit.

Note: this check can catch a problem although it doesn't guarantee correctness if the sol'n to (a) passes the test.

HW #1

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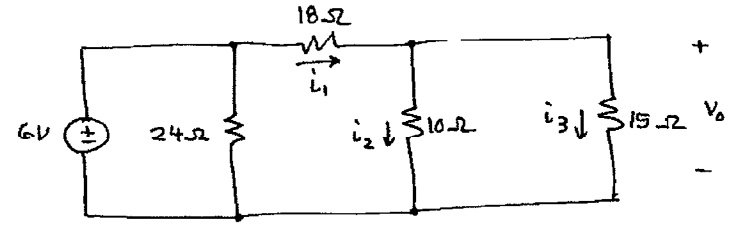
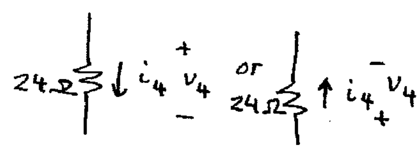
9.



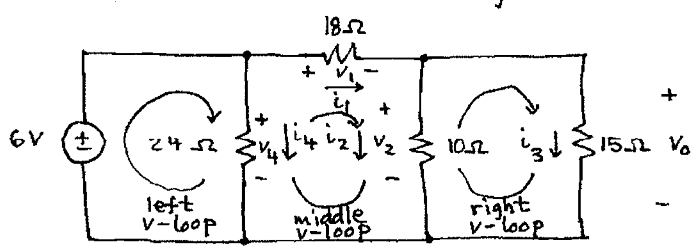
a. Calculate i_1, i_2, i_3, V_o.

b. Find power dissipated in the 24Ω resistor and the 6V supply.

sol'n: a) Using passive sign convention, label all currents and voltage drops. Note that we have two choices for the labeling of the 24Ω resistor:



I'll use the first labeling:



Write eq'ns for voltage loops for inner loops (unless that would force us to define a voltage for a current source).

Left V-loop: $+6V - V_4 = 0V \Rightarrow V_4 = 6V$

Makes sense since 24Ω resistor is across 6V source (battery).

Ohm's Law: $i_4 = \frac{V_4}{24\Omega} = \frac{6V}{24\Omega} = \frac{1}{4}$ A or 250 mA

Middle V-loop: $+V_4 - V_1 - V_2 = 0V$

Of course, $V_4 = 6V$. So $\boxed{V_1 + V_2} = \boxed{V_4} = 6V$

sol'n: 9.a) cont.

Right v-loop: $+v_2 - v_0 = 0V \Rightarrow \boxed{v_0 = v_2}$

Now write eq'ns for current sums at nodes. We do all but one node, (one is always redundant), unless we would have to define a current for a voltage source. In that case, we can skip that node.

Here, we observe that we can skip the node to left of the 18Ω resistor. (We don't want to define a current for the 6V source.)

For the node to the right of the 18Ω resistor, we sum currents measured in a direction away from the node:

$$\boxed{-i_1 + i_2 + i_3 = 0A}$$

We leave out one node, and I usually leave out the bottom rail. By the way, the two bottom nodes are connected by a wire so they are really just one node. (Redraw circuit so everything is connected to one point on the bottom.)

We are done with Kirchhoff's Laws. Now use Ohm's Law for all resistors: (already did 24Ω)

$$\boxed{v_1 = i_1 \cdot 18\Omega}$$
$$\boxed{v_2 = i_2 \cdot 10\Omega}$$
$$\boxed{v_0 = i_3 \cdot 15\Omega}$$

We have 6 eq'ns in 6 unknowns, (boxed eq'ns).

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sol'n: 9.a) cont.

One way to proceed is to replace all v's with $i \cdot R$'s:

	eq'n #
$i_1\, 18\Omega + i_2\, 10\Omega = 6V$	(1)
$i_3\, 15\Omega = i_2\, 10\Omega$	(2)
$-i_1 + i_2 + i_3 = 0A$	(3)

Solve the simultaneous eq'ns. I use the simplest eq'n first to isolate one variable in terms of others. Eq'n (2) tells us what to substitute for i_3: $i_3 = i_2 \cdot \frac{10\Omega}{15\Omega} = \frac{2}{3} i_2$

Using this for i_3 in eq'ns (1) and (3) gives

$$i_1\, 18\Omega + i_2\, 10\Omega = 6V$$

$$-i_1 + i_2 + \frac{2}{3} i_2 = 0A \quad \text{or} \quad -i_1 + i_2\left(1+\frac{2}{3}\right) = 0A$$

From this new eq'n (2) we have $i_1 = i_2\left(1+\frac{2}{3}\right) = \frac{5}{3} i_2$.

Eq'n (1) becomes $\frac{5}{3} i_2\, 18\Omega + i_2\, 10\Omega = 6V$

or $i_2 \cdot \left(\frac{5}{3} \cdot 18 + 10\right)\Omega = 6V$

$$i_2 \cdot 40\Omega = 6V$$

$$i_2 = \frac{6V}{40\Omega} = \frac{6V}{40\Omega} \frac{25}{\cdot 25} = \frac{150}{1k} \frac{V}{\Omega}$$

$$i_2 = 150\ mA$$

Substitute back into eq'n (1) to get

$$i_1 \cdot 18\Omega + 150mA \cdot 10\Omega = 6V$$

$$i_1 \cdot 18\Omega + 1.5V = 6V$$

$$i_1 \cdot 18\Omega = 4.5V$$

$$i_1 = \frac{4.5V}{18\Omega} = \frac{1}{4} A \text{ or } 250mA$$

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sol'n: 9. a) cont. Using eq'n (2), we have $i_3\,15\Omega = 150\text{mA}\cdot 10\Omega$.

$$i_3 = \frac{1.5V}{15\Omega} = \frac{1}{10}A \text{ or } 100 \text{ mA}$$

By Ohm's Law, $v_o = i_3 \cdot 15\Omega = 100\text{ mA} \cdot 15\Omega = 1.5V$.

Summary: $i_1 = 250$ mA $i_4 = 250$ mA
$i_2 = 150$ mA
$i_3 = 100$ mA
$v_o = 1.5V$

Consistency checks:

Outer v-loop correct?

$$+6V - v_1 - v_o \stackrel{?}{=} 0$$

$$+6V - 250\text{mA}\cdot 18\Omega - 100\text{ mA}\cdot 15\Omega \stackrel{?}{=} 0$$

$$+6V - 4.5V - 1.5V = 0 \checkmark$$ works!

Current flowing into upper right node equals current flowing out of upper right node?

Current flowing in $= i_1 = 250$ mA

Current flowing out $= i_2 + i_3 = 150\text{mA} + 100\text{mA}$
$= 250$ mA ✓ works!

b) $p = i \cdot v$ (value >0 means power dissipated
value >0 " " generated or sourced)

For resistors, $p = i\cdot v = i \cdot iR = i^2R$ by Ohm's Law
$= \frac{v}{R}\cdot v = \frac{v^2}{R}$ " " "

Thus, we can find p as i^2R or $\frac{v^2}{R}$ for R's.

$$P_{24\Omega} = i_4^2 \cdot 24\Omega = \left(\tfrac{1}{4}A\right)^2 \cdot 24\Omega = \tfrac{1}{16}\cdot 24 \text{ W} = 1.5 \text{ W}$$

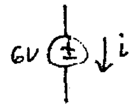
$$P_{18\Omega} = i_1^2 \cdot 18\Omega = \left(\tfrac{1}{4}A\right)^2 \cdot 18\Omega = \tfrac{1}{16}\cdot 18 \text{ W} = 1.125 \text{ W}$$

$$P_{10\Omega} = i_2^2 \cdot 10\Omega = \left(\tfrac{1.5}{10}A\right)^2 \cdot 10\Omega = \tfrac{2.25}{100}\cdot 10\text{W} = 225 \text{ mW}$$

$$P_{15\Omega} = i_3^2 \cdot 15\Omega = \left(\tfrac{1}{10}A\right)^2 \cdot 15\Omega = \tfrac{15}{100} \text{ W} = 150 \text{ mW}$$

soln: 9.b) cont.

Now we need the current for the 6V source:



$p = i \cdot 6V$

we avoided defining i earlier. <u>Now that we have found all the other i's and v's</u>, we can find i.

Use a current sum at the node to the left of the 18Ω:

$$i + i_4 + i_1 = 0A$$

$$i + 250\,mA + 250\,mA = 0A$$

$$i = -500\,mA \text{ or } -\tfrac{1}{2}A$$

$$P_{6V} = i \cdot 6V = -\tfrac{1}{2}A \cdot 6V = -3W$$

(negative means pwr <u>source</u>)

Consistency check:

Total pwr for all components should be zero:

$$-3W + 1.5W + 1.125W + 225\,mW + 150\,mW = 0W$$ ✓ works!

HW #1 sol'n (cont.)

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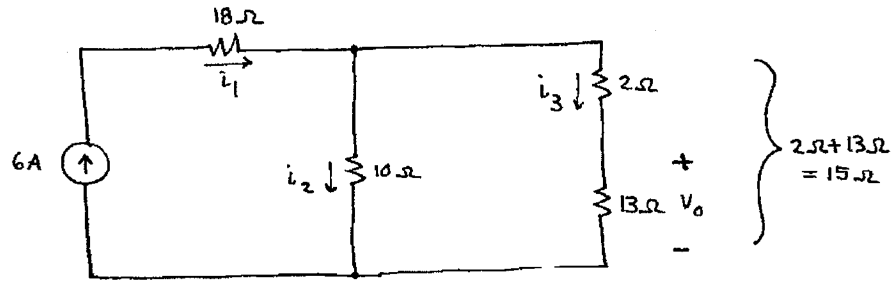
10.



Find i_1, i_2, i_3, and V_o

sol'n: Since 18Ω in series with 6A source, we must have $i_1 = 6A$. This is total i for right side.

Now we can use i-divider formula for the two branches on right:

$$i_2 = i_1 \cdot \frac{15\Omega}{10\Omega + 15\Omega} = i_1 \cdot \frac{3}{5} = 6A \cdot \frac{3}{5} = 3.6\ A$$

$$i_3 = i_1 \cdot \frac{10\Omega}{10\Omega + 15\Omega} = i_1 \cdot \frac{2}{5} = 6A \cdot \frac{2}{5} = 2.4\ A$$

To find V_o, we observe that i_3 flows thru 13Ω resistor. Thus, $V_o = i_3 \cdot 13\Omega = 2.4A \cdot 13\Omega$

$$= \frac{12}{5} A \cdot 13\Omega = \frac{156}{5} V = 31.2V$$

Summary: $i_1 = 6A$

$i_2 = 3.6A$

$i_3 = 2.4A$

$V_o = 31.2V$

Consistency checks: $i_1 \stackrel{?}{=} i_2 + i_3$ $\quad 6A = 3.6A + 2.4A$ ✓

$i_2 \cdot 10\Omega \stackrel{?}{=} i_3 \cdot 15\Omega$ $\quad 3.6A \cdot 10\Omega = 2.4A \cdot 15\Omega$ ✓

$\underbrace{i_3 \cdot 13\Omega}_{V_o} + i_3 \cdot 2\Omega \stackrel{?}{=} i_2 \cdot 10\Omega$ $\quad 31.2V + 2.4A \cdot 2\Omega \stackrel{?}{=} 3.6A \cdot 10$

$31.2V + 4.8V = 36V$ ✓