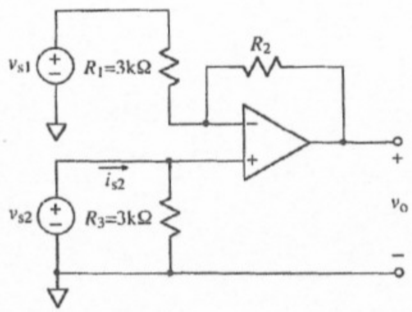
1.



Rail voltages = ±9 V

a) The above circuit operates in linear mode. Derive a symbolic expression for v_o. The expression must contain not more than the parameters v_{s1}, v_{s2}, R_1, R_2, and R_3.

b) If $v_{s2} = 0$ V, find the value of R_2 that will yield an output voltage of $v_o = -1$ V when $v_{s1} = 10$ mV.

c) Derive a symbolic expression for v_o in terms of common mode and differential input voltages:

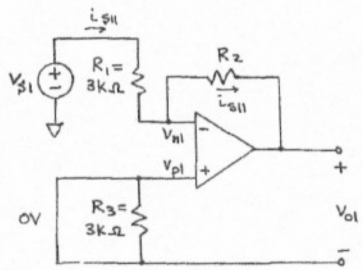
$$v_\Sigma = \frac{v_{s1} + v_{s2}}{2} \quad \text{and} \quad v_\Delta = \frac{v_{s1} - v_{s2}}{2}$$

The expression must contain not more than the parameters v_Σ, v_Δ, R_1, R_2, and R_3. Write the expression as v_Σ times a gain term plus v_Δ times a gain term. Hint: write v_{s1} and v_{s2} in terms of v_Σ and v_Δ.

d) Using the value of R_2 from part (b), if necessary, calculate the input resistance, $R_{in} = v_{s2}/i_{s2}$, seen by the v_{s2} source.

sol'n: a) Using superposition, we turn on one v-source at a time.

case I: v_{s1} on, v_{s2} off (= wire)



R_3 is bypassed by a wire and may be ignored. The circuit is then seen to be a simple negative-gain amplifier:

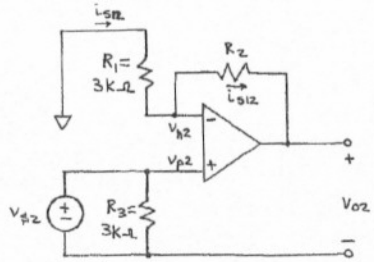
$v_{p1} = 0V \qquad v_{n1} = v_{p1} = 0V$

$i_{s1} = \dfrac{v_{s1} - 0V}{R_1}$ on left side

$i_{s1} = \dfrac{0V - v_{o1}}{R_2}$ on right side

$\therefore \; -\dfrac{v_{o1}}{R_2} = \dfrac{v_{s1}}{R_1}$ or $v_{o1} = -v_{s1}\dfrac{R_2}{R_1}$

case II: v_{S1} off (= wire), v_{S2} on



Since R_3 is across V-source v_{S2}, we may ignore it. The circuit is then seen to be a simple positive-gain amplifier:

$$v_{p2} = v_{S2} \qquad v_{n2} = v_{p2} = v_{S2}$$

$$i_{S12} = \frac{0 - v_{n2}}{R_1} = \frac{-v_{S2}}{R_1}$$

$$i_{S12} = \frac{v_{n2} - v_{o2}}{R_2} = \frac{v_{S2} - v_{o2}}{R_2}$$

$$\therefore \quad \frac{v_{S2} - v_{o2}}{R_2} = -\frac{v_{S2}}{R_1}$$

$$v_{o2} = v_{S2}\left(1 + \frac{R_2}{R_1}\right)$$

Summing results gives $v_o = v_{o1} + v_{o2} = -v_{S1}\frac{R_2}{R_1} + v_{S2}\left(1 + \frac{R_2}{R_1}\right)$.

b) Using $v_{S2} = 0V$ and $v_{S1} = 10mV$ in the formula from (a), we have

$$v_o = -1V = -10mV\,\frac{R_2}{3k\Omega} + \cancel{0\cdot\left(1 + \frac{R_2}{3k\Omega}\right)} \rightarrow 0$$

or $R_2 = \dfrac{-1V}{-10mV} \cdot 3k\Omega = 300\,k\Omega$

c) We have $v_{S1} = v_\Sigma + v_\Delta$ and $v_{S2} = v_\Sigma - v_\Delta$.

Substituting these into the formula for v_o from (a) yields the following:

$$v_o = -(v_\Sigma + v_\Delta)\frac{R_2}{R_1} + (v_\Sigma - v_\Delta)\left(1 + \frac{R_2}{R_1}\right)$$

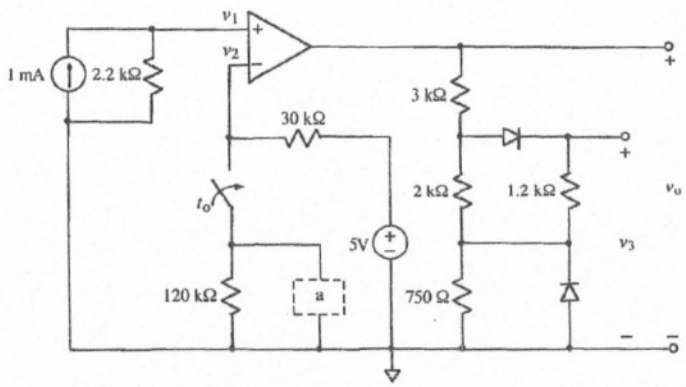
$$v_o = v_\Sigma\left(1 + \cancel{\frac{R_2}{R_1}} - \cancel{\frac{R_2}{R_1}}\right) - v_\Delta\left(\frac{R_2}{R_1} + 1 + \frac{R_2}{R_1}\right)$$

$$v_o = v_\Sigma - v_\Delta\left(1 + 2\frac{R_2}{R_1}\right)$$

d) $R_{in} = \dfrac{v_{S2}}{i_{S2}} = \dfrac{v_{S2}}{v_{S2}/R_3}$ (no current flows into op-amp, so $i_{S2} = v_{S2}/R_3$)

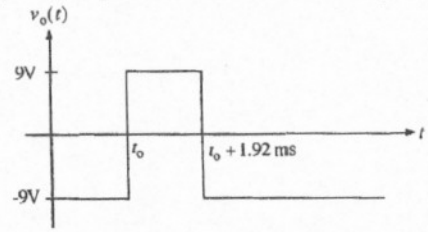
$$R_{in} = R_3$$

2.



Rail voltages = ±9 V

After being open for a long time, the switch closes at time $t = t_o$.



a) Choose either an L or a C to go in box **a** to produce the $v_o(t)$ shown above. Specify what component goes in the box and its value.

b) Sketch $v_1(t)$, showing numerical values appropriately.

c) Sketch $v_2(t)$, showing numerical values appropriately.

d) Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 1.92$ms, and for $t > t_o + 1.92$ms. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

sol'n: a) The op-amp acts as a comparator since their is no feedback. It measures the voltage drop across the inputs with the polarity as indicated by the + and − signs. If the voltage is positive, the output is $+V_{rail}$. If the voltage is negative, the output is $-V_{rail}$.

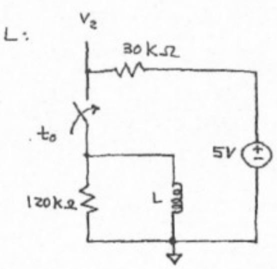
To produce the waveform shown in the problem, we must have $v_2 > v_1$ for $t < t_o$ and $t > t_o + 1.92$ms, and we must have $v_2 < v_1$ for $t_o < t < t_o + 1.92$ ms.

v_1 is constant. The value of v_1 is

$$v_1 = 1\,mA \cdot 2.2\,k\Omega = 2.2\,V.$$

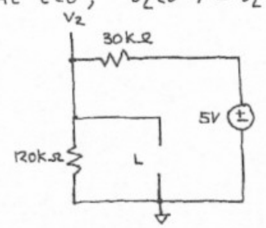
For v_2, we consider the behavior with $a = L$ and $a = C$ in turn. (Assume $t_o = 0$.)

$a = L$:



At $t = 0^-$, L = wire and $i_L(0^-) = 0$ A, $v_2 = 5V$.

At $t=0^+$, $i_L(0^+) = i_L(0^-) = 0A$ = open.



We have v-divider, with

$$v_2(0^+) = 5V \cdot \frac{120k}{120k+30k} = 4V.$$

For $t \to \infty$, L = wire and $v_2(t\to\infty) = 0V$.

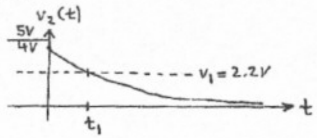
Using the general form of solution, we write an expression for $v_2(t>0)$:

$$v_2(t>0) = v_2(t\to\infty) + [v_2(0^+) - v_2(t\to\infty)]\, e^{-t/L/R_{Th}}$$

$$v_2(t>0) = 0V + [4V - 0V]\, e^{-t/L/R_{Th}}$$

$$v_2(t>0) = 4V\, e^{-t/L/R_{Th}}$$

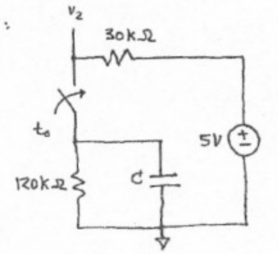
$v_2(t<0) = 5 V$ from earlier



The time constant, L/R_{Th}, is unknown, but the shape of the curve is exponential.

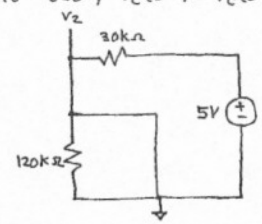
We see that $v_2 > v_1$ for $t<0$ and $t<t_1$, and $v_2 < v_1$ for $t > t_1$. Thus, this is not a viable waveform for v_2 to achieve the desired output pulse. Rather than proceeding with this solution, we consider the case of a=C.

a=C:



At $t=0^-$, C = open and $v_c(0^-) = 0V$, $v_2 = 5V$.

At $t=0^+$, $v_C(0^+) = v_C(0^-) = 0V$ = wire.



We have v_2 shorted to reference.
$\therefore\ v_2(0^+) = 0V$

For $t \to \infty$, C = open and we have V-divider:

$$v_2(t\to\infty) = 5V \cdot \frac{120\,k\Omega}{120k\Omega + 30k\Omega} = 4V$$

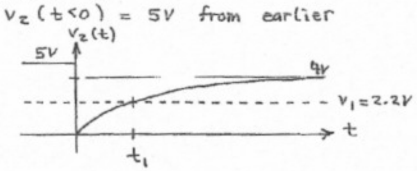
Using the general form of solution, we write an expression for $v_2(t>0)$:

$$v_2(t>0) = v_2(t\to\infty) + \left[v_2(0^+) - v_2(t\to\infty)\right] e^{-t/R_{Th}C}$$

$$v_2(t>0) = 4V + [0V - 4V]\, e^{-t/R_{Th}C}$$

$$v_2(t>0) = 4V - 4V e^{-t/R_{Th}C}$$

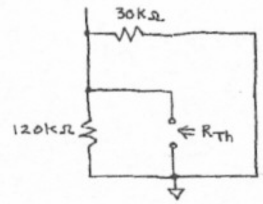
$$v_2(t<0) = 5V \text{ from earlier}$$



The time constant, $R_{Th}C$, is unknown at this point, but the curve is exponential.

We see that $v_2 > v_1$ for $t<0$ and $t>t_1$, and $v_2 < v_1$ for $0<t<t_1$. Thus, this is a viable waveform for v_2 to achieve the desired output pulse.

We choose C to make $t_1 = 1.92$ ms for proper pulse width. To find R_{Th}, we remove C and look into the terminals where C was connected—and turn off 5V src.



$$R_{Th} = 30k\Omega \parallel 120k\Omega$$

$$" = 30k\Omega \cdot 1 \parallel 4$$

$$" = 30\,k\Omega \cdot \frac{4}{5}$$

$$R_{Th} = 24\,k\Omega$$

For $t_1 = 1.92$ ms, we want $v_2(t_1) = V_1$.

$$V_1 = 2.2V = v_2(t_1 = 1.92\text{ms}) = 4V - 4V e^{-t_1/R_{Th}C}$$

or $\quad 2.2V - 4V = -4V e^{-t_1/R_{Th}C}$

or $\quad 1.8V = 4V e^{-t_1/R_{Th}C}$

or $\quad \dfrac{1.8V}{4V} = e^{-t_1/R_{Th}C}$

or $\quad \ln \dfrac{1.8V}{4V} = -t_1/R_{Th}C$

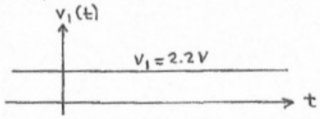
or $\quad R_{Th}C = -t_1/\ln\left(\dfrac{1.8}{4}\right) = \dfrac{-1.92\text{ ms}}{\ln(1.8/4)}$

or $\quad R_{Th}C = 2.4\text{ ms} = \text{time const, } \tau$

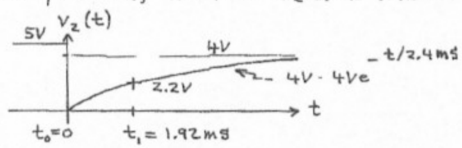
or $C = \frac{2.4\,ms}{R_{Th}} = \frac{2.4\,ms}{24\,k\Omega}$

or $C = 0.1\,\mu F$

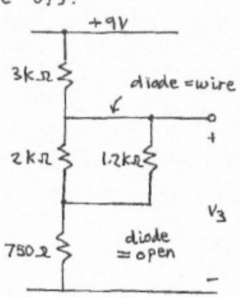
b) From part (a), we have $V_1 = 2.2V$ always.



c) From part (a), we have $V_2(t)$ as follows:



d) When V_o is high, the top diode will conduct and the bottom diode will be off.



We have a v-divider:

$$V_3 = +9V \cdot \frac{750\Omega + 1.2k\Omega \| 2k\Omega}{750\Omega + 1.2k\Omega \| 2k\Omega + 3k\Omega}$$

Note: $1.2k\Omega \| 2k\Omega = 0.2 \cdot 6 \| 10k\Omega$

$= 0.4 \cdot 3 \| 5k\Omega = 0.4 \cdot \frac{15}{8}\,k\Omega$

$= \frac{6}{8}\,k\Omega = 750\,\Omega$

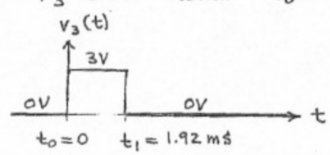
$$\therefore V_3 = 9V \cdot \frac{750 + 750}{750 + 750 + 3k} \frac{\Omega}{\Omega}$$

$$= 9V \cdot \frac{1.5k}{4.5k}$$

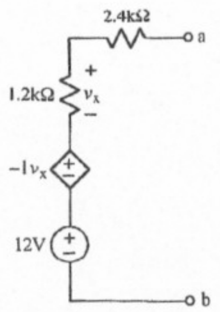
$V_3 = 3V$ when $V_o = +9V$

When $V_o = -9V$, the top diode will be off and the bottom diode will be on. Thus, the bottom diode will act like a wire shorting V_3 to ref.

$V_3 = 0V$ when $V_o = -9V$



3.

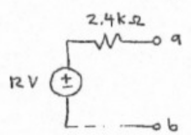


a) Find the Thevenin equivalent of the above circuit relative to terminals **a** and **b**.

b) If we attach R_L to terminals **a** and **b**, find the value of R_L that will absorb maximum power.

c) Calculate the value of that maximum power absorbed by R_L.

Sol'n: a) The dependent source and $1.2k\Omega$ resistor together produce a voltage drop of $-v_x + v_x = 0V$. Thus, they act like a wire and may be replaced by a wire:



This simplified circuit is the Thevenin equivalent.

$V_{Th} = 12V \qquad R_{Th} = 2.4k\Omega$

b) $R_L = R_{Th}$ for max power transfer

$R_L = 2.4\,k\Omega$

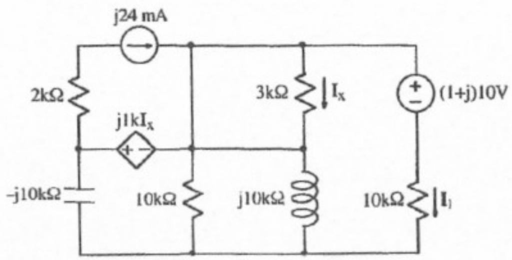
c) $$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(12V)^2}{4(2.4k\Omega)}$$

$$= \frac{12}{4} \cdot \frac{12}{2.4k} \; W$$

$$= 3.5 \; mW$$

$$P_{max} = 15 \; mW$$

4.



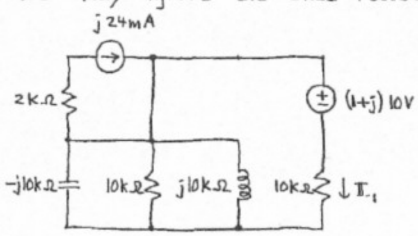
Rail voltages = ±9 V

a) A frequency-domain circuit is shown above. Write the value of phasor current $\mathbf{I}_1$ in rectangular form.

b) Given $\omega = 20k\ rad/s$, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

sol'n: a) First, we observe that $\mathbb{I}_x = 0$ since the 3kΩ resistor is shorted out by wires.

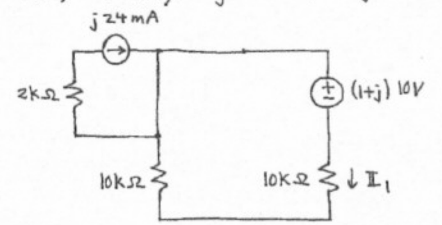
Thus, the dependent source = 0V, and we may ignore the 3kΩ resistor.



Second, we observe that the -j10kΩ and j10kΩ in parallel act like an open circuit:

$$-j10k\Omega \parallel j10k\Omega = j10k\cdot(-1\parallel 1)\,\Omega$$
$$'' \quad = j10k\left(\frac{-1}{0}\right)\Omega$$
$$'' \quad = \infty\ \Omega = \text{open}$$

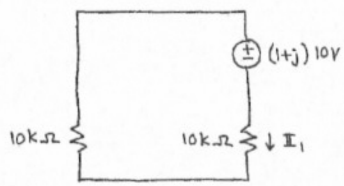
Thus, we may ignore the -j10kΩ and j10kΩ:



Third, we observe that the 2kΩ is in series with a current source and may be ignored.

Fourth, we observe that the current source is shorted by a wire that isolates it from the loop on the right. Thus, we may ignore the current source.

The circuit now consists of a voltage source and two resistors.



$$\mathbb{I}_1 = \frac{(1+j)\ 10V}{10k\Omega + 10k\Omega}$$

$$\mathbb{I}_1 = \frac{1+j}{2}\ mA = \frac{1}{2}\ mA + j\frac{1}{2}\ mA$$

b) In rectangular form $i_1(t)$ is a sum of a pure cos( ) and a pure sin( ):

$$i_1(t) = \frac{1}{2}\ mA \cos(20kt) - \frac{1}{2}\ mA \sin(20kt)$$

Note: $P^{-1}[j] = -\sin(\omega t)$

In polar form, $\mathbb{I}_1 = \frac{1}{2}\sqrt{2} \angle 45°\ mA$

$$i(t) = \frac{\sqrt{2}}{2}\ mA \cos(20kt + 45°)$$