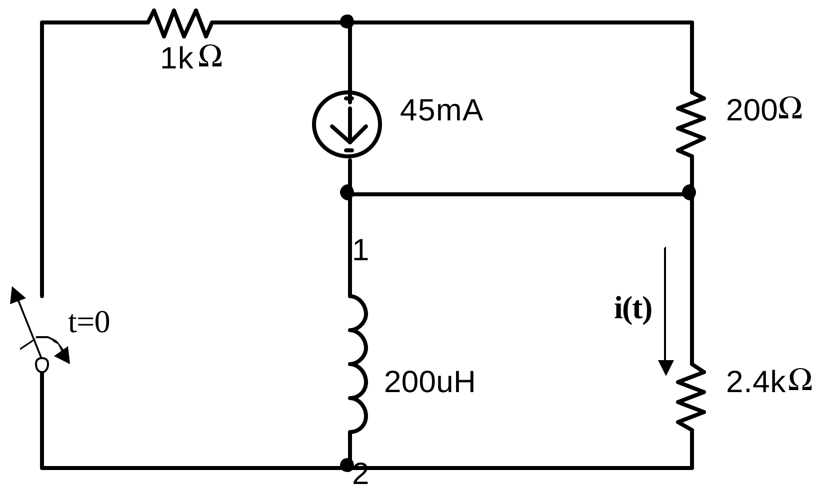
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

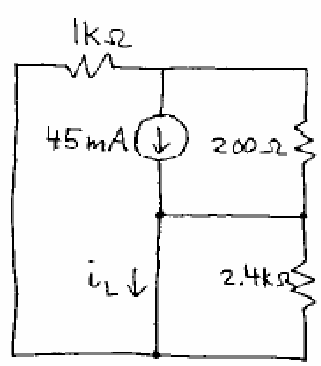
ECE 1270 HOMEWORK #6 Solution Summer 2007

1.

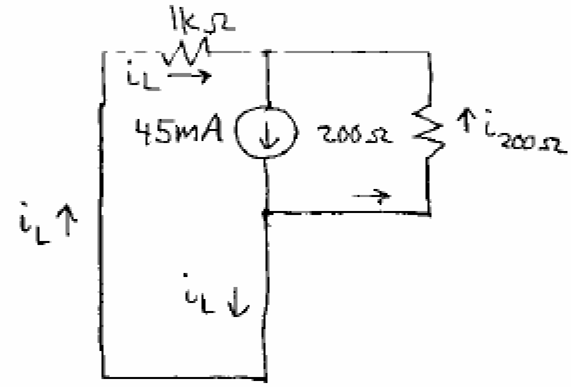


After being open for a long time, the switch closes at t=0.
a) Calculate the energy stored on the inductor as $t \to \infty$.
b) Write a numerical expression for i(t) for t>0.

sol'n: a) As $t \to \infty$, the L acts like a wire.



Since the 2.4 kΩ is shorted out by wires, we may ignore it. This leaves a current divider formed by the 1kΩ and 200Ω resistors for current from the 45 mA source.



$$i_L = 45\,mA \cdot \frac{200\,\Omega}{200\,\Omega + 1k\Omega} = 45\,mA \cdot \frac{1}{6}$$

$$i_L = 7.5\ mA$$

For an inductor, $w_L = \frac{1}{2} L i_L^2$:

$$w_L(t \to \infty) = \frac{1}{2} \cdot 200\,\mu H \cdot (7.5\,mA)^2$$

$$= 100\,\mu H \cdot \left(\frac{3}{4}\right)^2 (10\,mA)^2$$

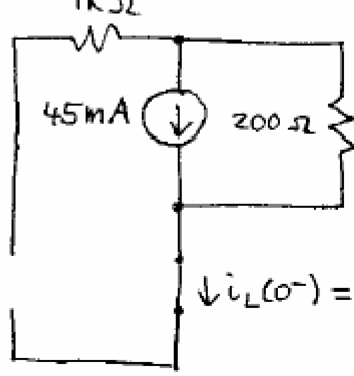
$$= \frac{9}{16} \cdot 100\,\mu \cdot 100\,\mu\ J$$

$$= \frac{9}{16} \cdot 10k\ \mu\mu\ J$$

$$= \frac{90}{16}\ nJ$$

$$w_L(t \to \infty) = 5.625\ nJ$$

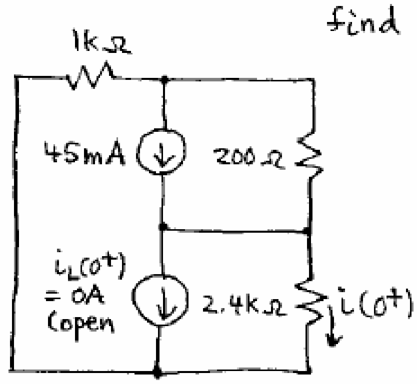
b) $t = 0^-$ model: L = wire, switch open
2.4kΩ shorted out (so ignore)
find $i_L(0^-)$

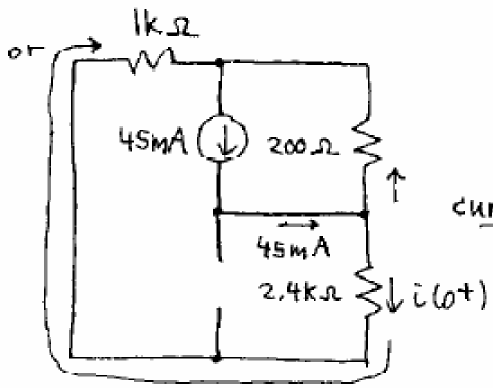


$\downarrow i_L(0^-) = 0$ (open circuit)

$t=0^+$ model: L = current source: $i_L(0^+) = i_L(0^-)$

find $i(0+)$ (switch closed)





current divider

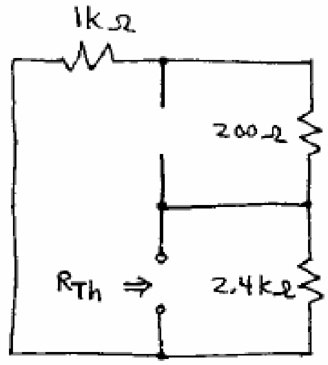
we have a current divider:

$$i(0^+) = 45mA \cdot \frac{200\Omega}{1k\Omega + 2.4k\Omega + 200\Omega} = \frac{45mA}{12}$$

or $i(0^+) = \frac{15}{4}$ mA

From part (a) we know $i(t\to\infty) = 0$ since the 2.4kΩ is shorted by the L and the wire on the middle right side.

$\tau = \frac{L}{R_{Th}}$ where R_{Th} = Thevenin R for terminals where L is connected.



We turn off the independent source and look into the terminals where L is connected.

$$R_{Th} = 2.4k\Omega \parallel (1k\Omega + 200\Omega)$$

$$= 2.4k\Omega \parallel 1.2k\Omega$$

$$= 1.2k\Omega \cdot 2 \parallel 1 = 1.2k\Omega \cdot \frac{2}{3}$$

$$R_{Th} = 800\Omega \qquad \tau = \frac{200\mu H}{800\Omega} = \frac{1}{4}\mu s$$

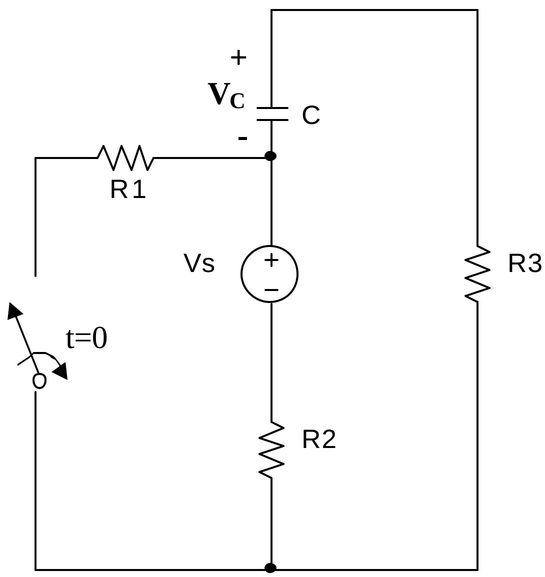
or $\tau = 250\ ns$

Use $i(t) = i(t\to\infty) + [i(0^+) - i(t\to\infty)]e^{-t/\tau} \qquad t>0$

$$\therefore\ i(t) = 0 + \left[\frac{15}{4}mA - 0\right]e^{-t/250ns} = 3.75mA e^{-t/250ns}$$

for $t>0$

2.



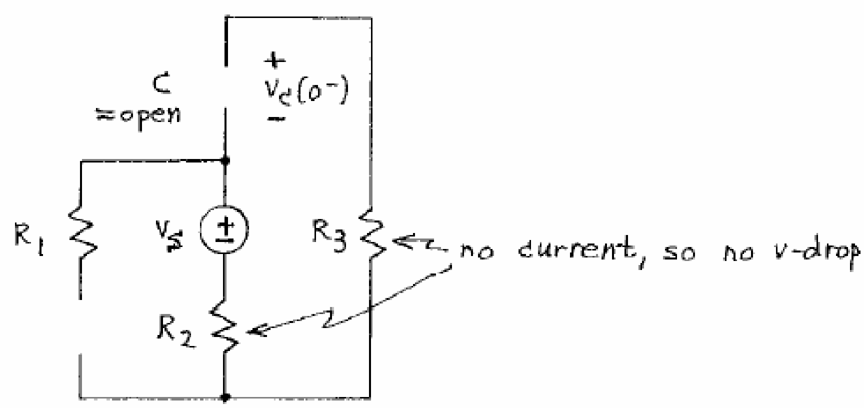
After being open for a long time, the switch becomes closed at t=0.

a) Write an expression for $V_c(t=0^+)$.

b) Write an expression for $V_c(t>0)$ in terms of R1, R2, R3, Vs, and C.

soln: a) We use $v_c(t=0^+) = v_c(0^-)$.

$t=0^-$ model: C acts like open circuit
switch is open



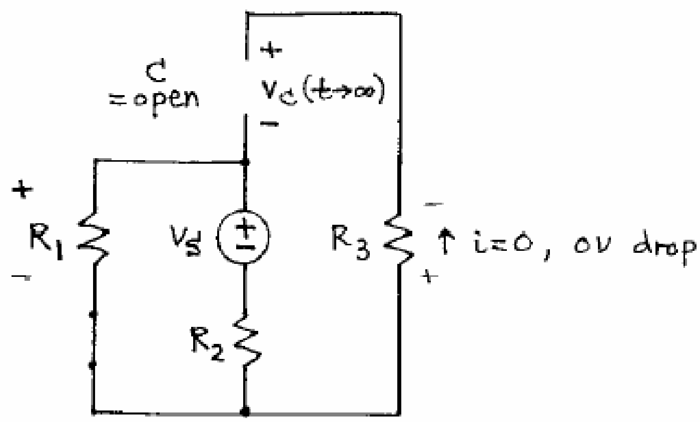
No current flows. $v_c(0^+) = v_c(0^-) = -V_s$

b) We use the general form of sol'n:

$$v_c(t>0) = v_c(t\to\infty) + \left[v_c(0^+) - v_c(t\to\infty)\right] e^{-t/\tau}$$

where $\tau = R_{Th} C$ (using Thevenin equiv where C connected)

$t\to\infty$ model: C = open, switch closed

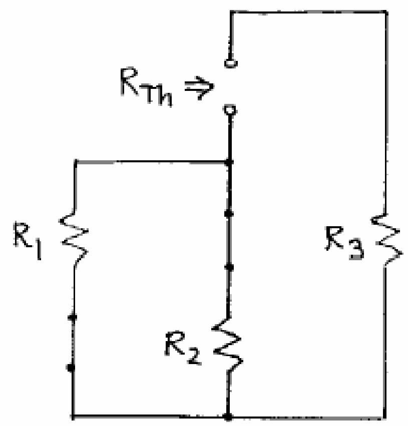


Since there is no v-drop across R_3, we have $v_c(t\to\infty) = -$ v-drop across R_1 from v-loop around outside of circuit.

v-drop across $R_1 = v_s \dfrac{R_1}{R_1+R_2}$ (v-divider)

$$\therefore\ v_c(t\to\infty) = -v_s \frac{R_1}{R_1+R_2}$$

$\tau = R_{Th}C$: We remove C and turn off v_s. Then we look into circuit from terminals where C connected.



We have $R_{Th} = R_1 \| R_2 + R_3$

Combining results, we have our solution:

$$v_C(t>0) = -v_g \frac{R_1}{R_1+R_2} + \left[-v_g + v_g \frac{R_1}{R_1+R_2}\right] e^{-t/(R_1 \| R_2 + R_3)C}$$

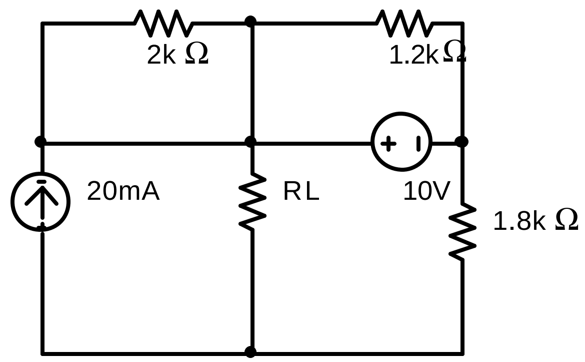
or

$$v_C(t>0) = -v_g \left\{ \frac{R_1}{R_1+R_2} + \frac{R_2}{R_1+R_2} e^{-t/(R_1 \| R_2 + R_3)C} \right\}$$

or

$$v_C(t>0) = -v_g + v_g \frac{R_2}{R_1+R_2}\left[1 - e^{-t/(R_1 \| R_2 + R_3)C}\right]$$

3.



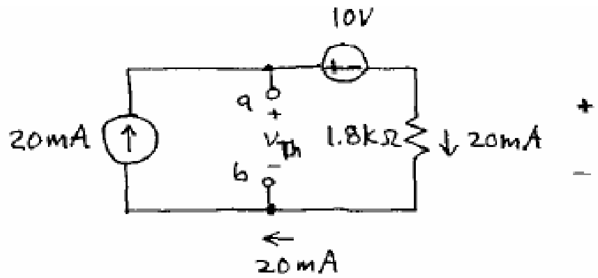
a) Calculate the value of RL that would absorb maximum power.
b) Calculate that value of maximum power RL could absorb.

soln: $R_L = R_{Th}$ for max power transfer where the Thevenin equivalent is with respect to terminals a and b (without R_L).

We observe that the 2kΩ resistor on top is shorted out by wires and may be ignored.

We also observe that the 1.2kΩ resistor on top is directly across the 10V source and may be treated as a separate circuit having no effect on the rest of the circuit other than to draw some current from the 10V source.

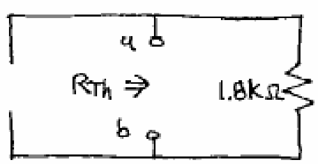
That leaves us with the following circuit:



V_{Th} = voltage across a, b terminals with no load from a to b.

We have $v_{ab} = 20mA \cdot 1.8k\Omega + 10V = 46V$

To find R_{Th}, we turn off the two independent sources and look into the circuit from the a and b terminals:



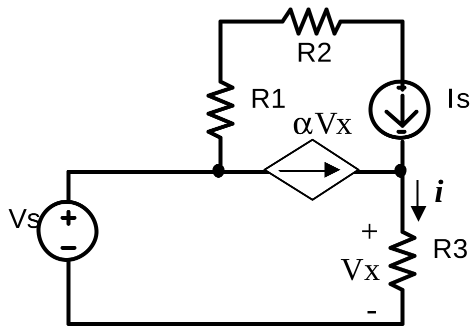
We see 1.8 kΩ across a and b.

$$R_{Th} = 1.8\ k\Omega$$

$$\therefore\ R_L = 1.8\, k\Omega$$

b) $$\text{max pwr} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(46V)^2}{4 \cdot 1.8k\Omega} \doteq 293.9\ mW$$

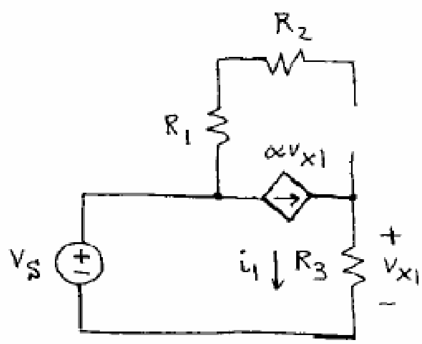
4.



Using superposition, derive an expression for ***i*** that contains no circuit quantities other than Is, Vs, R1, R2, R3, and α, where α>0.

sol'n: We turn on one independent source at a time, find i for each source, and sum the i's.

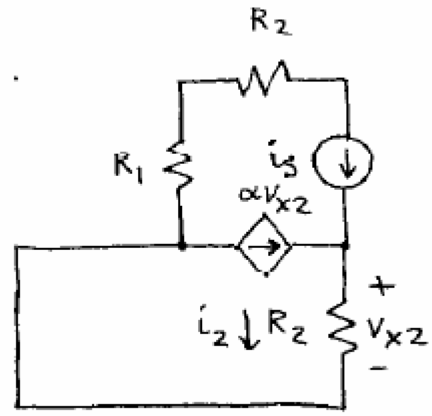
case I: v_s on and i_s off (= open)



We have $i_1 = \alpha v_{x1} = \dfrac{v_{x1}}{R_3}$

Unless $\alpha = \dfrac{1}{R_3}$ exactly, (which is impossible in practice), we must have $v_x = 0$, $i_1 = 0$.

case II: V_s off, i_s on
(=wire)



We have the following current summation at the node on the right side:

$$-i_s - \alpha i_2 R_2 + i_2 = 0$$

or

$$i_2(1 - \alpha R_2) = i_s$$

or

$$i_2 = \frac{i_s}{1 - \alpha R_2}$$

Now we sum the i_1 and i_2:

$$i = i_1 + i_2 = 0 + \frac{i_s}{1-\alpha R_2}$$

$$i = \frac{i_s}{1-\alpha R_2}$$