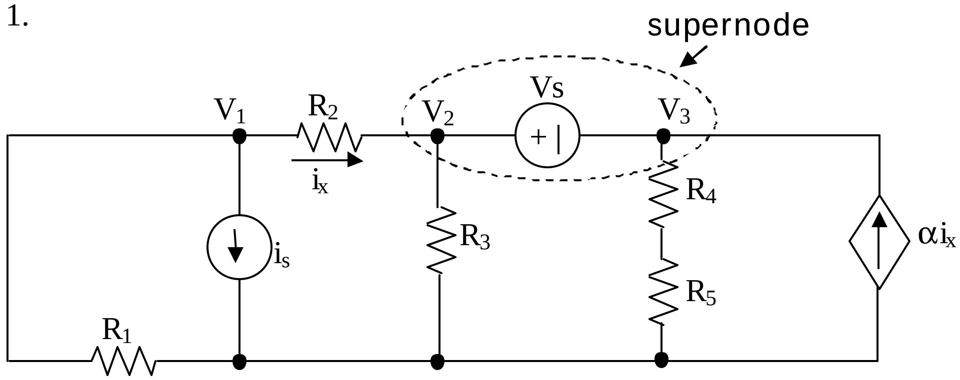
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1270 HOMEWORK #4 Solution Summer 2007

1.



For the circuit shown, write three independent equations for the node voltages V_1, V_2, and V_3. The quantity i_x must not appear in the equations.

First, we write i_x in terms of node v's:

$$i_x = \frac{v_1 - v_2}{R_2}$$

Second, we check whether node v_1 is part of a supernode, (i.e., is connected to another node by only a voltage source). v_1 is not part of a supernode, so we write a current-summation eq'n for it:

$$\frac{v_1}{R_1} + i_s + \frac{v_1 - v_2}{R_2} = 0A \qquad (1)$$

Third, we observe that v_2 and v_3 form a supernode. Thus, we write a eq'n for the summation of all currents out of the bubble shown on the circuit diagram:

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_3}{R_4 + R_5} - \alpha \underbrace{\left(\frac{v_1 - v_2}{R_2}\right)}_{i_x} = 0A \qquad (2)$$

Fourth, we add a voltage eq'n for nodes v_1 and v_2.

$$v_2 - v_3 = v_s \quad \text{or} \quad v_2 = v_3 + v_s \qquad (3)$$

2. Make a consistency check on your equations for Problem 1 by settings resistors and sources to values for which the values of V_1, V_2, and V_3 are obvious. State the values of resistors, sources, and for your consistency check, and show that your equations for Problem 1 are satisfied for these values. (In other words, plug in the values into your equations for Problem 1 and show that the left side and the right side of each equation are equal.)

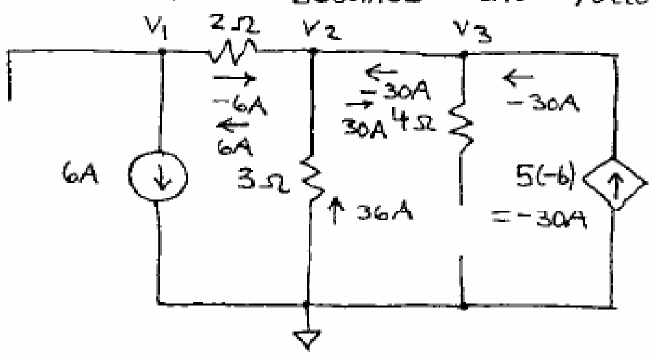
There are many possible consistency checks: one example is shown here.

Let $v_s = 0\,V$, $i_s = 6A$, $\alpha = 5$

$R_1 = \infty\,\Omega$ (open), $R_2 = 2\Omega$, $R_3 = 3\Omega$

$R_4 = 4\Omega$, $R_5 = \infty\,\Omega$ (open)

Our circuit becomes the following:



We have $i_x = -i_s = -6A$. Thus $\alpha i_x = -30A$.

From a current summation at node v_1, we conclude that 36A flows up thru the 3Ω resistor.

$\therefore\quad v_2 = -36A \cdot 3\Omega = -108\ V$

Since the v_2 and v_3 nodes are connected by a $0V$ source (a wire) we have

$$v_2 = v_3 = -108V.$$

For v_1, we have

$$v_1 = v_2 - 6A \cdot 2\Omega = v_2 - 12V = -120V.$$

Now we check to see if eq'ns (1), (2), and (3) from earlier hold when we substitute values from the consistency check:

$$(1)\quad \frac{-120V}{\infty\,\Omega} + 6A + \frac{-120V-(-108V)}{2\,\Omega}$$

$$= 6A - \frac{12V}{2\,\Omega} = 0V \quad \checkmark \text{ (consistent)}$$

$$(2)\quad -\frac{108V-(-120V)}{2\,\Omega} + \frac{-108V}{3\,\Omega} + \frac{-108V}{4\,\Omega+\infty\,\Omega} - 5\left(\frac{-120-(-108)}{2\,\Omega}\right)V$$

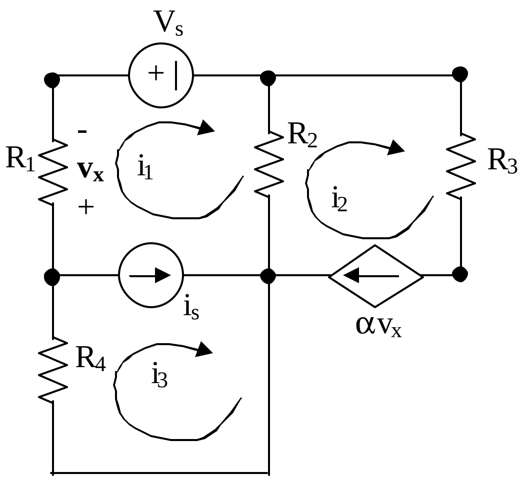
$$= \frac{12V}{2\,\Omega} - \frac{108V}{3\,\Omega} - 0A - 5\frac{(-12V)}{2\,\Omega}$$

$$= 6A - 36A + 30A = 0A \quad \checkmark \text{ (consistent)}$$

$$(3)\quad -108V-(-108V) = 0V = V_s \quad \checkmark \text{ (consistent)}$$

All three eq'ns are satisfied by values from the consistency check, giving us confidence that our node-voltage eq'ns are correct.

3.



For the circuit shown, write three independent equations for the three mesh currents i_1, i_2, and i_3. The quantity v_x must not appear in the equations.

sol'n: First, we write v_x in terms of mesh currents:

$$v_x = i_1 R_1$$

Note: current in R_1 is i_1 since R_1 is on the outside edge of the circuit.

Second, we observe that i_s is between the i_1 and i_3 loops, meaning that the i_1 and i_3 loops form a super mesh. Thus, we write a v-loop eq'n for the outside path around the i_1 and i_3 loops:

$$-i_1 R_1 + v_s - i_1 R_2 + i_2 R_2 - i_3 R_4 = 0\,\text{V} \qquad (1)$$

Third, we write a current summation eq'n for the source, i_s, between the i_1, i_3 loops:

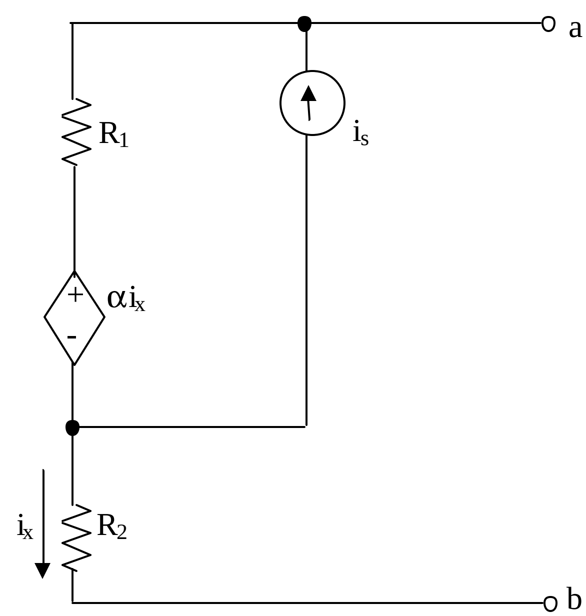
$$i_s = i_3 - i_2 \qquad (2)$$

Note: i_3 has a + sign because it flows in the same direction as i_s, whereas i_2 has a − sign because it flows in the opposite direction from i_s.

Fourth, we observe that the i_2 loop has a current source that is on the outside edge of the circuit. The loop current, i_2, must equal the current flowing in the current source:

$$i_2 = \alpha\,(\overset{v_x}{i_1 R_1}) \qquad (3)$$

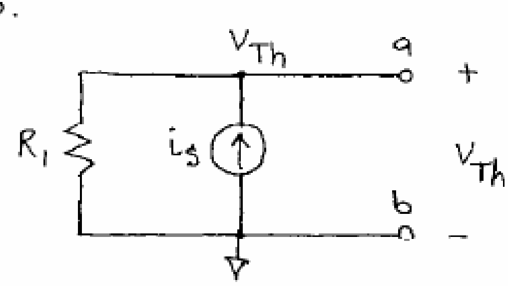
4.



Find the Thevenin equivalent circuit at terminals a-b. i_x must not appear in your solution.
Note: $0 < a < 1$.

$V_{Th} = V_{a,b}$ open circuit (always true)

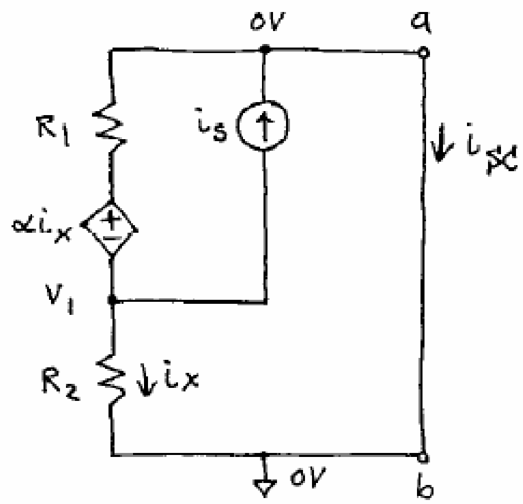
If we have an open circuit, $i_x = 0$ and there is no voltage drop across R_2 and $\alpha i_x = 0$ acts like a wire. An equivalent circuit model for finding v_{Th} is shown below.



We see by inspection that v_{Th} is across R_1 and $V_{Th} = i_s R_1$ by Ohm's law.

$$v_{Th} = i_s R_1$$

To find R_{Th}, we short the a, b terminals and measure the short-circuit current, i_{sc}.



One way to solve the circuit is to use the node-voltage method, as is done here.

First, we define i_x in terms of node-voltage:

$$i_x = \frac{v_1}{R_2}$$

Second, we write a current summation eq'n for node v_1:

$$\frac{v_1 + \alpha \frac{v_1}{R_2}}{R_1} + i_s + \frac{v_1}{R_2} = 0A$$

or

$$v_1\left(\frac{1}{R_1}+\frac{\alpha}{R_1R_2}+\frac{1}{R_2}\right) = -i_s$$

or $$v_1 = -i_s \cdot R_1 \,\Big\|\, \frac{R_1R_2}{\alpha} \,\Big\|\, R_2$$

Third, we observe that $i_{sc} = -i_x = -\dfrac{v_1}{R_2}$:

$$i_{sc} = i_s \cdot \left(R_1 \,\Big\|\, \frac{R_1R_2}{\alpha} \,\Big\|\, R_2\right) \cdot \frac{1}{R_2}$$

Fourth, we use $R_{Th} = \dfrac{v_{Th}}{i_{sc}}$:

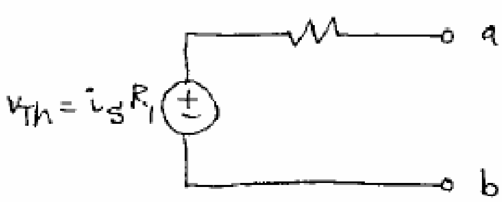
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{i_s\ R_1}{i_s \cdot R_1 \,\Big\|\, \frac{R_1R_2}{\alpha} \,\Big\|\, R_2 \cdot \frac{1}{R_2}}$$

$$R_{Th} = \frac{R_1\,R_2}{R_1 \,\Big\|\, \frac{R_1R_2}{\alpha} \,\Big\|\, R_2}$$

or $$R_{Th} = R_1R_2\left(\frac{1}{R_1}+\frac{\alpha}{R_1R_2}+\frac{1}{R_2}\right) = R_2+\alpha+R_1$$

Thevenin equivalent:

$R_{Th} = R_1 + R_2 + \alpha$



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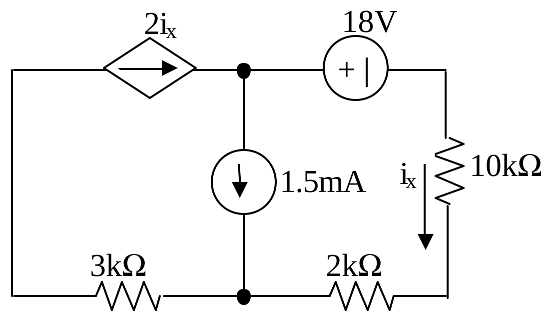

Calculate the power consumed (ie dissipated) by the 18V source. **Note:** If a source supplies power, the power it consumes is negative.

We may solve this circuit in a variety of ways, but a simple way is to write a current summation eq'n for the v_1 node:

$$-2i_x + 1.5\,mA + i_x = 0A \qquad (1)$$

Note: We normally avoid using i_x. To do so here, we would use the following eq'n for i_x:

$$i_x = \frac{v_1 + 18V}{10k\Omega + 2k\Omega}$$

This leads to a correct solution but requires more writing.

From eq'n (1), we have

$$i_x = 1.5\,mA$$

Our pwr is $p = i_x \cdot 18V = 1.5mA \cdot 18V = 27mW$