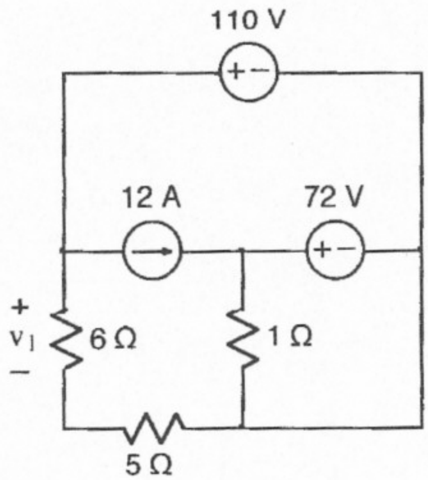
ECE 1270
Summer 2007

HW 2 Solution

1.



Calculate v_1.

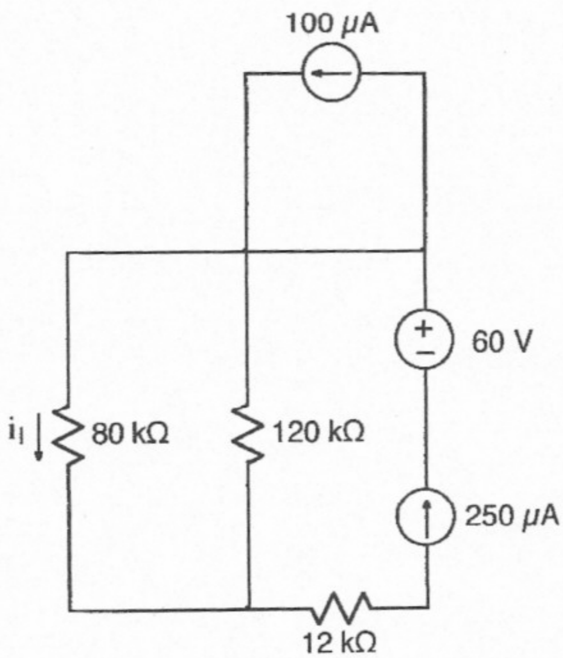
soln: The 5Ω and 6Ω R's are in series and are connected directly across the 110V source.

Thus, we have a voltage divider.

$$v_1 = 110V \cdot \frac{6\Omega}{5\Omega + 6\Omega} = 110V \cdot \frac{6}{11}$$

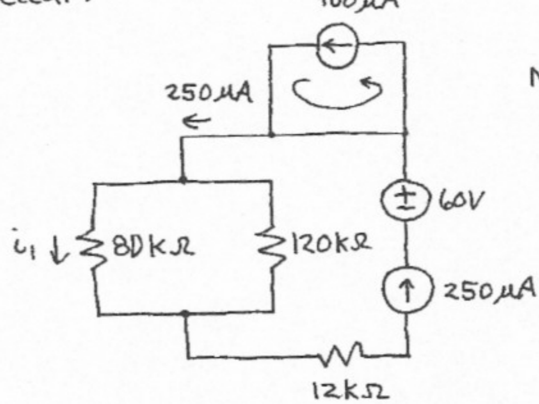
or $v_1 = 60V$

2.



Calculate i_1.

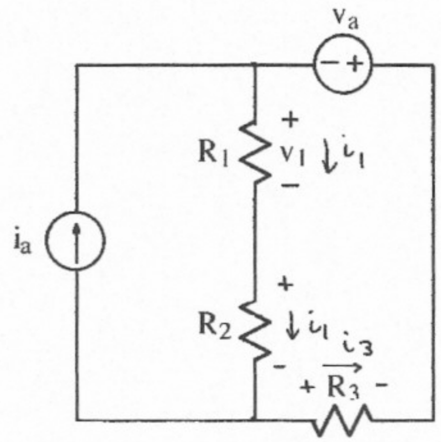
sol'n: The 80kΩ and 120kΩ R's are in parallel and carry all the current from the 250μA source. Redrawing the circuit makes this clear:



Note: The 100 μA circulates in a small loop. Only the 250μA flows around the lower loop.

We have a current divider. $i_1 = \frac{250\mu A \cdot 120k\Omega}{80k\Omega + 120k\Omega} = 150\mu A.$

3.



Derive an expression for v_1. The expression must not contain more than the circuit parameters v_a, i_a, R_1, R_2, and R_3.

sol'n: We add labels for voltages and currents as shown above. (Note that the labeling may vary, but we must obey the passive sign convention wherein current measurement arrows point toward − signs of voltage measurements.) We now apply Kirchhoff's laws, (and use Ohm's law to write v-drops as iR).

v-loop on right: $+i_1 R_2 + i_1 R_1 + v_a + i_3 R_3 = 0\text{V}$ (1)

No other v-loop eqn's since all other v-loops pass thru i_a current source.

i-sum at top node: $-i_a + i_1 - i_3 = 0\text{A}$ (2)

i-sum at bottom node is redundant.

From Eq'n (2) we can find i_3 in terms of i_1:

$$i_3 = i_1 - i_a \tag{3}$$

Using (3), we eliminate i_3 in (1):

$$i_1 (R_1 + R_2) + (i_1 - i_a) R_3 = -v_a \tag{4}$$

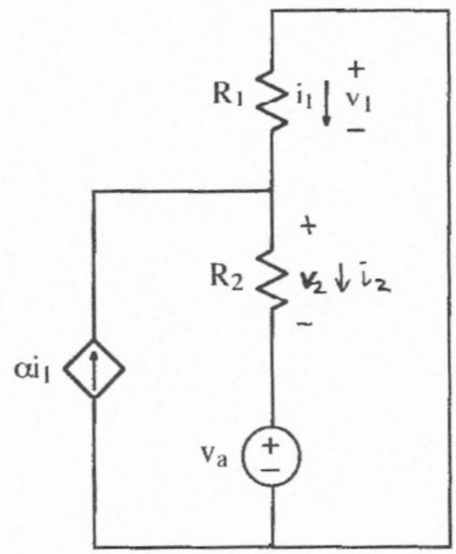
or $$i_1 (R_1 + R_2 + R_3) = i_a R_3 - v_a \tag{5}$$

or $$i_1 = \frac{i_a R_3 - v_a}{R_1 + R_2 + R_3} \tag{6}$$

By Ohm's law, $v_1 = i_1 R_1$:

$$v_1 = (i_a R_3 - v_a) \frac{R_1}{R_1 + R_2 + R_3} \tag{7}$$

4.



a) Derive an expression for v_1. The expression must not contain more than the circuit parameters α, v_a, R_1, and R_2. **Note:** $\alpha > 0$.

b) Make at least one consistency check (other than a units check) on your expression. In other words, choose component values that make it possible to solve the circuit by inspection, and verify that your answer to (a) gives that answer. Specify your consistency check by listing a numerical value for every source and resistor.

sol'n: a) After labeling i's and v's as shown above, we use Kirchhoff's laws and Ohm's laws to solve the circuit.

v-loop on right: $+v_a + i_2 R_2 + i_1 R_1 = 0V$

All other v-loops pass thru αi_1 source, so we ignore them.

i-sum at node between R_1 and R_2: $-\alpha i_1 - i_1 + i_2 = 0A$

Other i-sum at bottom node is redundant.

We use the i-sum eq'n to eliminate i_2:

$$i_2 = (\alpha+1)\, i_1$$

Substiting for i_2 in the v-loops yields the following result:

$$(\alpha+1)\, i_1 R_2 + i_1 R_1 = -v_a$$

$$\text{or} \quad i_1 \left[R_1 + (1+\alpha) R_2 \right] = -v_a$$

$$\text{or} \quad i_1 = \frac{-v_a}{R_1 + (1+\alpha) R_2}$$

Using Ohm's law, we obtain v_1:

$$v_1 = i_1 R_1 = -v_a \frac{R_1}{R_1 + (1+\alpha) R_2}$$

b) Many consistency checks are possible. Two examples are shown here.

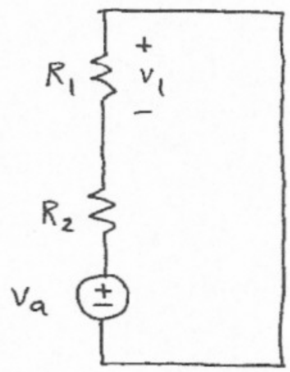
ex 1: $R_1 = 0\,\Omega$ (wire) so $\underline{v_1 = 0V}$
$R_2 = 2\,\Omega, \quad \alpha = 6, \quad v_a = 12V$

Now we check that formula in (a) gives the expected answer:

$$v_1 = -12V \cdot \frac{0\,\Omega}{0\,\Omega + (1+6)\cdot 2\,\Omega} = 0V \quad \checkmark \ \underline{\text{works}}$$

ex 2: $\alpha = 0$ (current source off - open circuit disappears)

We have a voltage divider left:



Let $R_1 = 1\Omega$, $R_2 = 2\Omega$, $v_a = 3V$.

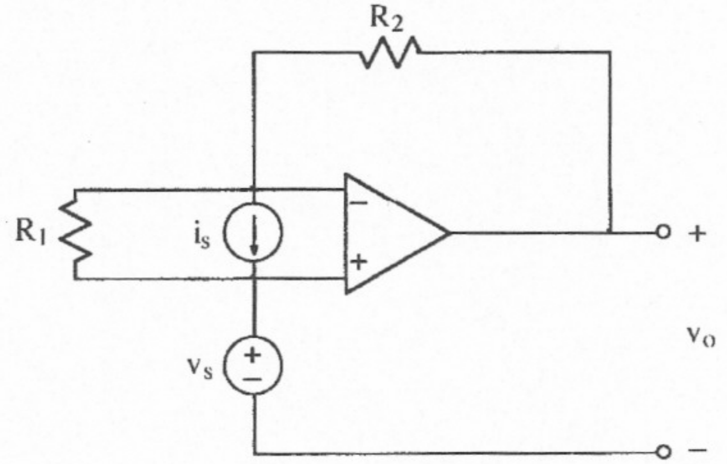
Then $$v_1 = -\,3V \cdot \frac{1\Omega}{1\Omega + 2\Omega} = \underline{-1V}.$$

Now we check whether formula from (a) agrees:

$$v_1 = -3V \frac{1\Omega}{1\Omega + (1+0)\,2\Omega} = -3V \cdot \frac{1}{3}$$

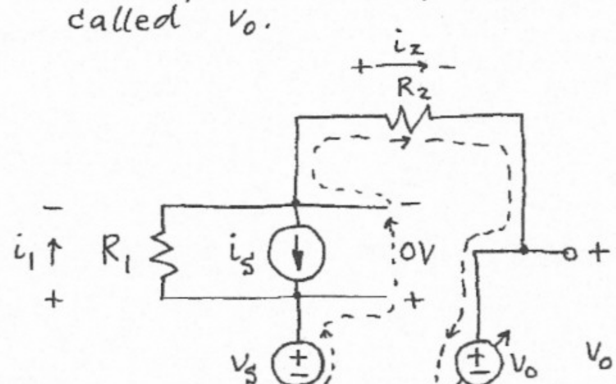
or $v_1 = -1V$ ✓ <u>works</u>

5.



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for v_o in terms of not more than v_s, i_s, R_1, and R_2.

sol'n: We assume a 0V drop across the op-amp inputs since it is in linear mode, and we replace the op-amp with a source called v_o.



We observe that the 0V drop across the op-amp inputs is also across R_1. Thus $i_1 = 0$ A.

There is only one other v-loop that avoids current source i_s and passes thru the 0V drop across the inputs. This is shown by the dashed line.

$$+v_s - 0V - i_2 R_2 - v_o = 0V$$

We have an i-sum above the i_s source:

$$\underset{\underset{0A}{\shortparallel}}{-i_1} + i_s + i_2 = 0\,A \quad \Rightarrow i_2 = -i_s$$

(The other i-sum below the i_s source is redundant.)

Substituting for i_2 in the v-loop eq'n yields our answer:

$$v_s - -i_s R_2 = v_o$$

or $\quad v_o = v_s + i_s R_2$