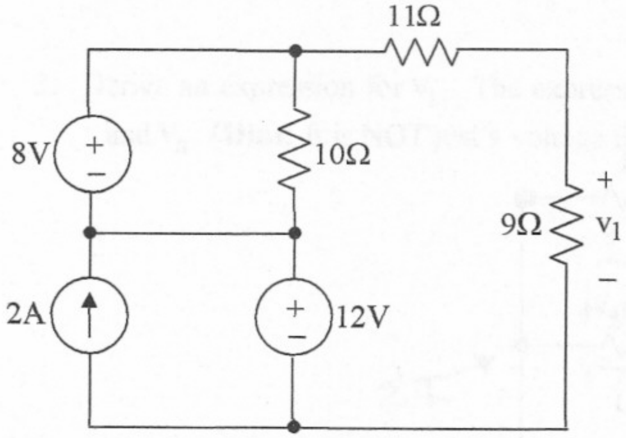
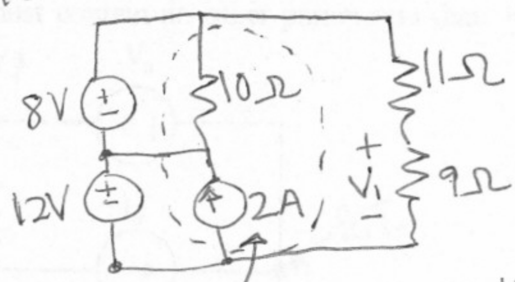
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1270 HOMEWORK #2 Spring 2008

1. Calculate V_1.



Notice that it can be redrawn as

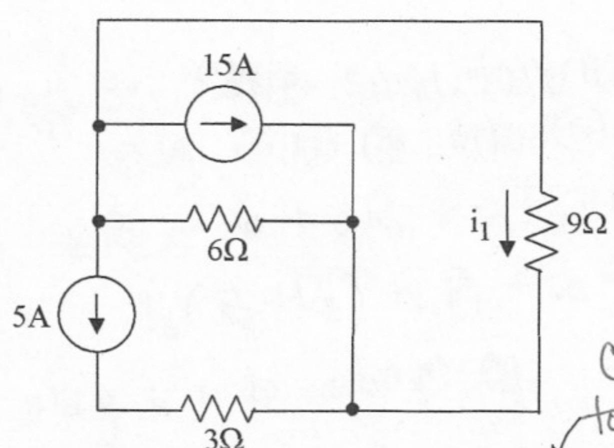


No effect on Voltage

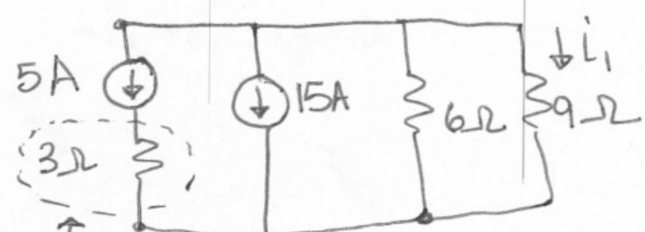
$\therefore$ V-divider :

$$V_1 = \frac{(12+8)9}{9+11} = \frac{20(9)}{20} = \boxed{9V}$$

2. Calculate i_1.



Can be redrawn:



This will change Voltage, no change in current. 5A fixes branch current

Can add both current sources together.

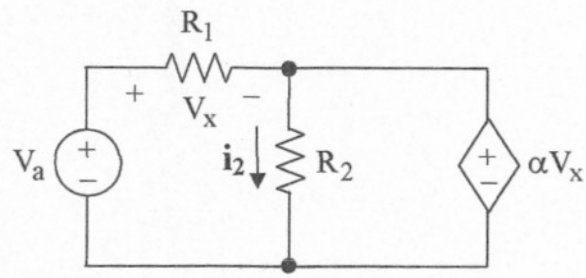
I-divider:

$$i_1 = -\frac{(5+15)(6)}{6+9} = \frac{-20(6)}{15} = \frac{-4(2)}{1} = \boxed{-8A}$$

$$i_1 = i_3 - i_a = \frac{(i_a R_1 - V_a)}{(R_1+R_2+R_3)} - i_a$$

$$i_1 = \frac{i_a(R_2+R_3) - V_a}{(R_1+R_2+R_3)}$$

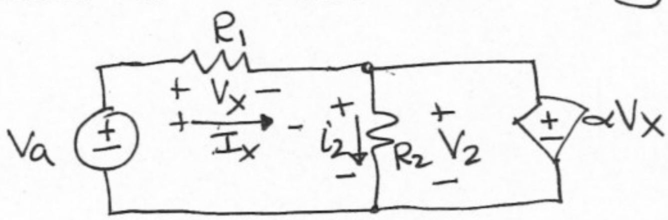
3. (30 points)



20 pts a) Drive the expression for i_2 containing not more than circuit parameters α, R_1, R_2, and V_a.

10 pts b) State the symbolic expression for power in the resistor, R_2. Use only α, R_1, R_2, and V_a in the expression.

a). ① Label all currents and voltages ⟹



② Ohm's Law ⟹

$V_x = I_x R_x$

$V_2 = i_2 R_2$

③ V loop ⟹

$+V_a - V_x - V_2 = 0$

$-V_2 + \alpha V_x = 0 \Rightarrow V_2 = \alpha V_x = \frac{\alpha V_a}{(1+\alpha)}$

(★) $+V_a - V_x - \alpha V_x = 0$

$V_a - V_x(1+\alpha) = 0$

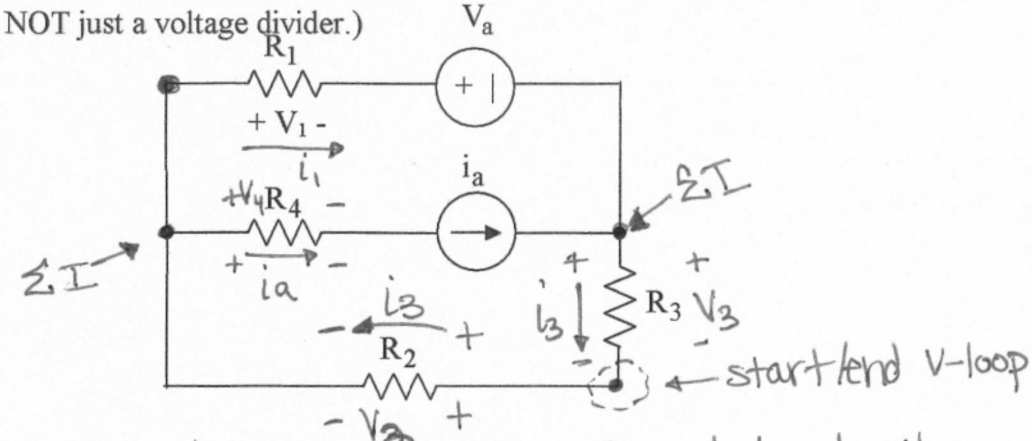
$\frac{V_a}{1+\alpha} = V_x$

$i_2 = \frac{V_2}{R_2} = \boxed{\frac{\alpha V_a}{(1+\alpha)R_2}}$

b) power $= V_2 \cdot i_2 = \boxed{\frac{\alpha^2 V_a^2}{(1+\alpha)^2 R_2}}$

If $(\alpha, V_a, R_2) > 0$ than <u>absorbs</u> power

3. Derive an expression for V_1. The expression must contain no other parameters than R_1, R_2, R_3, i_a, R_4 and V_a. (**Hint:** It is NOT just a voltage divider.)



Step 1: Label a I and V through every R. Label polarity

Step 2: a. Ohm's Laws:

① $V_1 = i_1 R_1$

② $V_2 = i_3 \cdot R_2$

③ $V_3 = i_3 \cdot R_3$

④ $V_4 = i_a R_4$

b. V-loop: (avoid taking loop through Isrc.)

⑤ $+V_3 + V_a + V_1 + V_2 = 0$

c. ΣI:

⑥ $+i_a + i_1 - i_3 = 0$

$-i_a - i_1 + i_3 = 0$

Step 3: Solve simultaneous eq.

plug ①, ②, ③ into ⑤

$i_3 R_3 + V_a + i_1 R_1 + i_3 R_2 = 0$

$i_3(R_2 + R_3) + i_1 R_1 + V_a = 0$ ← eq. with i_1 & i_3 unknown

⑥ also has i_1 & i_3 "

∴ Solve ⑥ for i_1 as function of i_3 ⇒ $i_1 = (i_3 - i_a)$

plug i_1 into above eq.

$i_3(R_2 + R_3) + (i_3 - i_a)R_1 + V_a = 0$

$i_3(R_1 + R_2 + R_3) - i_a R_1 + V_a = 0$

$\therefore\ i_3 = \dfrac{(i_a R_1 - V_a)}{R_1 + R_2 + R_3}$

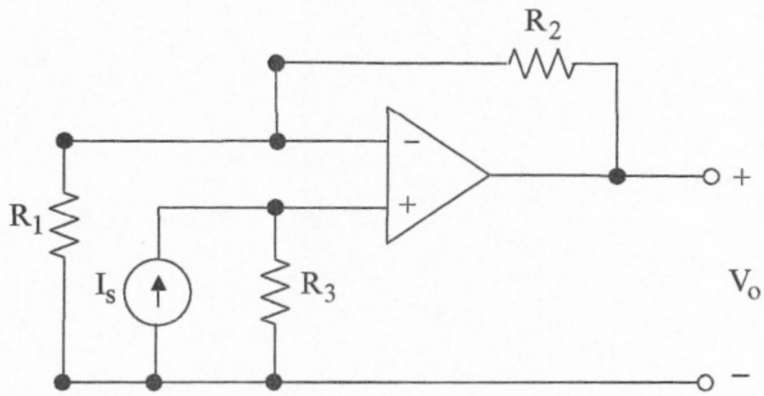
$i_1 = i_3 - i_a = \dfrac{(i_a R_1 - V_a)}{(R_1 + R_2 + R_3)} - i_a$

$i_1 = \dfrac{i_a(R_2 + R_3) - V_a}{(R_1 + R_2 + R_3)}$

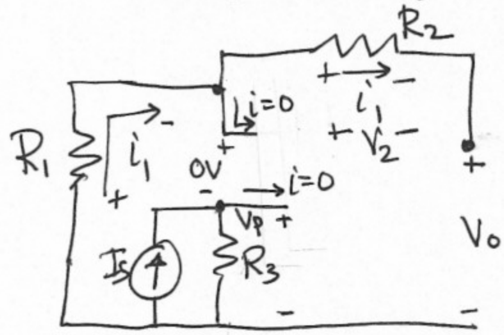
$V_1 = i_1 \cdot R_1$

$\therefore\ \boxed{V_1 = \dfrac{i_a R_1 (R_2 + R_3) - V_a R_1}{(R_1 + R_2 + R_3)}}$

4. (30 points)



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for V_o in terms of not more than $I_s, R_1, R_2,$ and R_3.



$$V_p = +I_s(R_3)$$

$$① \; +I_s(R_3) - V_2 - V_o = 0$$

$$V_2 = (i_1 \cdot R_2)$$

$$+I_s(R_3) - i_1 R_2 - V_o = 0$$

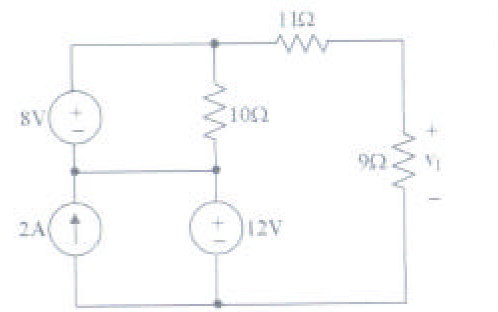
$$+i_1 R_1 + I_s R_3 = 0$$

$$i_1 = \frac{-I_s R_3}{R_1}$$

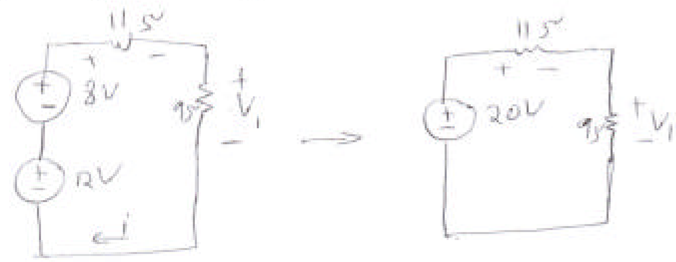
From ① : $I_s R_3 + \frac{I_s R_3 R_2}{R_1} = V_o$

$$\therefore \boxed{V_o = \frac{I_s R_3 (R_1 + R_2)}{R_1}}$$

1. Calculate V_1.



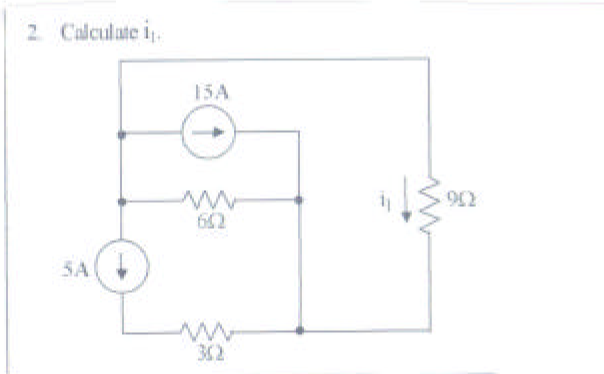
1) The voltage accross the 10Ω resistor is equal to 8V (parallel to the voltage source). We can redraw the right loop as follows



Then, it is a voltage divider

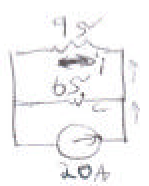
$$V_1 = 20 \times \frac{9}{9+11} = 20 \times \frac{9}{20} = 9V$$

2. Calculate i_1.



2.) We can redraw the circuit as follows

Since the two current sources contribute currents at the same direction to ~~node~~ node 1, and since we do not care about the voltage across the 3Ω resistor, we can again draw the circuit as follows:-

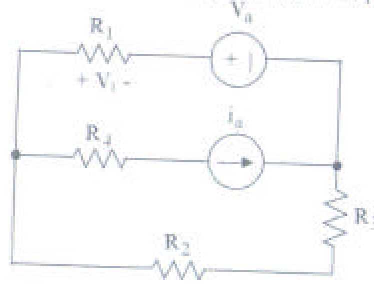


then using current divider:-

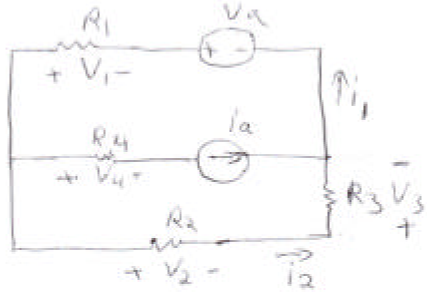
$$i = -20 \times \frac{6}{9+6} = -\cancel{20}^{4} \times \frac{\cancel{6}^{2}}{\cancel{15}_{3}} = -8A$$

The negative sign is because i is in the opposite direction of the actual current

3. Derive an expression for V_1. The expression must contain no other parameters than R_1, R_2, R_3, i_a, and V_a.
(Hint: It is NOT just a voltage divider.)



3. Name the voltages and currents..



V. loop:- take the outter loop because we don't know the Voltage accross the current source.

$$V_a + V_1 - V_3 - V_2 = 0 \quad \rightarrow 1$$

we need V_3 and V_2 in terms of R_3, R_2

Ohm's:- $V_3 = R_3 i_2$, $V_2 = R_2 i_2 \quad \rightarrow 2$

Now, we need i_2 in terms of i_a & i_1

$$i_2 + i_a - i_1 = 0 \Rightarrow i_2 = i_1 - i_a \rightarrow 3$$

We still need i_1 in terms of V_1 and R_1

$$i_1 = \frac{-V_1}{R_1}$$

substituting in 3

$$i_2 = \frac{-V_1}{R_1} - i_a$$

substituting for i_2 in 2

$$V_3 = R_3\left(\frac{-V_1}{R_1} - i_a\right), \quad V_2 = R_2\left(\frac{-V_1}{R_1} - i_a\right)$$

substituting for V_3 and V_2 in 1, we get

$$V_a + V_1 - R_3\left(\frac{-V_1}{R_1} - i_a\right) - R_2\left(\frac{-V_1}{R_1} - i_a\right) = 0$$

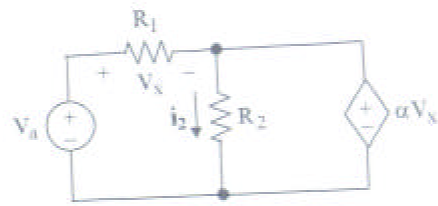
$$\Rightarrow V_1 - (R_3 + R_2)\left(\frac{-V_1}{R_1} - i_a\right) = -V_a$$

$$\Rightarrow V_1 + \frac{V_1(R_3+R_2)}{R_1} + i_a(R_3+R_2) = -V_a$$

$$\Rightarrow V_1\left(1 + \frac{R_3+R_2}{R_1}\right) = -V_a - i_a(R_3+R_2)$$

$$\Rightarrow V_1\left(\frac{R_1+R_2+R_3}{R_1}\right) = -V_a - i_a(R_2+R_3)$$

$$\Rightarrow V_1 = \frac{-R_1\left(V_a + i_a(R_2+R_3)\right)}{R_1 + R_2 + R_3}$$



a) Derive the expression for i_2 containing not more than circuit parameters α, R_1, R_2, and V_a.

b) State the symbolic expression for power in the resistor, R_2. Use only α, R_1, R_2, and V_a in the expression.

4. a) The voltage accross R_2 is αV_x because they are in parallel.

Using the outter V. loop

$$V_a - V_x - \alpha V_x = 0$$

$$\Rightarrow -V_x - \alpha V_x = -V_a$$

$$\Rightarrow V_x(1+\alpha) = V_a$$

$$\Rightarrow V_x = \frac{V_a}{1+\alpha} \longrightarrow 1$$

Ohm's law:- $i_2 = \frac{V_2}{R_2}$, $V_2 = \alpha V_x$

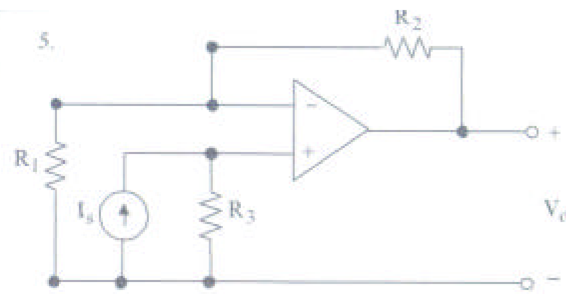
$i_2 = \frac{\alpha V_x}{R_2}$ using 1, we get

$$i_2 = \frac{\alpha V_a}{R_2(1+\alpha)}$$

b) $P_{R_2} = i_2 V_2 = \frac{\alpha V_a}{R_2(1+\alpha)} \times \alpha V_x$

$$= \frac{\alpha V_a}{R_2(1+\alpha)} \times \frac{\alpha V_a}{(1+\alpha)} = \left(\frac{\alpha V_a}{1+\alpha}\right)^2 / R_2$$

5.



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for V_O in terms of not more than I_s, R_1, R_2, and R_3.

5. $V_1 = V_2 = I_s R_3 \quad \text{— (1)}$

$$V_1 = V_o \times \frac{R_1}{R_1 + R_2} \quad \text{(voltage divider)}$$

$$\Rightarrow V_o = \frac{V_1 (R_1 + R_2)}{R_1}$$

using (1) and substituting for V_1

$$V_o = \frac{I_s R_3 (R_1 + R_2)}{R_1}$$