



Ex: The following equation describes the voltage, v_L , across an inductor as a function of time. Find an expression for the current, $i_L(t)$, through the inductor as a function of time. Assume that $i_L(t=0) = 0$ A.

$$v_L(t) = 2 + 6(1 - e^{-t/12.5\mu\text{s}}) \text{ kV}$$

SOL'N: We use the defining equation for an inductor and solve for i in terms of v .

$$v_L = L \frac{di_L}{dt}$$

First, we multiply both sides by dt .

$$v_L dt = L di_L$$

Second, we integrate both sides and use limits that correspond to the variable of integration for each side and are evaluated at the same points in time for both sides.

$$\int_0^t v_L dt = \int_{i_L(t=0)}^{i_L(t)} L di_L$$

The integral on the right side simplifies nicely.

$$\int_0^t v_L dt = Li_L \Big|_{i_L(t=0)}^{i_L(t)} = L[i_L(t) - i_L(t=0)]$$

or

$$i_L(t) = \frac{1}{L} \int_0^t v_L dt + i_L(t=0)$$

The above expression applies to any inductor in any circuit.

We now substitute the formula given for $v_L(t)$ and the value given for $i_L(t=0)$ to find $i_L(t)$:

$$i_L(t) = \frac{1}{L} \int_0^t [2 + 6(1 - e^{-t/12.5\mu\text{s}}) \text{ kV}] dt + 0 \text{ A}$$

or

$$i_L(t) = \frac{1}{L} \int_0^t [8 - 6e^{-t/12.5\mu\text{s}}] \text{ kV} dt + 0 \text{ A}$$

or

$$i_L(t) = \frac{1}{L} \left[8t \Big|_0^t + 6 \cdot 12.5 \mu\text{s} \cdot e^{-t/12.5 \mu\text{s}} \Big|_0^t \right] \text{kV}$$

or

$$i_L(t) = \frac{1}{L} \left[8t + 75 \mu\text{s} \cdot \left(e^{-t/12.5 \mu\text{s}} - 1 \right) \right] \text{kV}$$

or

$$i_L(t) = \frac{1}{L} \left[8 \text{kV} \cdot t + 75 \text{mV} \cdot \left(e^{-t/12.5 \mu\text{s}} - 1 \right) \right]$$