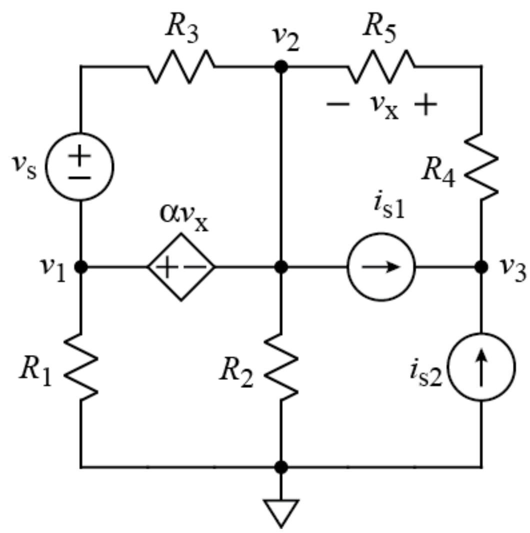
Ex:



For the circuit shown, start with three independent equations for the node-voltages, v_1, v_2, and v_3. The quantity v_x must not appear in the equations. Only component and source names may appear in answer.

Make at least one consistency check (other than a units check) on your three equations. In other words, choose component values that make the values of v_1, v_2, and v_3 obvious, and verify that your answer to problem 1 gives these values. State the values of resistors and sources for your consistency check.

SOL'N: Many solutions are possible. One possible check is as follows:

$$R_2 = \infty,\ R_3 = \infty,\ i_{s1} = 0,$$
$$R_1 = 1\,\Omega,\ R_4 = 4\,\Omega,\ R_5 = 5\,\Omega,$$
$$v_s = 24\,\text{V},\ \alpha = 12,\ i_{s2} = 6\,\text{A}$$

With these choices of values, we have open circuits for R_2 and R_3, and current source i_{s2} in series with R_4, R_5, and R_1. By Ohm's law, we get the following v-drops (with + on top of R_1 and + on bottom of R_4):

$$v_{R1} = 6\,\text{V},\ v_{R4} = 24\,\text{V},\ v_{R5} = 30\,\text{V}$$

We also have the v-drop for the dependent source as follows:

$$\alpha v_x = 12(30)\,\text{V} = 360\,\text{V}$$

Following v-drops from the reference = 0V, up through R_1, through the dependent source, through R_4 and R_5, we find the node v's:

$$v_1 = v_{R1} = 6\,\text{V}$$
$$\text{v}_2 = v_1 - \alpha v_\text{x} = 6\,\text{V} - 360\,\text{V} = -354\,\text{V}$$
$$v_3 = v_2 + v_{R5} + v_{R4} = -354\,\text{V} + 30\,\text{V} + 24\,\text{V} = -300\,\text{V}$$

Now we plug the numbers into the equations from node-v analysis to see if we get equality:

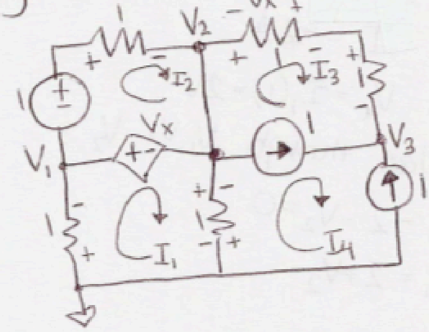
$$v_1 - \alpha(v_3 - v_2)\frac{R_5}{R_4 + R_5} = v_2$$
$$\frac{v_1}{R_1} + \frac{v_1 + v_s - v_2}{R_3} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_4 + R_5} = 0\,\text{V}$$
$$-i_{s1} - i_{s2} + \frac{v_3 - v_2}{R_4 + R_5} = 0\,\text{A}$$

or

$$6\text{V} - 12(-300 - -354)\frac{5}{4+5} \stackrel{?}{=} -354, \quad 6 - 12(54)\frac{5}{9} = 6 - 12(30) = -354 \ \surd$$
$$\frac{6\text{V}}{1\Omega} + \frac{6\text{V} + 24\text{V} - -354\text{V}}{\infty} + \frac{-354\text{V}}{\infty} + \frac{-354\text{V} - -300\text{V}}{4\Omega + 5\Omega} \stackrel{?}{=} 0\,\text{V}, \quad 6\text{A} - \frac{54\text{V}}{9\Omega} = 0\text{V} \ \surd$$
$$-0\,\text{A} - 6\,\text{A} + \frac{-300 - -354\text{V}}{4\Omega + 5\Omega} \stackrel{?}{=} 0\,\text{A}, \quad -6\,\text{A} + \frac{54\text{V}}{9\Omega} = 0\,\text{A} \ \surd$$

The equations hold true for this example. The node-v equations pass this consistency check.

(b) Using mesh currents:



$I_4 = -1$

$1 = I_4 - I_3$

$\therefore I_3 = I_4 - 1 = -2$

$V_x = -I_3(1) = +2$

$V_1 = -1(I_1)$

$V_2 = V_1 - V_x$

$V_3 = V_2 - I_3(2)$

$-1(I_1) - V_x - 1(I_1) + 1(I_4) = 0$

$I_1(2) = (-1) - 2 = \frac{-3}{2}$

$\therefore V_1 = +\frac{3}{2}$ ✓ same as eq. set $\left(V_1 = \frac{6}{4} = \frac{3}{2}\right)$

$V_2 = V_1 - V_x = \frac{3}{2} - \frac{4}{2} = -\frac{1}{2}$ ✓ same as eq. set $\left(V_2 = -\frac{2}{4} = -\frac{1}{2}\right)$

ΣI at V_3: $-I_3 - 1 - 1 = 0 \Rightarrow I_3 = -2$

$V_3 = V_2 - I_3(2) = -\frac{1}{2} + \frac{8}{2} = \frac{7}{2}$ ✓ same as eq. set $\left(V_3 = \frac{14}{4} = \frac{7}{2}\right)$

Alternative way: Create 1 pathway to ground $\Rightarrow$

$R_1 = 1,\ \alpha = 0,\ R_3 = \infty,\ I_{S1} = I_{S2} = 0,\ R_2 = \infty,\ R_4 = R_5 = 1$

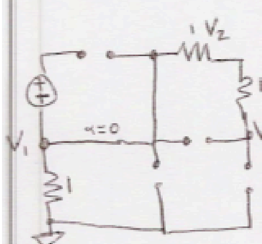
$\therefore$ From eq. set: ① $(V_1 - V_2) = 0 \Rightarrow V_1 = V_2$

② $V_1 + 0 - 0 + 0 + 0 + \frac{(V_2 - V_3)}{2} = 0$

$\left(V_2\frac{2}{2} + \frac{V_2}{2}\right) = \frac{V_3}{2} \Rightarrow V_2 = \frac{V_3}{2} \cdot \frac{2}{3}$

③ $-0 - 0 - \frac{(V_2 - V_3)}{2} = 0 \Rightarrow V_2 = V_3$

$\therefore V_2 = \frac{V_2}{3} \Rightarrow V_2\left(\frac{3}{3} - \frac{1}{3}\right) = 0 \Rightarrow V_2 = 0$ so $V_1 = V_3 = 0$



Since no source then $V_1 = V_2 = V_3$ because no I flow (open wire)