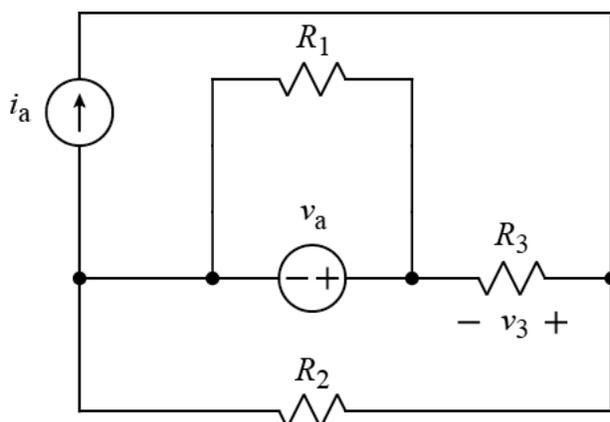
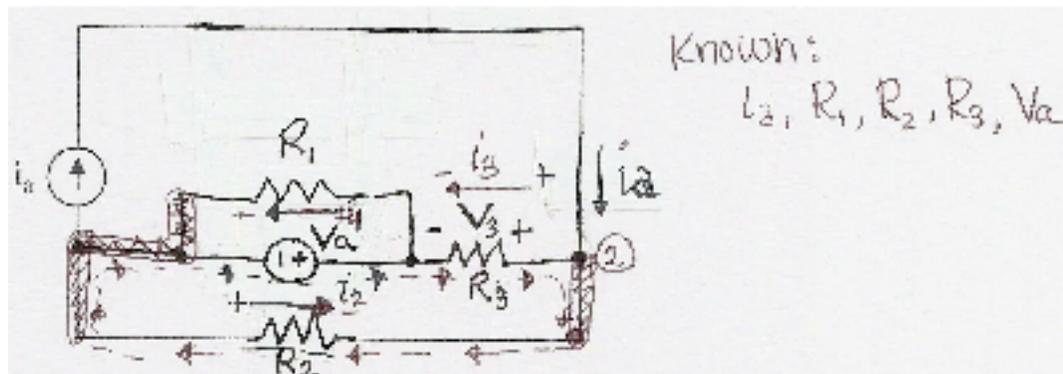


Ex:



- Derive an expression for  $v_3$  containing not more than circuit parameters  $v_a$ ,  $i_a$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .
- Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

SOL'N:



(a) V-loop:  $\textcircled{1} + i_2 R_2 + v_a + v_3 = 0$

Current Summation at  $\textcircled{2}$

$$-i_2 + i_3 - i_a = 0$$

$\textcircled{3} \quad i_2 = i_3 - i_a$

$\Omega$  Law:  $v_3 = i_3 R_3$

$\textcircled{4} \quad i_3 = \frac{v_3}{R_3}$

plug  $\textcircled{4}$  into  $\textcircled{3} \Rightarrow i_2 = \frac{v_3}{R_3} - i_a$

plug into ①  $\Rightarrow \frac{V_3(R_2)}{R_3} - i_a R_2 + V_a + V_3 = 0$

$$V_3 = \frac{i_a R_2 - V_a}{1 + \frac{R_2}{R_3}} \quad \text{OR} \quad \frac{(i_a R_2 - V_a) R_3}{R_3 + R_2}$$

- b) A consistency check is accomplished by making certain component values zero, in order to simplify the circuit enough that it may be solved by inspection. The zero values are then substituted into the solution given by the complete formula from (a) to verify that it yields the result found by inspection. If enough such checks are performed and are passed, then the solution in part (a) is probably correct.

There are many possible checks. For example, if we set  $R_3$  to zero, it becomes a wire with no voltage drop. Thus, the answer for  $V_3$  must be zero. For the complete solution, we would get the following calculation:

$$v_3 = \frac{(i_a R_2 - v_a) R_3}{R_2 + R_3} = \frac{(i_a R_2 - v_a) \cdot 0}{R_2 + R_3} = 0 \quad \text{solution verified}$$

Another possible check is  $v_a = 0$ , which turns the  $v_a$  source into a wire that bypasses  $R_1$  and reduces the circuit to a current divider involving only  $i_a$ ,  $R_2$ , and  $R_3$ . We can write down a formula for the current in  $R_3$  and then use Ohm's law to find  $v_3$ :

$$i_3 = \frac{i_a R_2}{R_2 + R_3}$$

$$v_3 = i_3 R_3 = \frac{i_a R_2}{R_2 + R_3} R_3$$

If we plug  $v_a = 0$  into the solution from (a) we get the same result, and the solution from is verified as satisfying this special case.

Yet another possible check is  $i_a = 0$ , which turns the  $i_a$  source into an open circuit, leaving  $v_a$  across  $R_2$ , and  $R_3$  and forming a voltage divider. (Notice that  $R_1$  is a second circuit across the  $v_a$  source that may be solved

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separately.) Using a voltage divider formula, we have the following value for  $v_3$ :

$$v_3 = \frac{v_a R_3}{R_2 + R_3}$$

If we plug  $i_a = 0$  into the solution from (a) we get the same result, and the solution from is again verified as satisfying this special case.