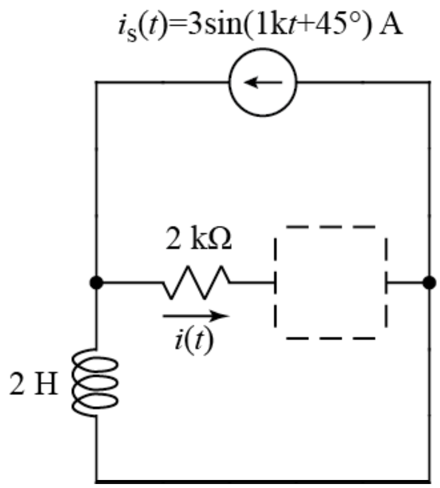
HOMEWORK #16 prob 2 solution

Ex:



a) Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = \mathrm{I}_0 \cos(1\mathrm{k}t + 45°)$$

where I_0 is a positive, (i.e., nonzero and non-negative), real constant with units of Amps. State the value of the component you choose.

b) Using the value of the component you chose for part (a), calculate the resulting value of I_0.

SOL'N: a) We start by finding the s-domain model.

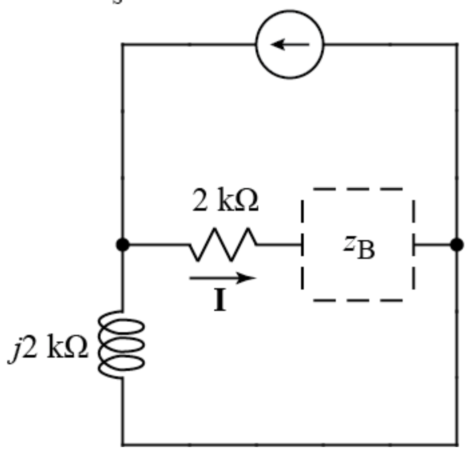
$$I_s = P[3\sin(1kt + 45°)A] = 3\cos(1kt + 45° - 90°)A$$

$$= 3\cos(1kt - 45°)A = 3\angle -45°$$

$$I = P[I_0\cos(1kt + 45°)] = I_0\angle +45°$$

$$Z_L = j\omega L = j(1k)(2H) = j2k$$

$\mathbf{I}_S = 3\angle -90° + 45°$ A $= 3\angle -45°$ A



we have a current Divider

$$\Rightarrow I = I_S \frac{Z_L}{Z_L + (Z_R + Z_B)} = 3\angle -45^\circ \frac{j2K}{j2K + 2K + Z_B} = I_o \angle 45^\circ$$

$$\Rightarrow \frac{\angle\, 3\angle -45^\circ (2K\angle 90^\circ)}{\angle\, j2K + 2K + Z_B} = \angle\,(I_o \angle 45^\circ)$$

$$\Rightarrow \frac{\angle -45^\circ + 90^\circ}{\angle (j2K + 2K + Z_B)} = \angle 45^\circ \Rightarrow \angle (j2K + 2K + Z_B) = \frac{\angle 45^\circ}{\angle 45^\circ} = \angle 0^\circ$$

we need $\angle 0^\circ$ and if we have $-j2K$ in the box

we are left with the resistor creating $\angle 0^\circ$

$$\Rightarrow j2K + 2K + \frac{-j}{wC} = 2K \Rightarrow j2K = \frac{+j}{wC}$$

$$\Rightarrow 1/wC = 2K \Rightarrow \frac{1}{(K)(C)} = 2K \Rightarrow 1/C = 2M$$

$$\Rightarrow C = 1/2M = 0.5\mu f = 500nf \Rightarrow \boxed{Box = C = 500nf}$$

$$\Rightarrow Z_C = -j/wC = -j2K$$

b) $\Rightarrow I = \dfrac{(3\angle -45^\circ)(+j2k)}{\cancel{j2k} + 2k - \cancel{j2k}} = \dfrac{(3\angle -45^\circ)(2k\angle +90^\circ)}{2k}$

$\Rightarrow I_o = Re[I] = Re\left[\dfrac{6k\angle +45^\circ}{2k}\right] = \dfrac{6k}{2k} = \boxed{3A}$

$\Rightarrow \boxed{I_o = 3A}$