HOMEWORK #14 prob 3 solution

EX: Derive a symbolic expression for the impedance of an R, an L, and a C in parallel at frequency ω. Rationalize the expression so the denominator is real.

SOL'N: When working with parallel impedances, it is typically easier to use the summation of conductance form of parallel impedance.

$$z = \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{j\omega C}} + \frac{1}{j\omega L}} = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}}$$

Keeping real parts and imaginary terms together reduces the complexity, so we sum the imaginary terms.

$$z = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

At this point, we may choose from several different approaches. One choice would be to rationalize the expression. Another choice would be to write z as a ratio of polynomials in ω. A third choice would be to make the denominator unitless by multiplying by R. Their are more choices available as well, and which is chosen is a matter of personal preference. Here, we make the third choice and make the denominator unitless.

$$z = \frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)}$$

Now we rationalize the expression to obtain our final answer.

$$z = \frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} \frac{1 - jR\left(\omega C - \frac{1}{\omega L}\right)}{1 - jR\left(\omega C - \frac{1}{\omega L}\right)} = \frac{R - jR^2\left(\omega C - \frac{1}{\omega L}\right)}{1^2 + R^2\left(\omega C - \frac{1}{\omega L}\right)^2}$$