HOMEWORK #13 prob 4 solution

Ex: Write phasors (as both $Ae^{j\phi}$ and $A\angle\phi$) for each of the following signals:

a) $v(t) = 4\cos(100t + 30°)$ V

b) $i(t) = 7\sin(\omega t - 45°)$ mA

c) $i(t) = 50 \text{ nF} \cdot \frac{d}{dt} 4\cos(100t + 30°)$ V

d) $v(t) = 17\ \mu\text{H} \cdot \frac{d}{dt} 7\sin(60t - 45°)$ mA

e) $v(t) = 4\cos(100t + 30°) \text{ V} + 3\sin(100t - 150°)$ V

SOL'N: a) The magnitude of the phasor is the magnitude of the sinusoid, and the phase angle in the exponent of the phasor is the phase shift of the cosine waveform.

$$\text{P}[v(t) = 4\cos(100t + 30°) \text{ V}] = 4e^{j30°} \text{ V}$$

b) The phasor of $\sin(\omega t)$ is $-j$.

$$\text{P}[i(t) = 7\sin(\omega t - 45°) \text{ mA}] = 7(-j)e^{-j45°} \text{ mA}$$

or

$$\text{P}[i(t)] = 7e^{-j90°}e^{-j45°} \text{ mA} = 7e^{-135°} \text{ mA}$$

or

$$\text{P}[i(t)] = 7\angle -135° \text{ mA}$$

c) When we take a derivative, we multiply by $j\omega$.

$$\text{P}\left[i(t) = 50 \text{ nF} \cdot \frac{d}{dt} 4\cos(100t + 30°) \text{ V}\right] = 50\text{nF} \cdot \text{j}\omega \cdot 4e^{j30°}\text{V}$$

or

$$\text{P}[i(t)] = 50\text{nF} \cdot \text{j}100\text{s}^{-1} \cdot 4e^{j30°}\text{V} = j20e^{j30°}\mu\text{A} = 20e^{j90°}e^{j30°}\mu\text{A}$$

or

$$\text{P}[i(t)] = 20e^{j120°}\mu A$$

d) Here, we have multiplication by $j\omega$ for the derivative and $-j$ for sin().

$$\mathrm{P}\left[v(t)=17\ \mu\mathrm{H}\cdot\frac{d}{dt}7\sin(60t-45°)\ \mathrm{mA}\right]=17\mu\mathrm{H}\cdot j60\mathrm{s}^{-1}\cdot(-j7)e^{-j45°}\mathrm{mA}$$

$$\mathrm{P}[v(t)]=7.14e^{-j45°}\mu\mathrm{V}$$

e) We convert the two waveforms to phasors before adding.

$$\mathrm{P}[v(t)=4\cos(100t+30°)\ \mathrm{V}+3\sin(100t-150°)\ \mathrm{V}]$$
$$=4e^{j30°}+3(-j)e^{-j150°}\ \mathrm{V}$$

or

$$\mathrm{P}[v(t)]=4e^{j30°}+3e^{-j90°}e^{-j150°}\ \mathrm{V}=4e^{j30°}+3e^{-j240°}\ \mathrm{V}$$

We could covert each term to rectangular form and sum, but a more efficient approach is to observe that the vectors are perpendicular. The first phasor is a vector of length 4 at an angle of 30° to the real axis. Adding the second phasor to the first creates a 4, 3, 5 triangle. The hypotenuse is the total phasor and has length 5. The angle of the phasor is 30° from the 1st phasor plus the angle in the 4, 3, 5 triangle between the sides of length 4 and 5.

$$\mathrm{P}[v(t)]=5\angle[30°+\tan^{-1}(3/4)]\mathrm{V}=5\angle30°+36.9°\mathrm{V}=5\angle66.9°\mathrm{V}$$