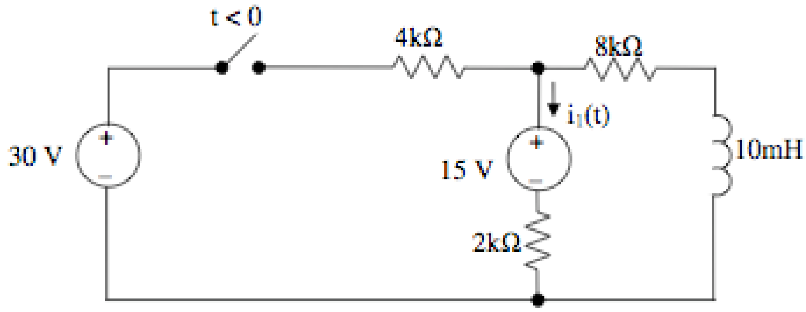
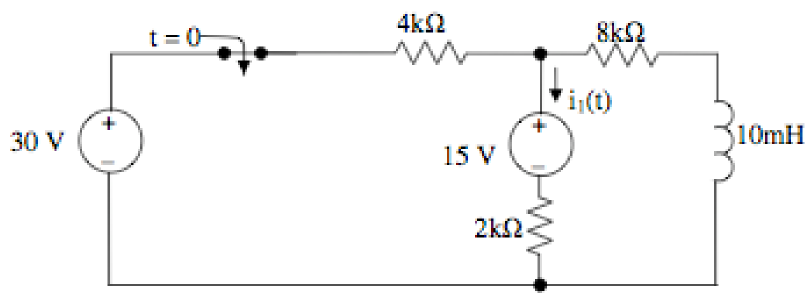
HOMEWORK #12 probs 1 and 2 solution

Ex:





After being open for a long time, the switch closes at $t = 0$.

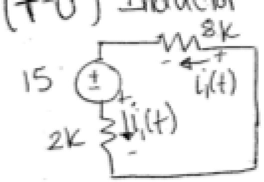
a) Calculate the energy stored on the inductor as $t \to \infty$.

b) Write a numerical expression for $i_1(t)$ for $t > 0$.

SOL'N:

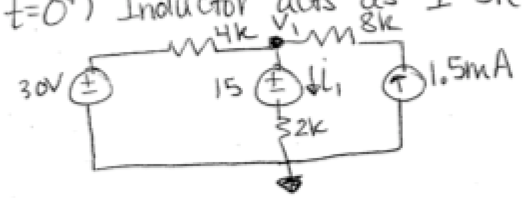
($t=0^-$) Inductor acts as a wire. switch is open

V-loop: $+2k i_1 + 15 + 8k i_1 = 0 \quad \therefore i_1 = \frac{-15}{10k} = -1.5mA$



($t=0^+$) Inductor acts as I src. Switch closed.

node-V: $\frac{V_1 - 30}{4k} + \frac{V_1 - 15}{2k} + (+1.5m) = 0$



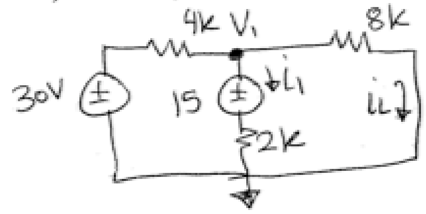
$V_1\left(\frac{1}{4k} + \frac{2}{4k}\right) = \frac{30}{4k} + \frac{15(2)}{2k(2)} - 1.5m$

$V_1 = \frac{4k\left(\frac{60}{4k} - 1.5m\right)}{3} =$

$i_1 = \frac{(V_1 - 15)}{2k} = \frac{(18-15)}{2k} = \frac{3}{2k} = \boxed{1.5mA}$

Initial value

($t \to \infty$) Inductor acts as a wire. switch closed.



using node-V:

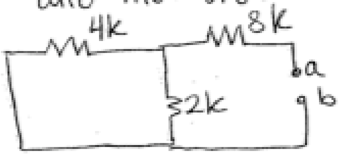
$$\frac{(V_1-30)}{4k} + \frac{(V_1-15)}{2k} + \frac{V_1}{8k} = 0$$

$$V_1\left(\frac{2(1)}{2(4k)} + \frac{4(1)}{4(2k)} + \frac{1}{8k}\right) = \frac{30}{4k} + \frac{15(2)}{2k(2)}$$

$$V_1 = \frac{60}{4k} \cdot \left(\frac{8k}{7}\right) = \frac{120}{7}$$

$$i_1 = \frac{(V_1-15)}{2k} = \frac{\left(\frac{120}{7}-15\right)}{2k} = \boxed{1.1mA}$$ final value

τ : (Remove inductor and ind. src's) $\tau = \frac{L}{R_{eq}}$



$$R_{eq} = 8k + 4k \parallel 2k$$

$$R_{eq} = 8k + \frac{4k(2k)}{6k} = \frac{48k}{6} + \frac{8k}{6}$$

$$R_{eq} = \frac{56k}{6} \quad \Rightarrow \tau = \frac{10m}{56k} \cdot 6 \cong 1.1\mu$$

($t>0$)

$$\boxed{i_1(t) = 1.1m + (1.5m - 1.1m)e^{-t/1.1\mu sec.}\ A}$$

Energy at $t \to \infty$

$$i_L = \frac{V_1}{8k} = \frac{120}{7(8k)} \approx 2.1mA$$

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(10m)(2.1m)^2 = \boxed{22.05nJ}$$