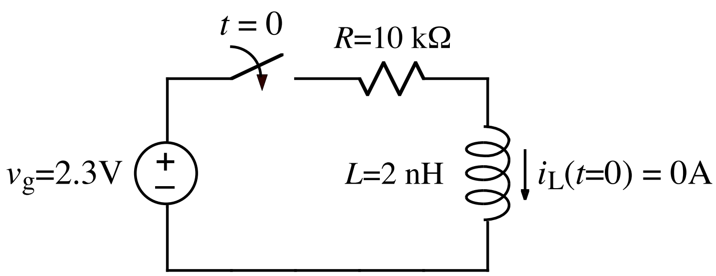
Ex:



a) Find an expression for $i_L(t)$ for $t \geq 0$.

b) Find the energy stored in the inductor at time t = 30 µs.

SOL'N: a) The following general form of solution applies to any RL circuit with a single inductor:

$$i_L(t \geq 0) = i_L(t \to \infty) + [i_L(t = 0^+) - i_L(t \to \infty)]e^{-t/(L/R_{Th})}$$

The Thevenin resistance, R_{Th}, is for the circuit after t = 0 (with the L removed) as seen from the terminals where the L is connected. In the present case, we have R_{Th} = 10 kΩ.

$$L/R_{Th} = 2 \text{ nH}/10 \text{ k}\Omega = 0.2 \text{ ps}$$

The value of $i_L(t{=}0)$ is given in the problem as 0 V. Note that the L could not have any current before t = 0 in this circuit since there is no closed path to carry current.

As time approaches infinity, the L current will converge to its final value, and the voltage across the L will cease to change. Thus, di_L/dt = 0 and $v_L = 0$, meaning the L will act like a wire. It follows that the current through the L will equal the current through the R, which will equal 2.3 V/10 kΩ = 0.23 mA.

$$i_L(t \to \infty) = 0.23 \text{ mA}$$

Substituting values, we have the following result:

$$i_L(t \geq 0) = 0.23 \text{ mA} + [0 - 0.23 \text{ mA}]e^{-t/0.2\text{ps}}$$

b) The energy in an inductor is given by the following formula:

$$w_L = \frac{1}{2} L i_L^2$$

We use the solution to (a) to evaluate $i_L(t)$ at t = 30 µs.

$$i_L(t = 30\mu s) = 0.23\text{ mA V} - 0.23\text{ mA} \cdot e^{-30\mu s/0.2\text{ps}} \approx 0.23\text{ mA}$$

Using this voltage, we evaluate the energy on the capacitor.

$$w_L = \frac{1}{2} 2\text{nH} \cdot (0.23\text{mA})^2 = 0.053\text{ fJ} = 53\text{ aJ}$$