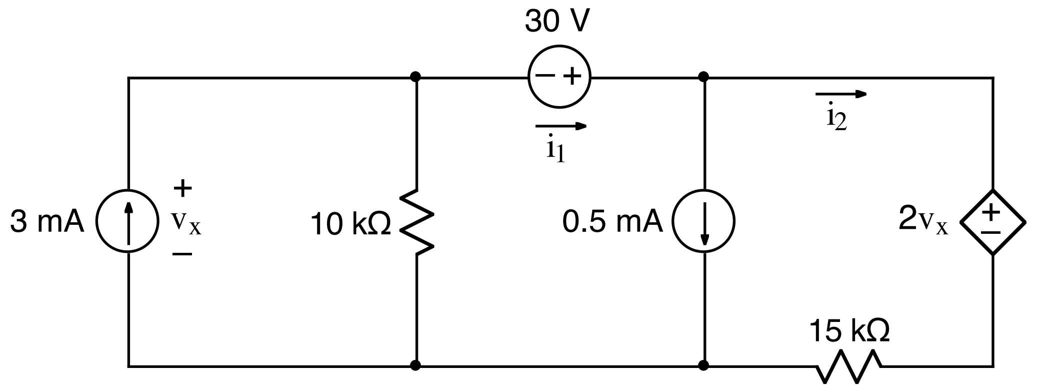
HOMEWORK #7 solution

Ex:



a) Use the mesh-current method to find i_1 and i_2.

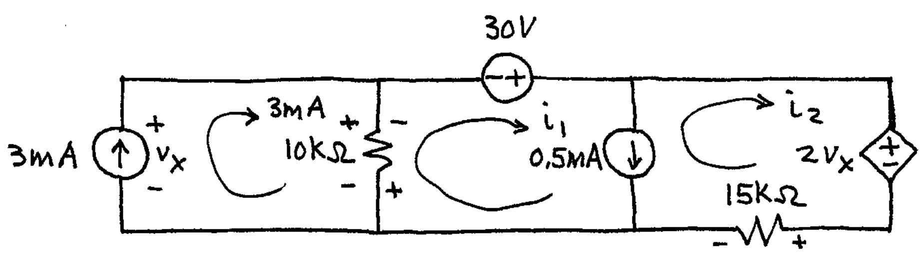
b) Find the power dissipated by the dependent source.

soln: a) We follow a step-by-step procedure:

1) We define mesh currents. If, however, we have any current sources on outside edges of the circuit, the mesh currents for those loops will be the same as the current source.

In this circuit, we have a current source on the left edge. Thus, the mesh current for the left loop is 3mA.

Since i_1 and i_2, as defined, are on the outside edge of the circuit, we may use them as our mesh currents.



2) We define the voltage from the dependent src, v_x, in terms of mesh currents. Here, we observe that v_x is across the $10k\Omega$ resistor, too. For the $10k\Omega$ resistor, we have

$$v_x = 3\,mA \cdot 10k\Omega - i_1 \cdot 10\,k\Omega$$

3) We look for loops with a current source in between, meaning we have a super mesh. This is the case for the i_1, i_2 loops. For the i_1, i_2 supermesh, we take a v-loop around the outside edge of the i_1 and i_2 loops, (bypassing the 0.5 mA src).

$$i_1, i_2 \text{ v-loop: } -i_1 \cdot 10\,k\Omega + 30V - 2\overbrace{(3mA - i_1)\,10k\Omega}^{v_x} + 3mA \cdot 10k\Omega$$

$$- i_2 \cdot 15\,K\Omega = 0V$$

Add a current eq'n for the 0.5 mA src between the loops:

$$i_1 - i_2 = 0.5\,mA = \tfrac{1}{2}\,mA$$

Note: we have $-i_2$ for current measured opposite the arrow in the current src.

4) We solve our eq'ns for i_1 and i_2.

We group i_1 and i_2 terms on the left and move constant to the right side.

$$i_1 (\underbrace{-10\text{k}\Omega + 2\cdot 10\text{k}\Omega}_{=10\text{k}\Omega}) + i_2(-15\text{k}\Omega) = -60\text{V} + 60\text{V}$$

$$i_1 \qquad - i_2 \qquad = \tfrac{1}{2}\text{ mA}$$

Solving the 2nd eq'n for i_1, we have

$$i_1 = i_2 + \tfrac{1}{2}\text{ mA}$$

Substituting into 1st eq'n, we have

$$\left(i_2 + \tfrac{1}{2}\text{ mA}\right) 10\text{k}\Omega + i_2(-15\text{k}\Omega) = 30\text{V}$$

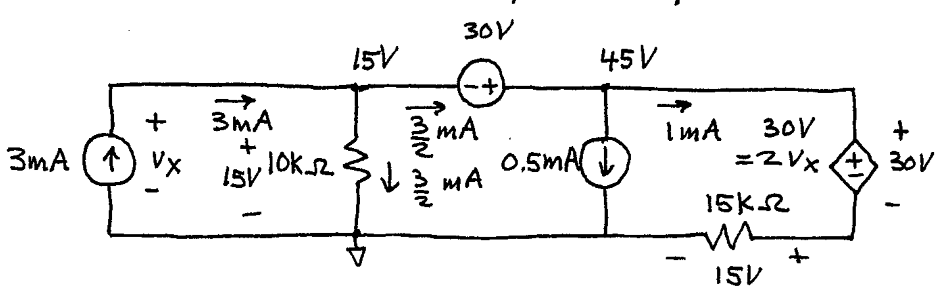
or $\quad i_2 (10\text{k}\Omega - 15\text{k}\Omega) = 0\text{V} - \tfrac{1}{2}\text{mA}\cdot 10\text{k}\Omega$

or $\quad -i_2 (5\text{k}\Omega) = -5\text{ V}$

or $\quad i_2 = 1\text{ mA}$

Then $\quad i_1 = 1\text{mA} + \tfrac{1}{2}\text{ mA} = \tfrac{3}{2}\text{ mA}.$

Consistency check: calculate v-drops for i_1, i_2 and verify v-loops.



$$v_X = \tfrac{3}{2}\text{mA}\cdot 10\text{k}\Omega = 15\text{ V}$$

All v-loops sum to 0V, and all current sums at nodes = 0A. ✓

b) We know $v_x = (3mA - i_1)\, 10k\Omega$

$$'' = \frac{3}{2} mA \cdot 10k\Omega$$

$$v_x = 15V$$

The current for the dependent src is i_2.

$$i_2 = 1\ mA$$

Thus, power for the dependent src is

$$p = v \cdot i = 2v_x\, i_2 = 2(15V) \cdot 1mA$$

or $p = 30\ mW$.