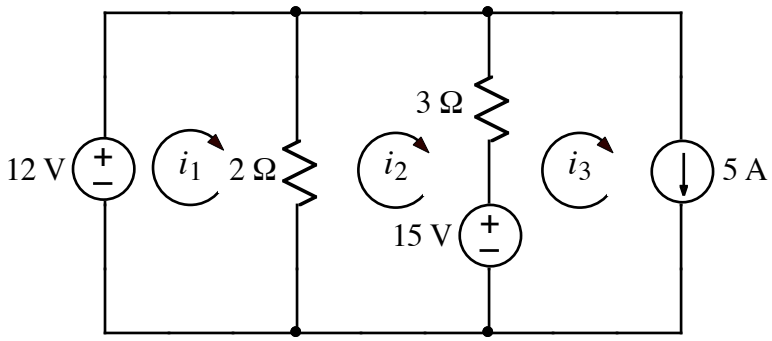
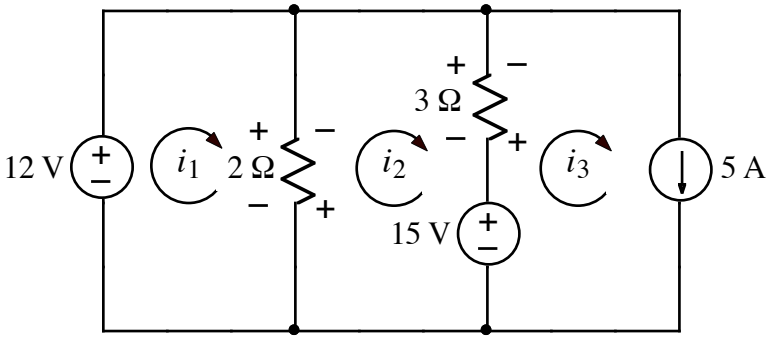
HOMEWORK #7 solution

Ex:



Use the mesh-current method to find i_1, i_2, and i_3.

Sol'n: The mesh-current method is based on the idea of calculating voltage loops in terms of mesh currents in inner loops that add up to give the total current through components that are shared by loops. The diagram below shows the polarities of voltage drops for mesh currents. These voltage drops follow the passive sign convention for their respective mesh current.



The current through the 2Ω resistor (measured with an arrow pointing down) is $i_1 - i_2$. The current through the 3Ω resistor (measured with an arrow pointing down) is $i_2 - i_3$.

On the right side, we have a current source on the outside edge of the loop. It follows that mesh current, i_3, is determined by the current of the source:

$$i_3 = 5 \text{ A}$$

For the other loops, we use voltage loops. The voltage loop for mesh current i_1 is as follows:

$$12 \text{ V} - i_1 \cdot 2\ \Omega + i_2 \cdot 2\ \Omega = 0 \text{ V} \qquad (1)$$

The voltage loop for mesh current i_2 is as follows:

$$-i_2 \cdot 2\ \Omega + i_1 \cdot 2\ \Omega - i_2 \cdot 3\ \Omega + i_3 \cdot 3\ \Omega - 15\ \text{V} = 0\text{V} \qquad (2)$$

We substitute i_3 = 5 A and solve the simultaneous equations by first combining terms multiplying *i*'s and moving constant terms to the right side.

$$-i_1 \cdot 2\ \Omega + i_2 \cdot 2\ \Omega = -12\ \text{V} \qquad (1')$$

$$i_1 \cdot 2\ \Omega - i_2 \cdot (2\ \Omega + 3\ \Omega) + 5\ \text{A} \cdot 3\ \Omega = 15\ \text{V}$$

or

$$i_1 \cdot 2\ \Omega - i_2 \cdot (2\ \Omega + 3\ \Omega) = 15\ \text{V} - 5\ \text{A} \cdot 3\ \Omega = 0\ \text{V} \qquad (2')$$

Summing the two equations eliminates the i_1 term.

$$i_2 \cdot 2\ \Omega - i_2 \cdot (2\ \Omega + 3\ \Omega) = -12\text{V} + 0\ \text{V} = -12\ \text{V} \qquad (3)$$

or

$$-i_2 \cdot 3\ \Omega = -12\text{V}$$

or

$$i_2 = 4\ \text{A}$$

Using equation 1' and the value of i_2, we find the value of i_1:

$$-i_1 \cdot 2\ \Omega + i_2 \cdot 2\ \Omega = -12\ \text{V}$$

or

$$-i_1 \cdot 2\ \Omega = -12\ \text{V} - i_2 \cdot 2\ \Omega = -12\ \text{V} - 4\ \text{A} \cdot 2\ \Omega = -20\ \text{V}$$

or

$$i_1 = \frac{20\ \text{V}}{2\ \Omega} = 10\ \text{A}$$