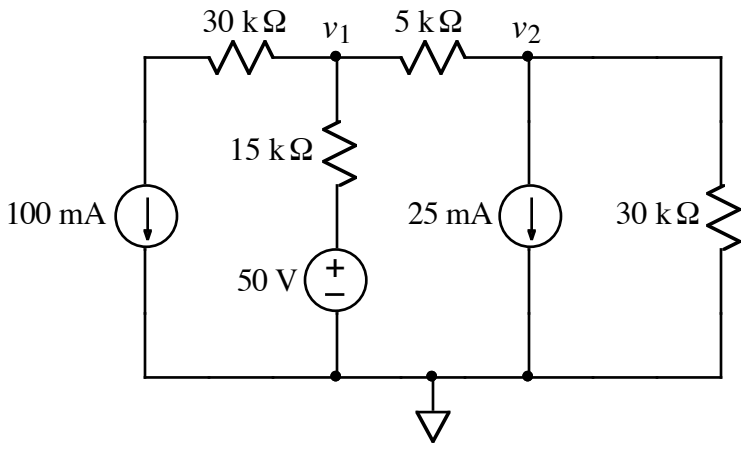
HOMEWORK #6 solution

Ex:



Use the node-voltage method to find v_1 and v_2.

SOL'N: We derive two equations by setting the sum of currents out each node equal to zero.

$$100\text{ mA} + \frac{v_1 - 50\text{ V}}{15\text{ k}\Omega} + \frac{v_1 - v_2}{5\text{ k}\Omega} = 0\text{ A} \qquad (v_1 \text{ node})$$

$$\frac{v_2 - v_1}{5\text{ k}\Omega} + 25\text{ mA} + \frac{v_2 - 0\text{ V}}{30\text{ k}\Omega} = 0\text{ A} \qquad (v_2 \text{ node})$$

A convenient format for the equations has terms multiplying v_1 and v_2 grouped together and constant terms on the right side.

$$v_1\left(\frac{1}{15\text{ k}\Omega} + \frac{1}{5\text{ k}\Omega}\right) - v_2\frac{1}{5\text{ k}\Omega} = -100\text{ mA} + \frac{50\text{ V}}{15\text{ k}\Omega} \qquad (v_1 \text{ node})$$

$$v_1\frac{-1}{5\text{ k}\Omega} + v_2\left(\frac{1}{5\text{ k}\Omega} + \frac{1}{30\text{ k}\Omega}\right) = -25\text{ mA} \qquad (v_2 \text{ node})$$

We obtain some simplification by multiplying both sides by the common denominator:

$$15\text{ k}\Omega \cdot v_1\left(\frac{1}{15\text{ k}\Omega} + \frac{1}{5\text{ k}\Omega}\right) - v_2\frac{1}{5\text{ k}\Omega} = 15\text{ k}\Omega \cdot \left(-100\text{ mA} + \frac{50\text{ V}}{15\text{ k}\Omega}\right)$$

or

$$v_1(1+3) - v_2 \cdot 3 = -1500\text{ V} + 50\text{ V}$$

or

$$4v_1 - 3v_2 = -1450 \text{ V} \quad (v_1 \text{ node})$$

and

$$30 \text{ k}\Omega\left[v_1 \frac{-1}{5 \text{ k}\Omega} + v_2\left(\frac{1}{5 \text{ k}\Omega} + \frac{1}{30 \text{ k}\Omega}\right)\right] = (30 \text{ k}\Omega)(-25 \text{ mA})$$

or

$$\left[v_1(-6) + v_2(6+1)\right] = -750 \text{ V}$$

or

$$-6v_1 + 7v_2 = -750 \text{ V}$$

Solving the first equation for v_2 yields the following:

$$v_2 = \frac{4v_1 + 1450 \text{ V}}{3}$$

Substituting into the second equation, we obtain one equation in one unknown:

$$-6v_1 + 7\frac{4v_1 + 1450 \text{ mA}}{3} = -750 \text{ V}$$

or, multiplying both side by 3,

$$-18v_1 + 7 \cdot (4v_1 + 1450 \text{ V}) = -2250 \text{ V}$$

or

$$10v_1 = -12.4 \text{ kV}$$

or

$$v_1 = -1240 \text{ V}$$

Using an earlier equation, we find v_2 from v_1:

$$v_2 = \frac{4v_1 + 1450 \text{ V}}{3} = \frac{4(-1240) + 1450 \text{ V}}{3} = -1170 \text{ V}$$