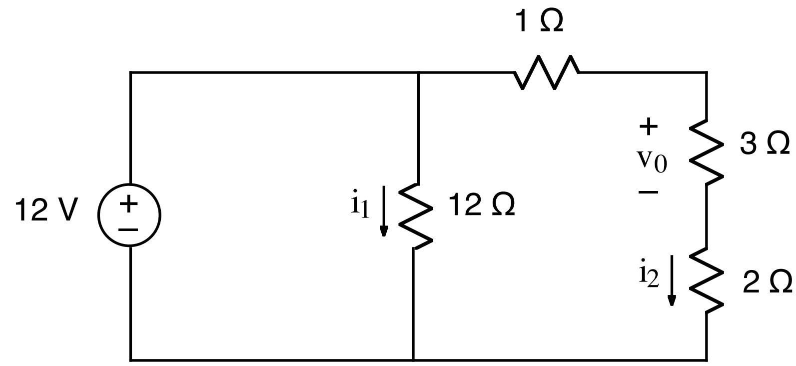
HOMEWORK #3 solution

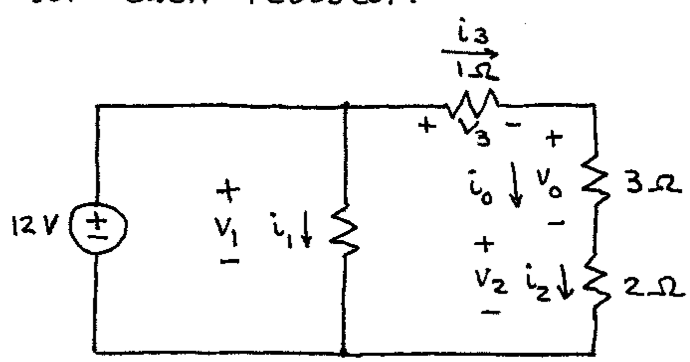
Ex:



a) Calculate i_1, i_2, and v_0.

b) Find the power dissipated for every component, including the voltage source.

soln: a) We first label voltage and current for each resistor.



Starting with voltage loops, we have the following equations:

v-loop on left: $+12V - V_1 = 0V$ or $V_1 = 12V$

This means that a resistor across a voltage source has that voltage drop across it.

v-loop on right: $+V_1 - V_3 - V_0 - V_2 = 0V$

This loop is in the clockwise direction.

Since we have eq'ns for the two inner loops, the outside v-loop would be redundant.

Now we consider i-sums at nodes.

At the top center node, we discover that we lack a current for the 12V source. If we define a current for the voltage source, we add another unknown and another eq'n. Consequently, this gets us no closer to solving for the currents and voltages. Thus, we avoid writing a current-sum eq'n for the top center node.

The same argument applies to the bottom center node. Thus, this problem requires no current-sum eq'ns.

The next step is to equate currents in series components. Here, the same current must flow in 1Ω, 3Ω, and 2Ω resistors:

$$i_3 = i_o = i_2$$

From this point forward, we use i_2 in place of i_3 and i_o. Note: if we look for such series currents at the outset, then we may eliminate some currents immediately.

Last, we use Ohm's law.

$v_1 = i_1 \cdot 12\Omega$ or $12V = i_1 \cdot 12\Omega \Rightarrow i_1 = \frac{12V}{12\Omega} = 1A$

$v_0 = i_2 \cdot 3\Omega$

$v_2 = i_2 \cdot 2\Omega$

$v_3 = i_2 \cdot 1\Omega$

Note that we can solve for v_1 and i_1 separately. This will happen whenever we have different parts of the circuit that are connected in parallel directly across a V-source.

For right side of the circuit, we can substitute the Ohm's law expressions into the voltage eq'n and solve for i_2:

$$v_1 - v_3 - v_0 - v_2 = 0V$$

or $12V - i_2 \cdot 1\Omega - i_2 \cdot 3\Omega - i_2 \cdot 2\Omega = 0V$

or $i_2(1\Omega + 3\Omega + 2\Omega) = 12V$

or $i_2 = \frac{12V}{1\Omega + 3\Omega + 2\Omega} = \frac{12V}{6\Omega} = 2A$

$$i_2 = 2A$$

For v_0, we use Ohm's law:

$$v_0 = i_2 \cdot 3\Omega = 2A \cdot 3\Omega = 6V.$$

b) power $= i \cdot v$

For resistors, $p = iv = i^2 R = \frac{v^2}{R}$.

$$p_{12\Omega} = i_1^2 \cdot 12\Omega = (1A)^2 \cdot 12\Omega = 12W$$

$$p_{1\Omega} = i_2^2 \cdot 1\Omega = (2A)^2 \cdot 1\Omega = 4W$$

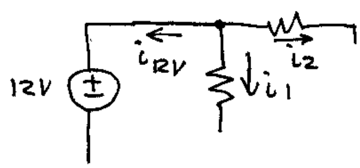
$$p_{3\Omega} = i_2^2 \cdot 3\Omega = (2A)^2 \cdot 3\Omega = 12W$$

$$p_{2\Omega} = i_2^2 \cdot 2\Omega = (2A)^2 \cdot 2\Omega = 8W$$

Total R pwr = 36W

For the 12V source, we need the current. Now that we have solved the circuit, we can use Kirchhoff's laws to find the current. Using a current source for the top center node, we have the following eq'n:

$$i_{12V} + i_1 + i_2 = 0\,A$$



$$i_{12} = -(i_1 + i_2) = -(1A + 2A) = -3A$$

So $\quad p_{12V} = -3A \cdot 12V = -36W$

Total power for circuit is $-36W + 36W = 0W$.

Note: a negative power means a source is <u>supplying</u> power.