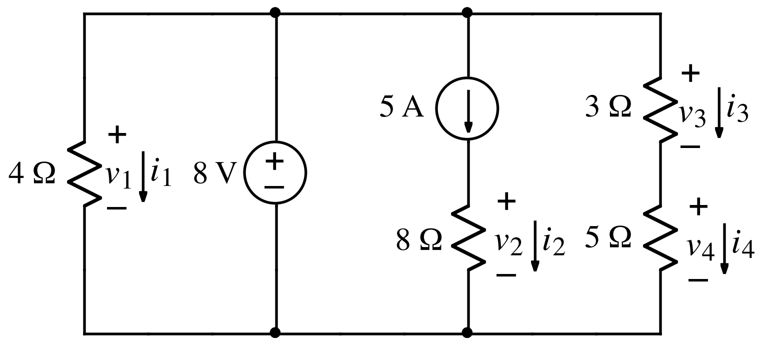
HOMEWORK #2 solution

Ex:



a) For the circuit shown above, use Kirchhoff's laws to write equations relating voltages and currents.

b) Find the value of v_1 and i_2.

SOL'N: a) We look for voltage loops that are as small as possible without containing current sources, (since we avoid defining voltage drops for current sources). A voltage loop through the 4 Ω resistor and 8 V source, proceeding in the clockwise direction, yields the following equation:

$$v_1 - 8 \text{ V} = 0 \text{ V}$$

or

$$v_1 = 8 \text{ V} \tag{1}$$

A second loop through the 8 V source and the resistors on the right side yields a second equation:

$$8 \text{ V} - v_3 + v_4 = 0 \text{ V} \tag{2}$$

Next, we write equations expressing the idea that currents in components connected in series are the same. For the two branches on the right side, we obtain the following equations:

$$-5 \text{ A} + i_2 = 0 \text{ A} \tag{3}$$

$$i_3 - i_4 = 0 \text{ A} \tag{4}$$

Turning to current summations, if we try to sum the currents out of the top left node, we have a problem defining the current out of the node through the voltage source. The same is true of any node in the circuit, since nodes connected by wires are really single nodes. (We would sum all the currents out of both nodes.) To resolve this difficulty, we may argue that the current flowing out of any box drawn around some part of a circuit must sum to zero. We may consider a box drawn around the two nodes on the top rail and the voltage source and the two nodes on the bottom rail of the circuit and sum the currents flowing out of it.

Having made this argument, we discover that the currents flowing out of this box all cancel out:

$$i_1 + 5\text{ A} + \mathrm{i}_3 - i_1 - i_2 - i_4 (\text{but } i_4 = i_3) = 0\text{ A}$$

Thus, this equation (though correct) fails to add any information that will help us solve the circuit. We conclude that this equation is unnecessary, and we ignore it.

Have we written enough equations to solve the circuit? The answer is yes, because the equations numbered (1)-(4) above, plus four equations resulting from Ohm's law, we can solve for all resistor currents and voltages, of which there are eight, (two for each of the four resistors).

b) Two of the above equations yield values of voltage or current:

$$v_1 - 8\text{ V} = 0\text{ V} \Rightarrow v_1 = 8\text{ V}$$

$$-5\text{ A} + i_2 = 0\text{ A} \Rightarrow i_2 = 5\text{ A}$$