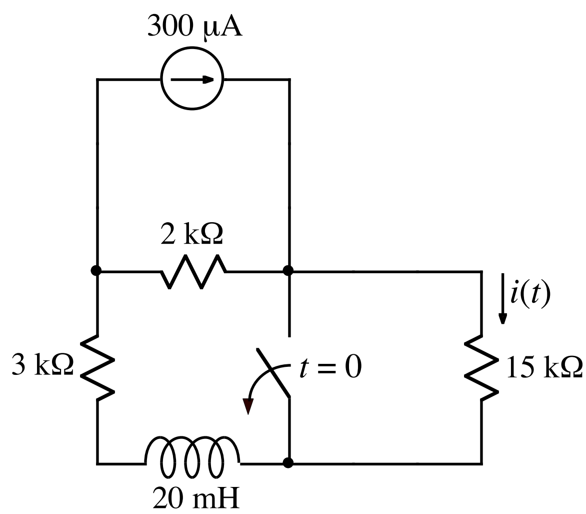
HOMEWORK 12 Solution Prob 1 and 2

Ex:



After being closed for a long time, the switch opens at $t = 0$.

a) Calculate the energy stored on the inductor as $t \to \infty$.

b) Write a numerical expression for $i(t)$ for $t > 0$.

SOL'N: a) As t approaches infinity, the switch is open and the inductor acts like a wire. The 3 kΩ and 15 kΩ sum to act like an 18 kΩ resistor. This 18 kΩ resistor is in parallel with the 2 kΩ resistor, forming a current divider. The current in the inductor is the same as the current in the 3 kΩ and 15 kΩ resistors:

$$i_L(t \to \infty) = 300\ \mu\text{A} \frac{2\ \text{k}\Omega}{2\ \text{k}\Omega + 18\ \text{k}\Omega} = 30\ \mu\text{A}$$

The energy for an inductor is the current squared times half the inductance:

$$w_L(t \to \infty) = \frac{1}{2} L\, i_L^2(t \to \infty)$$

Using the final current we have the energy:

$$w_L(t \to \infty) = \frac{1}{2} 20\ \text{mH}\ (30\ \mu\text{A})^2 = 9\ \text{pJ}$$

b) We use the general form of solution for RL circuits:

$$i(t > 0) = i(t \to \infty) + [(i(0^+) - i(t \to \infty)]e^{-t/(L/R_{Th})}$$

We find the initial condition for i by determining the inductor current at $t = 0^+$. With the switch closed for a long time, the 15 kΩ resistor on the right side is bypassed, and the inductor looks like a wire. The circuit becomes a current divider, with the 2 kΩ and 3 kΩ resistors in parallel. The current flowing through L is the same as the current flowing through the 3 kΩ resistor:

$$i_L(t = 0^-) = 300\ \mu\text{A} \frac{2\ \text{k}\Omega}{2\ \text{k}\Omega + 3\ \text{k}\Omega} = 120\ \mu\text{A}$$

Because the energy stored by the inductor cannot change instantly, the value of i_L at time $t = 0^+$ is the same as at time $t = 0^-$:

$$i_L(t = 0^+) = i_L(t = 0^-) = 120\ \mu\text{A}$$

Since the 15 kΩ resistor is in series with the inductor, it carries the same current as the inductor:

$$i(0^+) = i_L(t = 0^+) = 120\ \mu\text{A}$$

Finally, we find the value of R_{Th} for time $t > 0$. We remove the L and look into the terminals to find R_{Th}. Here, we may simply turn off the current source, leaving a resistance of 3 kΩ + 2 kΩ + 15 kΩ:

$$R_{Th} = 20\ \text{k}\Omega$$

Our time constant is L/R_{Th}:

$$\tau = \frac{L}{R_{Th}} = \frac{20\ \text{mH}}{20\ \text{k}\Omega} = 1\ \mu\text{s}$$

Substituting values yields our final answer:

$$i(t > 0) = 30\ \mu\text{A} + [120\ \mu\text{A} - 30\ \mu\text{A}]e^{-t/1\ \mu\text{s}}$$

or

$$i(t > 0) = 30\ \mu\text{A} + 90\ \mu\text{A}e^{-t/1\ \mu\text{s}}$$