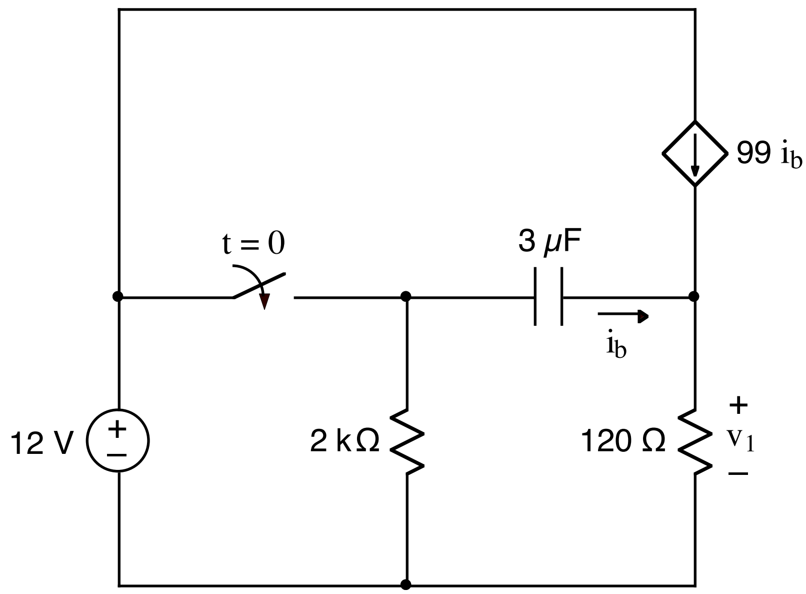
HOMEWORK #11 Solution Probs 4 and 5

Ex:



After being open for a long time, the switch closes at $t = 0$.

a) Find $v_C(0-)$ for the above circuit.

b) For $t > 0$, find the Thevenin equivalent of the above circuit as seen from the terminals where the capacitor is attached.

c) For the above circuit, find find $v_1(t)$ for $t > 0$.

NOTE: The original problem shows the current in the capacitor as $99i_b$. The dependent source in the problem was intended to represent a bipolar transistor, in which case i_b is the base current that is approximately 1/100 times the collector current shown in the problem as $99i_b$. The solution below assumes the base (and capacitor) current is i_b. The method of solution using the capacitor current as $99i_b$ is the same, but R_{Th} changes, and the answers obtained are as follows:

a) 0 V (same as below)

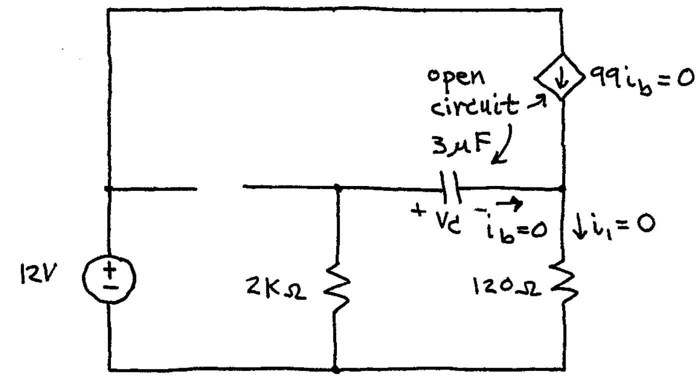
b) $R_{TH} = 240\ \Omega$ ($2 \cdot 120\ \Omega$ instead of $100 \cdot 120\ \Omega$)

c) $v_1(t > 0) = 12e^{-t/720\mu s}$ V

soln: Use general form of solution for RC problems:

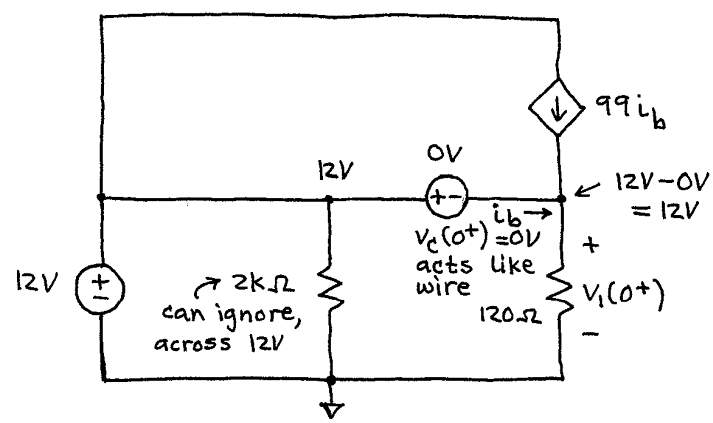
$$v_1(t>0) = v_1(t\to\infty) + \left[v_1(0^+) - v_1(t\to\infty)\right] e^{-t/R_{Th}C}$$

$t=0^-$: C acts like open circuit $\Rightarrow i_b = 0$, $99 i_b = 0$
switch is open



Since no power is connected to C, $v_C(0^-) = 0V$

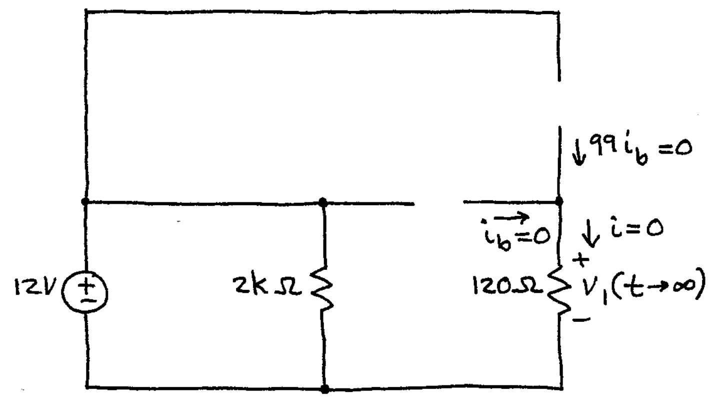
$t=0^+$: C acts like v src, $v_C(0^+) = v_C(0^-)$.
switch is closed



If we consider a v-loop around the outside of the bottom half of the circuit, we find that we have 12V across the 120Ω resistor:

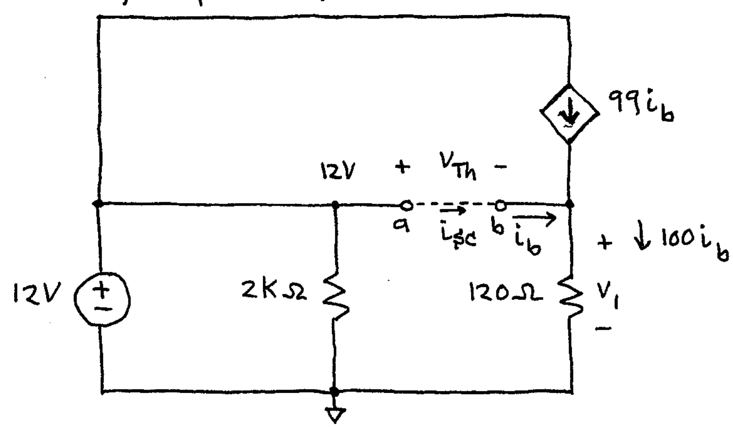
$$v_1(0^+) = 12V$$

$t \to \infty$: C acts like open circuit $\Rightarrow i_b = 0$, $99 i_b = 0$
switch is closed



No power is connected to 120Ω.
Thus, $v_1(t \to \infty) = 0V$

R_{Th}:



$R_{Th} = \frac{V_{Th}}{i_{sc}}$ $\qquad$ $V_{Th} = v_{a,b}$ with C removed (a,b open circuit)

With open circuit a,b we have $i_b = 0$ and $99 i_b = 0$. Thus, $v_1 = 0V$.

$$V_{Th} = 12V - v_1 = 12V - 0V = 12V$$

Now connect wire from a to b and measure current, i_{sc}.

We have $v_1 = 12V$ since it is now connected across 12V source by wires.

The current thru the 120Ω resistor is $100 i_b$:

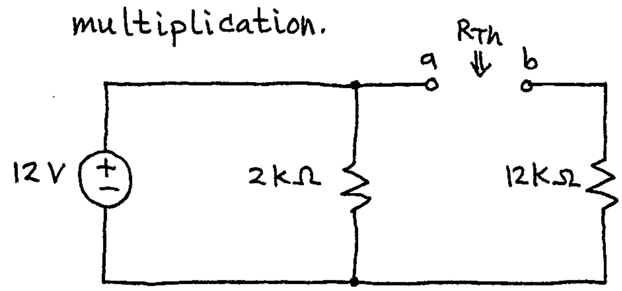
$$\frac{v_1}{120\Omega} = \frac{12V}{120\Omega} = 100\,mA = 100\, i_b$$

Thus, $i_{sc} = i_b = \frac{100\,mA}{100} = 1\,mA$

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{12V}{1\,mA} = 12k\Omega$$

Note: We get the same result if we remove the dependent source and multiply the 120Ω resistor by 100 to account for the $100 i_b$ flowing thru it.

This is the concept of impedance multiplication.



We find R_{Th} by turning off the 12V source (which becomes a wire) and determining R seen looking into a, b.

We have $R_{Th} = 12k\Omega$, as before.

Plugging values into the general solution yields our final answer:

$$v_1(t) = 0V + [12V - 0V]\, e^{-t/12k\Omega \cdot 3\mu F}, \qquad t > 0$$

or

$$v_1(t>0) = 12V\, e^{-t/36ms}$$