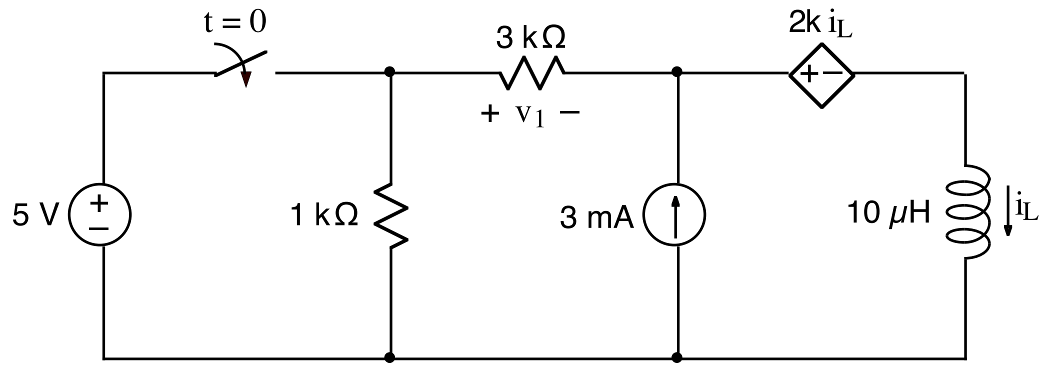
HOMEWORK #11 Solution Prob 3

Ex:



After being open for a long time, the switch closes at t = 0. Find $v_1(t)$ for t > 0.

soln: To find $v_1(t>0)$, we use the general form of solution, (which applies to any current or voltage):

$$v_1(t>0) = v_1(t\to\infty) + [v_1(0^+) - v_1(t\to\infty)]\, e^{-t/\frac{L}{R_{Th}}}$$

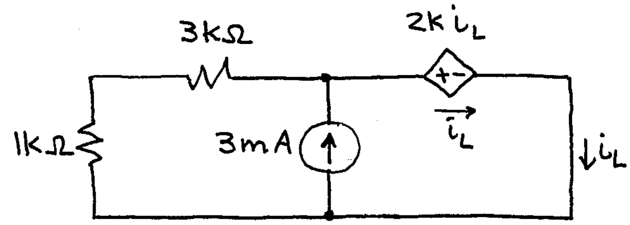
We have an inductor whose behavior at time $t=0^+$ will affect the value of $v_1(0^+)$.

We find the value of i_L at time $t=0^-$ and employ the concept that i_L, being an energy variable, cannot change instantly. Thus, $i_L(0^+) = i_L(0^-)$.

At $t=0^+$, currents and voltages have stabilized, and time derivatives $= 0$. Thus, $v_L = L\,di/dt = 0\text{V}$ and the L acts like a wire: it has no v drop but it can carry current.

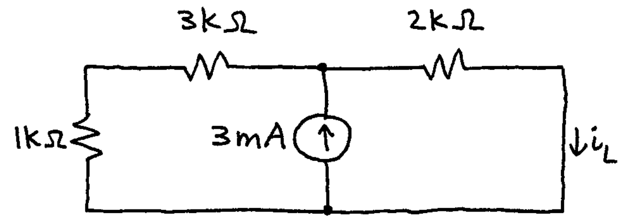
At $t=0^-$, the switch is open, removing the 5V source from the circuit.

$t=0^-$:



We observe that the dependent source is equivalent to a resistor:

$$R_{eq} = \frac{v}{i} = \frac{2k\, i_L}{i_L} = 2k\Omega$$



This is a current divider.

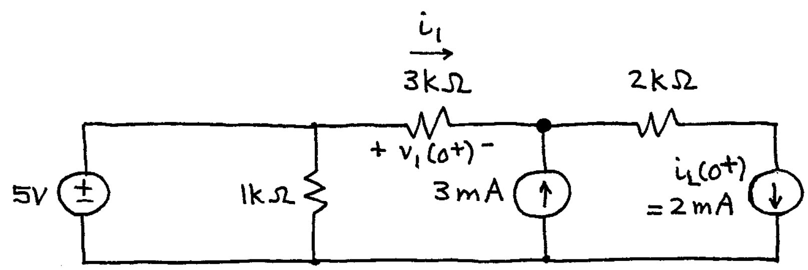
$$i_L(0^-) = 3\,mA \frac{1k\Omega + 3k\Omega}{1k\Omega + 3k\Omega + 2k\Omega} = 2\,mA$$

Note that we do <u>not</u> find $v_1(0^-)$ since it may change instantly when the switch closes.

$t=0^+$: We model the L as a current source with $i_L(0^+) = i_L(0^-)$.

The switch is closed for $t>0$.

As before the dependent source acts like a $2k\Omega$ resistor.



A current summation at the node shown as a large dot gives the current thru the 3kΩ resistor:

$$-i_1 - 3\,mA + 2\,mA = 0\,A$$

or

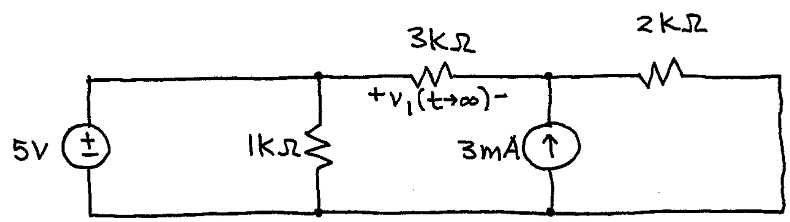
$$i_1 = -1\,mA$$

By Ohm's law, we have

$$v_1(0^+) = i_1 \cdot 3k\Omega = -1\,mA \cdot 3k\Omega = -3V$$

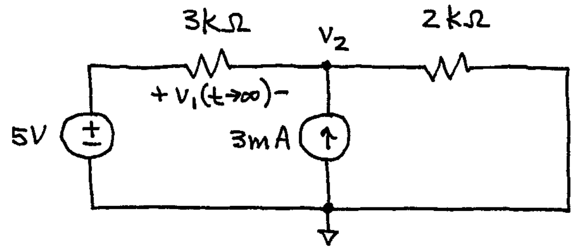
Now we find $v_1(t\to\infty)$.

$t\to\infty$: The L again acts like a wire, and the switch is closed.



We may ignore the 1kΩ resistor since it acts like a separate circuit across the 5V source. (The other circuit across the 5V source consists of 3kΩ, 2kΩ, and i_{src}.

HOMEWORK #11 Solution Prob 3 (cont.)



Using the node-voltage method, we have

$$\frac{v_2 - 5V}{3k\Omega} - 3mA + \frac{v_2}{2k\Omega} = 0\,A$$

Multiplying both sides by $6k\Omega$ yields

$$v_2(2+3) = 2\cdot 5V + 6k\Omega \cdot 3mA$$

or

$$5v_2 = 28V$$

or

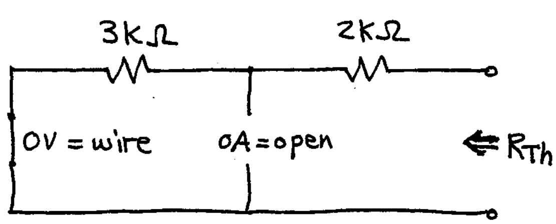
$$v_2 = \frac{28V}{5}$$

Thus, $v_1(t\to\infty) = 5V - v_2 = 5V - \frac{28V}{5} = -\frac{3}{5}V$

R_{Th}: We can use the circuit at the top of the page with the independent sources set to zero and the L (i.e., wire) removed.

We have

$R_{Th} = 2k\Omega + 3k\Omega$



$$\therefore\ v_1(t>0) = -\frac{3}{5}V + \left[-3V - -\frac{3V}{5}\right]e^{\frac{-t}{10\mu/5k}} = -0.6 - 2.4e^{-t/50ms}\ V$$