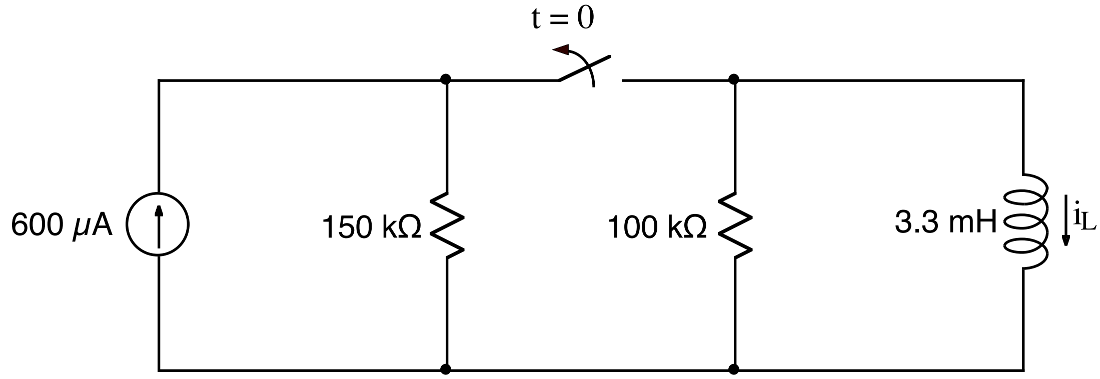
HOMEWORK #11 Solution Prob 1

Ex:



After being closed for a long time, the switch opens at t = 0. Find $i_L(t)$ for $t > 0$.

sol'n: Use the general form of solution for RL problems:

$$i_L(t>0) = i_L(t\to\infty) + [i_L(t=0^+) - i_L(t\to\infty)]e^{-t/\frac{L}{R_{Th}}}$$

To find $i_L(0^+)$, we consider $t=0^-$.

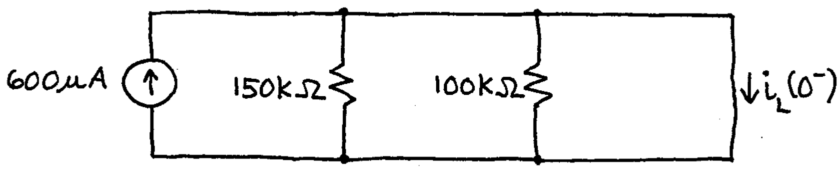
At $t=0^-$, the circuit has reached stable values, and all time derivatives of i and v are zero. Thus,

$$v_L = L\frac{di_L}{dt} = L\cdot 0 = 0$$ and L acts like a wire.

Since the switch is closed at $t=0^-$, we have a current source shorted by a wire.

We are only interested in the energy variable, $i_L(0^-)$. All other currents and voltages may change instantly when the switch opens.

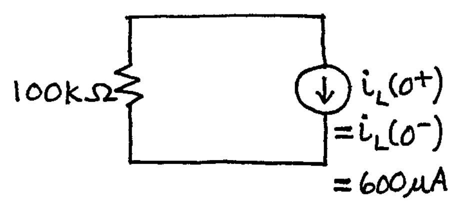
$t = 0^-$: L = wire, switch closed, find $i_L(0^-)$



$i_L(0^-) = 600\,\mu A$ since all the current will flow thru the L = wire

$i_L(0^+) = i_L(0^-)$ since energy variables like i_L and v_C cannot change instantly

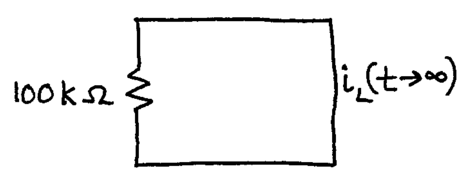
$t = 0^+$: L = current source, $i_L(0^+) = i_L(0^-)$, switch open (left side of circuit disconnected)



$i_L(0^+) = i_L(0^-) = 600\,\mu A$

Now we find $i_L(t \to \infty)$. As $t \to \infty$, the circuit again reaches stable values, and the L again acts like a wire.

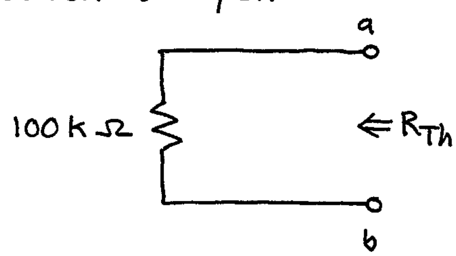
$t \to \infty$:



$i_L(t \to \infty) = 0$ since there is no power source.

The last quantity we need is R_{Th}, the Thevenin equivalent resistance of the circuit as seen from the terminals where the L is connected. In other words, we remove the L and find the Thevenin equivalent of the remaining circuit.

Since $t>0$, the switch is open.



The circuit is already in Thevenin equivalent form with $V_{Th} = 0V$ and $R_{Th} = 100k\Omega$.

Thus, the time constant of the circuit is

$$\frac{L}{R_{Th}} = \frac{3.3\,mH}{100\,k\Omega} = \frac{33\,mH}{1\,M\Omega} = 33\,ns$$

Substituting values into the general form of solution, we have our desired answer:

$$i_L(t>0) = 0A + (600\mu A - 0A)\,e^{-t/33ns}$$

or

$$i_L(t>0) = 600\mu A \cdot e^{-t/33\,ns}$$