Indefinite Focusing

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Physics

by

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2002
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2002
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ACKNOWLEDGEMENTS

Each chapter of this thesis has or will be submitted for publication as listed below:


D. Schurig, D. R. Smith, S. Schultz, S. A. Ramakrishna, J. B. Pendry; Focusing Properties of a Slab of Negative Index Media, not yet submitted.


D. Schurig and D. R. Smith; Sub-Diffraction Imaging with Compensated Bilayers of Indefinite Media, not yet submitted.

D. Schurig and D. R. Smith; Electromagnetic Spatial Filtering Using Media with Negative Properties, not yet submitted.

I would like to thank the following people: Daniel Schurig, Diane Schurig, Darren Schurig, Sheldon Schultz, David Smith, John Pendry, Alan Schneider, Norman Kroll, the rest of my thesis defense committee, Andy Pommer, George Kassabian, Matt Pufall, Sang Park, Willie Padilla, Joan Marler, Jason Goldberg, Erik Stauber, Jason Feinberg, Syrus Nemat-Nasser, Stephen Wolfram, John Jackson, James Maxwell, Tony Starr, Tatiana Starr, Douglas Paulson, everyone at Tristan Technologies Inc., the UCSD cycling team, the UCSD triathlon team, CalSpace, Kevin the Hari Krishna, and the UCSD physics graduate class of 1991.
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ABSTRACT OF THE DISSERTATION

Indefinite Focusing

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Doctor of Philosophy in Physics

University of California, San Diego, 2002

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It is shown that a modulated Gaussian beam undergoes negative refraction at the interface between a positive and negative refractive index material. While the refraction of the beam is clearly negative, the modulation interference fronts are not normal to the group velocity, and thus exhibit a sideways motion relative to the beam—an effect due to the inherent frequency dispersion associated with the negative index medium.

A slab of negative index media has been shown to focus both the far-field and the near-field of an electromagnetic source and form an image with resolution better than what is possible with diffraction limited, conventional optics. However, deviations from the ideal material properties, such as dissipation, limit the image resolution. A wide range of the material property parameter space is studied, demonstrating the limits of resolution with numerical results and approximate analytical formulas. The effects of a finite aperture on the imaging are also examined.

Wave propagation in materials for which not all of the principle elements of the permeability and permittivity tensors have the same sign is studied. These tensors are neither positive nor negative definite. It is found that a wide variety of effects can be realized in such media, including negative refraction, near-field focusing and high impedance surface reflection.
A new type of near field focusing device is analyzed that consists of a bilayer of media with indefinite electromagnetic material property tensors. The dispersion of the media supports propagating waves for any transverse wave vector, which results in a weaker dependence of spatial bandwidth on media lossiness.

We show how bilayers of media with negative electromagnetic property tensor elements can be used to construct low, high and band pass spatial filters. These filters possess sharp adjustable roll offs and can operate in the near and far field regimes to select specific spatial variation components or beam angles.
Chapter I

Negative Refraction of Modulated Electromagnetic Waves

I.A Abstract

We show that a modulated Gaussian beam undergoes negative refraction at the interface between a positive and negative refractive index material. While the refraction of the beam is clearly negative, the modulation interference fronts are not normal to the group velocity, and thus exhibit a sideways motion relative to the beam—an effect due to the inherent frequency dispersion associated with the negative index medium. In particular, the interference fronts appear to bend in a manner suggesting positive refraction, such that for a plane wave, the true direction of the energy flow associated with the refracted beam is not obvious.

I.B Letter

Since the experimental demonstrations of a structured metamaterial with simultaneously negative permittivity and negative permeability [1, 2]-often referred to as a left-handed material, the phenomenon of negative refraction, predicted to occur in left-handed materials [3, 4], has become of increasing interest. The consequence of a modulated plane wave incident on an infinite half-space of negative
refractive index material has been considered by Valanju et al. [5], who have noted that the interference fronts due to the modulation appear to undergo positive refraction, despite the underlying negative refraction of the component waves. Valanju et al. further associate the direction of the advancing interference pattern with that of the group velocity, and draw a broad set of conclusions regarding the validity of negative refraction.

In this letter, we examine the effect of a finite-width beam refracting at the interface between two semi-infinite media, one with positive refractive index and the other with negative refractive index. We find that the transmitted beam does indeed refract in the negative direction, although the interference fronts do not necessarily point in the same direction. We first describe the refraction of a modulated plane wave incident on a half-space with negative index, which demonstrates in a straightforward manner, that the interference fronts of the transmitted wave display interesting behavior.

We assume the geometry shown in Fig. I.1, in which a modulated electromagnetic wave is incident from vacuum onto a material with frequency dependent, negative index of refraction, \( n(\omega) \). We choose the z-axis normal to the interface, the x-axis in the plane of the figure, and the y-axis out of the plane of the figure. While our arguments will apply to a wave with arbitrary polarization, we assume here that the electric field is polarized along the y-axis (out of plane). For convenience, we use identical causal plasmonic forms for the permittivity (\( \varepsilon \)) and the permeability (\( \mu \)), so that the refractive index is given by

\[
\begin{align*}
n(\omega) = \sqrt{\varepsilon \mu} &= 1 - \frac{\omega_p^2}{\omega (\omega + i\Gamma)}. \tag{I.1}
\end{align*}
\]

Note that the refractive index is negative below the plasma frequency \( \omega_p \), and has the value of approximately \( -1 \) when \( \omega = \omega_p/\sqrt{2} \) and losses are small, \( \Gamma/\omega \ll 1 \).

A modulated plane wave has the form

\[
E(\mathbf{r}, t) = e^{i\mathbf{k} \cdot \mathbf{r}} e^{-i\omega t} \cos(\Delta k \cdot \mathbf{r} - \Delta \omega t), \tag{I.2}
\]
Figure I.1: Normal incidence (A). The group velocity and interference wave vector are parallel. Incidence at 45 degree angle (B). On the incident side (left) the component wave vectors, interference wave vector and group velocity all point in the same direction. In the NIM (right) the interference wave vector has refracted positively and the component wave vectors and group velocity have refracted negatively with the component wave vectors approximately anti-parallel to the group velocity. The dashed lines indicate conservation of the transverse component of the various wave vectors.
where \( \Omega \) is the carrier frequency of the wave, \( \Delta \omega \) is the modulation frequency and the wave vector, \( k \), is given in terms of the incident angle,

\[
k = \sin(\theta) \hat{x} + \cos(\theta) \hat{z} \quad (I.3)
\]

where \( k = \omega/c \). Eq. I.3 can be equivalently viewed as the superposition of two plane waves with different frequencies, or

\[
E(r, t) = \frac{1}{2} e^{ik_+ \cdot r} e^{-i\omega_+ t} + \frac{1}{2} e^{ik_- \cdot r} e^{-i\omega_- t}
\]

where the frequencies of the component waves are \( \omega_\pm = \omega \pm \Delta \omega \) with corresponding propagation vectors \( k_\pm = k \pm \Delta k \). \( \Delta k \) is parallel to \( k \) and is given by

\[
\Delta k = \frac{\Delta \omega}{\omega} k \quad (I.4)
\]

Because the negative index medium is assumed to be frequency dispersive, the transmitted waves will refract at slightly different angles, thus introducing a difference in the phase versus interference propagation directions. The transmitted wave can be expressed in the form

\[
E(r, t) = e^{iq \cdot r} e^{-i\Omega t} \cos(\Delta q \cdot r - \Delta \omega t), \quad (I.5)
\]

which again is composed of two propagating plane waves having propagation vectors \( q_+ \) and \( q_- \), the values of which can be found using the conservation of the parallel wave vector across the interface

\[
q_x = k_x \quad (I.6)
\]

and the constancy of frequency,

\[
q \cdot q = n^2 k^2 \quad (I.7)
\]

resulting in

\[
q = k_x \hat{x} \pm \sqrt{n^2 k^2 - k_x^2} \hat{z} \quad (I.8)
\]
The correct sign of this expression for individual plane waves is well established elsewhere [6, 7]. Using $\Delta q_x = \Delta k_x$ and Eq. I.7, the wave vector of the interference pattern is obtained.

$$\Delta q = \left[ k_x \hat{x} + \frac{1}{q_z} \left( k^2 n n_g - k_x^2 \right) \hat{z} \right] \frac{\Delta \omega}{\Omega} \quad (I.9)$$

The *group index*, given by

$$n_g \equiv \frac{\partial (n \omega)}{\partial \omega} \geq 1$$

is constrained by causality to be greater than unity regardless of the sign of the refractive index of the medium. Thus, we see that for a negative refractive index the $z$-component of the interference wave vector has opposite sign to the $z$-component of the individual wave vectors, so though the individual wave vectors refract negatively, the interference pattern refracts positively. Furthermore, the apparent velocity of the interference fronts is directed away from the interface, as would be expected if the direction of the interference pattern were coincident with the direction of the group velocity. Note that Eq. I.9 also implies that the spacing between interference fronts (or beats) in the negative index medium is set by the dispersion characteristics of the medium rather than just the frequencies of the two component waves. This effect is illustrated in Fig. I.1A for a wave normally incident from vacuum on a medium with a frequency dispersive negative refractive index. In this example, the carrier frequency is assumed to be at the frequency where $n = -1$. For our choice of frequency dispersion (Eq. I.1), $\frac{\partial (n \omega)}{\partial \omega} = 3$, and Eq. I.9 shows that the spacing between interference fronts is one-third that in free space, as seen in the figure.

When a monochromatic wave is incident on the interface between positive and negative media at an angle, previous calculations and theory [1, 3] have shown that the phase velocity is negative, and the transmitted ray in the medium is directed toward the interface and negatively refracted. Eq. I.9, however, indicates that the interference fronts associated with a modulated plane wave should undergo positive refraction. A calculation of the refraction of a modulated plane wave shows
that this is indeed the case (Fig. I.1B): the incident interference pattern refracts to the opposite side of the normal, consistent with Valanju et al. [5].

The question thus naturally arises: what is the relationship between the interference fronts associated with a modulated wave and its group velocity? The group velocity is defined as [8]

\[ \mathbf{v}_g \equiv \nabla_q \omega (\mathbf{q}), \]  

\( \text{(I.10)} \)

which, for isotropic, low loss materials becomes

\[ \mathbf{v}_g = \frac{\mathbf{q}}{q} \frac{d\omega}{dq} (q) = \frac{\mathbf{q}}{q} \text{sign}(n) \frac{c}{n_g}. \]  

\( \text{(I.11)} \)

The scalar part of this expression is a material property and the unit vector pre-factor is a determined by the direction of the incoming wave. Note that when the index is negative, Eq. I.11 predicts that the direction of the group velocity is anti-parallel to the phase velocity, and thus refracts negatively-unlike the interference wave vector. This would seem to be a paradox, as the interference velocity of a modulated beam is frequently equated with the group velocity [8]. To resolve this dilemma, we examine the manner in which a point of constant phase on the interference patterns moves. Setting the argument of the Cosine in Eq. I.5 equal to a constant, we find that the velocity of such a point, \( \mathbf{v}_{int} \), must satisfy

\[ \frac{\Delta \mathbf{q}}{\Delta \omega} \cdot \mathbf{v}_{int} = 1 \]  

\( \text{(I.12)} \)

The smallest possible such velocity is the one parallel to \( \Delta \mathbf{q} \) (the interference direction), which has been interpreted as the group velocity by Valanju et al. [5]; however, \( \mathbf{v}_{int} \) can have an arbitrary component perpendicular to \( \Delta \mathbf{q} \). For non-normal incidence, the group velocity has such a component. Using Eqs. I.9 and I.11, we can verify that the group velocity does indeed satisfy Eq. I.12. We thus conclude that a \textit{component} of the group velocity follows the interference pattern, but that there is also a component \textit{parallel} to the interference wave fronts.

Since we have concluded that the interference velocity need not be in the same direction as the group velocity, we cannot easily ascertain the group velocity.
from the field pattern of the transmitted part of a modulated plane wave refracting at the interface of a negative index material. To obtain a better picture of the refraction phenomenon, we consider a modulated beam of finite width. Following the method outlined by Kong et al. [9], we calculate the refraction associated with an incident modulated beam with a finite (Gaussian) profile. The electric field as a function of position is determined from the inverse Fourier transform of the \( k \)-space field.

\[
E(x, z) = F^{-1}\{E(k_x, z)\} = \int_{-\infty}^{+\infty} dk_x e^{ik_xx} E(k_x, z)
\]

where the \( k \)-space field is determined by solving the boundary value problem

\[
E(k_x, z) = E_i(k_x) \begin{cases} 
e e^{ik_xz} + \rho e^{-ik_xz} & z < 0 \\
\tau e^{i\delta z} & z > 0 \end{cases} \tag{I.13}
\]

and the expression

\[
E_i(k_x) = \frac{g}{2\sqrt{\pi}} e^{-\frac{1}{4}g^2(k_x-k_{xc})^2} \tag{I.14}
\]

provides the Gaussian shape of the incident beam. The incident beam is centered about the wave vector with \( k_x = k_{xc} \). When the wave interacts with the interface, reflected and transmitted waves are generated. Expressions for the reflection and transmission coefficients are determined by matching the electric and magnetic fields at the interface between the negative and positive index media. The transmission coefficient is

\[
\tau = \frac{2\mu k_z}{\mu k_z + q_z}, \tag{I.15}
\]

and the reflection coefficient is

\[
\rho = \frac{\mu k_z - q_z}{\mu k_z + q_z}. \tag{I.16}
\]

To produce a modulated beam, we superpose two beams with different frequencies, and hence different incident wave vectors \( k_x \).

Figure I.2 shows the intensity pattern of the electric field in a beam undergoing refraction and reflection at the interface between vacuum and a negative
Figure I.2: Refraction of a Gaussian beam into a NIM. The angle of incidence is 30 degrees. $n(\omega_-) = -1.66 + 0.003i$ and $n(\omega_+) = -1.00 + 0.002i$. $\Delta\omega/\omega = 0.07$.

refractive index material. The modulation is evident in the spacing between intensity maxima (the carrier wave is not shown in the figure).

Although the interference fronts associated with the modulation are not perpendicular to the direction of the refracted beam, the modulation is indeed carried through. As would be expected, as the time step is advanced the transmitted modulation fronts advance in the direction of the beam, even though the phase velocity of the carrier is antiparallel to the beam propagation direction. This behavior is both consistent with the notion of negative refraction, as well as the results obtained by Valanju et al [5].

Negative refractive materials are necessarily frequency dispersive, so that the various frequency components of a modulated beam are refracted at different angles within the medium. This effect is made clear in Fig. I.2, where further from the interface, within the medium, the two component beams have separated, and are also damped by the losses included in the calculation. Nevertheless, were the beam to interact with a finite section of negative index material, the modulation would be preserved in the transmitted beam. Note also that we have simulated
very large modulation frequencies in these simulations to demonstrate these effects—much larger than would be necessary for the transfer of information in typical communications applications.

Through an analysis of the points of constant phase of a modulated plane wave, we have shown that both the group and phase velocities undergo negative refraction at the interface between a positive and a negative index material. The interference fronts of the modulated wave are not normal to the group velocity, and exhibit a sideways motion as they move at the group velocity. Consideration of a modulated beam of finite extent clearly resolves the difference between the group velocity and normal to the interference fronts, and shows that recent observations [5] are consistent with negative refraction [3]. These conclusions hold also for certain bands in photonic band gap structures that exhibit negative effective refractive index [10, 11].

I.C Acknowledgement

We thank Professors S. Schultz and N. Kroll for valuable discussions. This work was supported under by DARPA through a grant from ONR (Contract No. N00014-00-1-0632).

This chapter, in full, has been submitted for publication to *Applied Physics Letters*, with authors D. R. Smith, D. Schurig, J. B. Pendry.
Chapter II

Focusing Properties of a Slab of Negative Index Media

II.A Abstract

A slab of negative index media has been shown to focus both the far-field and the near-field of an electromagnetic source and form an image with resolution better than what is possible with diffraction limited, conventional optics. However, deviations from the ideal material properties, such as dissipation, limit the image resolution. We explore a wide range of the material property parameter space demonstrating the limits of resolution with numerical results and approximate analytical formulas. We also examine the effects of a finite aperture on the imaging.

II.B Introduction

Negative index media (NIM) is defined as a material in which the real parts of the dielectric permittivity ($\varepsilon$) and the magnetic permeability ($\mu$) are simultaneously negative. Since Veselago’s original treatment [3] this subject remained dormant until the recent experimental demonstration of NIM at microwave frequencies[1, 2]. This field received more attention when Pendry[6] demonstrated that it was theoretically possible for a slab of NIM to “focus” or restore the near-
field, evanescent components of a source as in addition to the far-field, propagating components (as originally shown by Veselago). Thus the image formed by this “lens” is not subject to the traditional diffraction limit on image resolution of conventional optics. We will refer to an object capable of sub-diffraction limit focusing as a super lens. If the material properties of NIM are ideal it is even possible for the image to be identical to the source. Pendry referred to this possibility as a perfect lens. Motivated by this and other possible applications, there have been many investigations into the physics of NIM[1, 2, 6, 12, 13, 14, 15, 7, 16, 17, 18, 19].

Current experimental implementations of NIM operate at microwave frequencies and employ structured composites consisting of two interpenetrating arrays of copper elements[1]. Magnetic response is supplied by an array of split ring resonators (SRRs)[20], and electric response by an array of wires[21]. When probed by wavelengths larger than the unit cell size these structures behave as materials with effective, bulk electromagnetic properties. In this case the SRR array gives rise to an effective magnetic permeability that is negative, and the wire array gives rise to an effective electric permittivity that is negative. When the arrays are combined into a single medium, the resulting composite has a negative index of refraction. This engineered composite, which exhibits a property not found in nature, can be classified as a metamaterial. The current NIM implementation does not have the required material properties to function as a super lens, but adequately demonstrate the process of designing and building a material with specific, desired, negative electromagnetic properties. Experimental demonstration of a super lens should be a feasible if difficult task.

In the original paper on the perfect lens [6], the central result was derived by summing a series representing multiple reflections within the slab. An implementation using a silver slab with realistic dissipation was discussed in the electrostatic limit. In Ref. [22], we began to consider the effect of deviations of the material properties from their ideal values and concluded that sub-wavelength resolutions can only be obtained within a very restricted range. We also determined
the effects of the finite unit cell size required for composite material implementations. In Ref. [19], we studied an asymmetric extension to the theory that is useful for analyzing thin film on substrate implementations.

In this paper, we solve for the fields inside and outside of a slab with arbitrary values of the material properties. We use a boundary value problem approach which should eliminate concern over series convergence and square-root branch ambiguity. We derive simple approximate, analytic formulas for the maximum parameter deviation allowed for a given resolution. The k-space transfer function and the spatial image for a specific sub-wavelength object are computed numerically for a wide range of parameter values. We show how property deviations can lead to resolution limiting resonance behavior, and make the connection to plasmon resonance phenomena. Finally, we discuss the effects of the finite aperture that is required for any physically realizable device.

II.C Description of Perfect Image Evolution

In Fig. II.1, we plot the evolution of a spatial field distributions through an NIM slab with ideal properties $\mu = \epsilon = -1$. The object is a Gaussian field distribution. We begin outside the slab, at the object plane. As we move downstream through $\mu = \epsilon = 1$ media from the object we wish to image, the image evolves normally. Sharp features blur as the near-field attenuates and the far-field diverges. Then, when crossing into $\mu = \epsilon = -1$ media, the z-component of the propagation constant changes sign. Now as we continue downstream (energy flow direction), the waves reverse their evolution. We begin replaying the image evolution observed in the right handed media - but in reverse. This applies to both the near- and far-field components. For the near-field, attenuation becomes growth. Eventually we converge to the original object we wished to image. This is the first focus we observe, and it is internal to the NIM slab. Continuing downstream we continue the reverse evolution. The image evolves into the precursors of the ob-
ject. These precursor images are just images that would evolve into the object, if evolution proceeded in the forward direction. These images are identical to images upstream from the object. Crossing a second interface and back into $\mu = \epsilon = 1$ media reverses the sign (of the $z$ component of the propagation constant) again, back to its original sign. Now continuing downstream the precursor images will evolve back into the object, for a second focus. This focus is beyond the slab and occurs exactly where the amount of forward evolution equals the amount of reverse evolution. If the slab has thickness $d$, the second focus occurs $2d$ from the object. Using the same reasoning, if the object is $\Delta$ in front of the slab, then the first focus, (the internal one), occurs at the plane $\Delta$ within the slab, figure II.3. It is also clear, that if the desired object is further away from the slab than the slab’s thickness, $d$, no real image will be formed either within or beyond the slab.

It is important to note that the above description employing the reversal of the propagation constant at the interfaces applies only to a slab. An interface between semi-infinite media behaves quite differently. As has been pointed out by other authors[5], an evanescent field incident on a semi-infinite NIM cannot undergo a sign change of the propagation constant because this results in growth in an infinite media which violates causality, energy conservation or normalizability depending on your preference. In the slab problem, the second interface interrupts this runaway growth. In solving the slab boundary value problem below, we must include all valid basis functions (growing and decaying) and allow the boundary conditions to uniquely determine the solution.

II.D The Boundary Value Problem

We start with a plane wave which may be inhomogenous (the wave vector $\mathbf{k}$ may have complex components.) For our two dimensional treatment we can choose the $y$-axis to be the direction of invariance, i.e. $\mathbf{k} = k_x \mathbf{\hat{x}} + k_z \mathbf{\hat{z}}$. It is traditional, in reflection/transmission problems to refer to the two polarizations as
Figure II.1: Spatial evolution of field distribution through a perfect lens slab. Both slab interfaces are mirror planes for the field distribution. (Field profiles are not uniformly scaled.)
s and p. For s-polarization, the electric polarization vector is defined to lie along the invariant axis of the fields. We will work out the results for s-polarization. The results for p-polarization can then be obtained by a simple substitution, (\( \mu \leftrightarrow \epsilon \) and \( \mathbf{H} \leftrightarrow \mathbf{E} \).) The electric field is given by

\[
\mathbf{E} = \hat{y} e^{i(k \cdot r - i\omega t)}
\]

Maxwell’s curl equation then gives the magnetic field as

\[
\mathbf{H} = \frac{1}{\omega \mu} (k_x \hat{z} - k_z \hat{x}) e^{i(k \cdot r - i\omega t)}
\]

where the wave equation dictates the dispersion

\[
k \cdot k = k_x^2 + k_z^2 = \omega^2 \mu \epsilon
\]

Due to material invariance along the layers (x-axis), \( k_x \), will be conserved across the layers.

We will discuss, at length, a single layer with material constants \( \mu_1, \epsilon_1 \) surrounded on both sides by regions with material constants \( \mu_0, \epsilon_0 \). The z-axis origin is located in the middle of the slab. The solution consists of an incident, reflected, transmitted and internal plane waves. The wave vector for each piece of the solution is determined by the parameter, \( k_x \), as follows:

\[
\begin{align*}
\mathbf{k}_i &= k_x \hat{x} + k_z \hat{z} \\
\mathbf{k}_r &= k_x \hat{x} - k_z \hat{z} \\
\mathbf{k}_a &= k_x \hat{x} + q_z \hat{z} \\
\mathbf{k}_b &= k_x \hat{x} - q_z \hat{z} \\
\mathbf{k}_r &= k_x \hat{x} + k_z \hat{z}
\end{align*}
\]

where
Figure II.2: Labeling conventions for boundary value problem.
\[
\begin{align*}
k_z &= \sqrt{k_0^2 - k_x^2} \\
q_z &= \sqrt{\mu k_0^2 - k_x^2} \\
k_0^2 &= \omega^2 \varepsilon_0 \\
\mu &= \frac{\mu_1}{\mu_0}, \quad \varepsilon = \frac{\varepsilon_1}{\varepsilon_0}
\end{align*}
\]

We desire the incident and transmitted wave to propagate or decay in the positive \( z \) direction and the reflected wave to propagate or decay in the minus \( z \) direction. These conditions are all met if we choose a branch of the square root for \( k_z \) that yields positive, real values for positive, real arguments and positive, imaginary values for negative, real arguments. The default branch used on most computer systems satisfies the criteria. This is the routine choice always employed in reflection and transmission problems and has nothing to do with NIM. However, the choice of branch for \( q_z \) is not routine and has everything to do with NIM. Fortunately, we need not make any a priori choice since both signs of \( q_z \) appear in our trial solution.

The transverse component of the electric field in the three regions (figure II.2) with unknown coefficients is given

\[
E_y = e^{ik_x x} e^{-iwt} \begin{cases} 
  e^{ik_z z} + \rho e^{-ik_z z} & I \\
  ae^{iq_z z} + be^{-iq_z z} & II \\
  \tau e^{ik_z z} & III
\end{cases}
\] (II.1)

This function must be continuous. Matching at the boundaries gives the first two equations needed to solve the system of four unknowns.

\[
\begin{align*}
e^{-ik_z d/2} + \rho e^{ik_z d/2} &= ae^{-iq_z d/2} + be^{iq_z d/2} \\
abe^{iq_z d/2} + be^{-iq_z d/2} &= \tau e^{ik_z d/2}
\end{align*}
\]

The transverse component of the magnetic field is given by
$$H_x = -\frac{e^{ik_x x}}{\omega \mu_1} e^{-i\omega t} \begin{cases} 
\mu k_z e^{ik_z z} - \rho \mu k_z e^{-ik_z z} & \text{I} \\
a q_z e^{i q_z z} - b q_z e^{-i q_z z} & \text{II} \\
\tau \mu k_z e^{ik_z z} & \text{III} \end{cases}$$

Matching at the boundaries gives the second two equations required to solve for the four unknown coefficients.

\[
\begin{align*}
\mu k_z e^{-ik_z d/2} - \rho \mu k_z e^{ik_z d/2} &= a q_z e^{-iq_z d/2} - b q_z e^{iq_z d/2} \\
a q_z e^{iq_z d/2} - b q_z e^{-iq_z d/2} &= \tau \mu k_z e^{ik_z d/2}
\end{align*}
\]

Solving the system of four equations yields the coefficients

\[
\begin{align*}
\tau &= e^{-ik_z d} 4 \mu q_z k_z \delta^{-1} \quad \text{(II.3a)} \\
\rho &= e^{-ik_z d}(q_z + \mu k_z)(q_z - \mu k_z)(e^{iq_z d} - e^{-iq_z d})\delta^{-1} \quad \text{(II.3b)} \\
a &= e^{-ik_z d/2} 2 \mu k_z(q_z + \mu k_z) e^{-iq_z d/2} \delta^{-1} \quad \text{(II.3c)} \\
b &= e^{-ik_z d/2} 2 \mu k_z(q_z - \mu k_z) e^{iq_z d/2} \delta^{-1} \quad \text{(II.3d)}
\end{align*}
\]

where

\[
\delta \equiv (q_z + \mu k_z)^2 e^{-iq_z d} - (q_z - \mu k_z)^2 e^{iq_z d} \quad \text{(II.4)}
\]

The coefficients have definite symmetry with respect to \(q_z\)

\[
\begin{align*}
\delta(-q_z) &= -\delta(q_z) \\
\tau(-q_z) &= \tau(q_z) \\
\rho(-q_z) &= \rho(q_z) \\
a(-q_z) &= b(q_z)
\end{align*}
\]

The symmetry of the these coefficients implies that the solution is invariant with respect to choice of branch for the square root used to compute \(q_z\).
There is a resonant mode whenever $\delta = 0$. This condition occurs when

$$e^{2iqzd} \left( \frac{q_z - \mu k_z}{q_z + \mu k_z} \right)^2 = 1 \quad (\text{II.5})$$

Note that when $\mu = \epsilon = -1$, this condition can never be satisfied, but whenever there is a deviation from this value, resonant modes occur. This condition has an intuitive interpretation if one compares the above with the coefficient for single interface reflection going from region II material to region I/III material.

$$\rho_1 = \frac{q_z - \mu k_z}{q_z + \mu k_z}$$

Then equation II.5 becomes

$$(e^{iqzd} \rho_1)^2 = 1$$

and the resonance condition says that one round trip within the slab returns the plane wave to its original magnitude and phase. Thus the multiply reflected waves within the slab constructively interfere with each other. It is worth noting that if $\mu = \epsilon = -1$ exactly, then $q_z = k_z$, $\rho_1$ is divergent, and there will be no resonance for any finite value of $q_z$. Taking the square root

$$e^{iqzd} \rho_1 = \pm 1 \quad (\text{II.6})$$

we see that the two types of solutions are possible. For the plus sign, a one way trip across the slab returns the plane wave to its original phase, yielding a symmetric mode. For the minus sign a one way trip across the slab results in a $\pi$ phase shift, yielding an asymmetric mode. These slab modes can be decomposed into symmetrically and anti-symmetrically coupled surface plasmons. Plasmons are treated thoroughly in Ruppin[13], where equivalent forms of equation II.6 are used

$$\mu k_z + q_z \tanh \frac{qzd}{2t} = 0 \quad (\text{II.7})$$
$$\mu k_z + q_z \coth \frac{qzd}{2t} = 0 \quad (\text{II.8})$$
Equation II.7 has solutions that are symmetric modes and equation II.8 has solutions that are antisymmetric modes.

The $k$-space transfer function is defined as the ratio of the total field, $E_y$, at some position, $z_1$, to the incident field, $E_{yi}$, at some other position, $z_0$ for a given transverse wave vector, $k_x$. $z_0$ must lie in region I where the incident field is defined. The transfer function depends implicitly on $k_x$ through $k_z$ and $q_z$.

$$T_{k_z}(z_0, z_1) \equiv \frac{E_y(z_1)}{E_{yi}(z_0)}$$

The transfer function is given by

$$T(z_0, z_1) = \delta^{-1} \begin{cases} 
e^{ik_z(z_1-z_0)} + (q_z + \mu k_z)(q_z - \mu k_z)(e^{iq_zd} - e^{-iq_zd})e^{-ik_z(z_1+z_0+d)} & \text{I} \\
2\mu k_z e^{-ik_z(z_0+d/2)} [(q_z + \mu k_z)e^{-iq_z(d/2-z_1)} + (q_z - \mu k_z)e^{iq_z(d/2-z_1)}] & \text{II} \\
4\mu q_z k_z e^{ik_z(z_1-z_0-d)} & \text{III} 
\end{cases}
$$

\text{(II.9)}

**Special Cases**

When the material constants are both unity, $\mu = \epsilon = 1$, it is easy to show that the transfer function reduces to

$$T_{k_z}(z_0, z_1) = e^{ik_z(z_1-z_0)}$$

which is the correct transfer function for a plane wave in free space.

For the perfect lens, $\mu = \epsilon = -1$, the electric field reduces to

$$E_y = e^{ik_zx} \begin{cases} e^{ik_zz} & \text{I} \\
e^{-ik_z(z+d)} & \text{II} \\
e^{ik_z(z-2d)} & \text{III} \end{cases}$$

It is worth noting that, though no a priori choice was made for the wave vector in the slab, only the $-k_z$ solution is present. This is the solution with negative phase velocity for propagating wave vectors and exponential growth behavior for
Figure II.3: Location of foci. The two foci occur at the locations marked $z_1$. The source is at $z_0$.

Evanescent wave vectors. The electric field is, of course, continuous and the transfer function is given by

$$T_{k_x}(z_0, z_1) = \begin{cases} 
eq \frac{e^{ik_x(z_1-z_0)}}{z_1 \in I} \\ \frac{e^{-ik_x(z_1+z_0+d)}}{z_1 \in II} \\ \frac{e^{ik_x(z_1-z_0-2d)}}{z_1 \in III} \end{cases}$$

If $z_0$ is within distance $d$ of the slab $(-3d/2 \leq z_0 \leq -d/2)$ then

$$T_{k_x}(z_0, z_0 + 2d) = 1$$

$$T_{k_x}(z_0, -z_0 - d) = 1$$

This is perfect focusing of the $z$-plane at $z_0$ onto a $z$-plane in region III and also a plane in region II within the slab. The locations of these foci are shown diagrammatically in figure II.3.
II.E Image Resolution: Approximate Analytical Results

Based on the transfer function derived above, it is possible to develop a useful approximate analytical expression for material parameter deviation vs. image resolution. The transfer function for the ideal object to image distance is

\[ T_{k_z}(z_0, z_0 + 2d) = \frac{4\mu q_z k_z e^{ik_z d}}{(q_z + \mu k_z)^2 e^{-iq_z d} - (q_z - \mu k_z)^2 e^{iq_z d}} \]

The exponential in the numerator arises from translation outside the slab. The exponentials in the denominator arise from the solutions inside the slab. If \( \mu = \epsilon = -1 \) then \( q_z = \pm k_z \) and one of the terms in the denominator is nulled out. The remaining term exactly cancels the numerator and we have perfect focusing. If we do not have \( \mu = \epsilon = -1 \) then both terms are non-zero. An approximate condition for marginal focusing occurs when the two terms have equal magnitude.

This condition is

\[ \left| \frac{q_z + \mu k_z}{q_z - \mu k_z} \right| = |e^{iq_z d}| \]

This is the equation we will use to obtain an expression for the resolution. We define the following measure of resolution

\[ R \equiv \frac{|k_x|}{k_0} = \frac{\lambda_0}{\lambda_x} \]

This is just the number of spatial variations per free space wavelength. We wish to find an approximation in the regime where this resolution is significantly greater than one. This is the regime unavailable to conventional optics. We begin by approximating \( q_z \) and \( k_z \) in this limit

\[ q_z \approx i |k_x| \left[ 1 - \frac{1}{2} \mu \varepsilon \left( \frac{k_0}{k_x} \right)^2 \right] \]

\[ k_z \approx i |k_x| \left[ 1 - \frac{1}{2} \left( \frac{k_0}{k_x} \right)^2 \right] \]
The rational expression above becomes

\[
\frac{q_z + \mu k_z}{q_z - \mu k_z} \approx \frac{(\mu + 1) - \frac{1}{2}\mu (\varepsilon + 1) \left( \frac{k_0}{k_x} \right)^2}{(1 - \mu) + \frac{1}{2}\mu (1 - \varepsilon) \left( \frac{k_0}{k_x} \right)^2}
\]

If we deviate from the ideal permeability by a small amount \( \mu = -1 + \delta \mu \) with \( \varepsilon = -1 \) then we obtain

\[
\frac{q_z + \mu k_z}{q_z - \mu k_z} \approx \frac{1}{2} \delta \mu
\]

Similarly for \( \varepsilon = -1 + \delta \varepsilon \) and \( \mu = -1 \) we find

\[
\frac{q_z + \mu k_z}{q_z - \mu k_z} \approx \frac{1}{4} \delta \varepsilon \left( \frac{k_0}{k_x} \right)^2
\]

We use a zeroth order approximation for \( q_z \) where it appears in the exponential

\[
e^{iqz d} \approx e^{-|k_x|d} = e^{-Rk_0 d}
\]

and finally obtain

\[
|\delta \mu| = 2e^{-Rk_0 d} \quad \text{(II.10a)}
\]

\[
|\delta \varepsilon| = 4R^2 e^{-Rk_0 d} \quad \text{(II.10b)}
\]

These expression give the allowed parameter deviation from minus one to obtain spatial resolution \( R \). The allowable deviation is also strongly dependent on \( k_0 d \), the phase advance of a free space wave across the slab thickness \( d \). Note that the allowable deviation in the permittivity is greater than that allowed in the permeability by the large factor \( 2R^2 \). These roles are reversed for p-polarization.

The expression for \( \delta \mu \) can be inverted to an explicit expression for the resolution.

\[
R = -\ln \left( \frac{|\delta \mu|}{2} \right) \frac{1}{k_0 d} \quad \text{(II.11)}
\]

The validity of these approximate expression for the resolution enhancement is supported by comparing Eq. II.11 with Figure II.5. For example, for a deviation of \( \delta \mu = 0.005 \), the numerically computed transfer function has a limiting resonance at \( k_x/k_0 \sim 9.5 \), which is the same value predicted by Eq. II.11.
The dependence of the resolution enhancement $R$ on the deviation from the perfect lens condition is critical. The factor $k_0d$ dominates the behavior of the resolution, the logarithm term being a relatively weakly varying function. For example, if $\frac{k_0d}{2\pi} = 1$, we find that to achieve an $R$ of 10, $|\delta \mu|$ must be less than $10^{-27}$! However, for $\frac{k_0d}{2\pi} = 0.1$, $\delta \mu$ can vary by as much as 0.004 to achieve the same resolution enhancement. Similar deviations in either the real or the imaginary part of the permeability will result in approximately the same resolution, although the slab resonances occur only when the deviation occurs in the real part.

II.F Source Object

The source object is not literally a material object. Creating a focused image of an object is the recreation, at some distance from the object, of the fields that were produced or reflected by the object. So in the context of image formation, the object is defined by the fields it produces. In our two dimensional geometry, an object is a field distribution at an x-y plane, the object plane, at some $z_0$ location. An image is said to be formed if this same x-y field distribution is recreated at some other location, $z$, the image plane.

Since we have chosen the y-axis to be homogenous, an x-y plane field distribution is actually one dimensional. This one dimensional distribution can be expanded in a one dimensional Fourier series.

$$E_{yi}(z_0\hat{Z} + x\hat{x}) = \sum_{k_x} c_{k_x} e^{ik_xx}$$

Each Fourier coefficient determines the phase and magnitude of a plane wave as it passes through the object plane. The wave vector of this plane wave has $x$ component, $k_x$, the transform variable, and $z$ component, $k_z$, determined from equation ???. The coefficients are determined by the usual integral.

$$c_{k_x} = \frac{1}{a} \int_{-a/2}^{a/2} E_{yi}(z_0\hat{Z} + x\hat{x})e^{-ik_xx} dx$$
The periodicity, \( a \), determines the \( k_x \) interval as discussed in the next section. The field distribution at any \( z \) plane is determined using \( T_{k_x} \) to transfer the Fourier coefficients to that plane.

\[
E_y(z\hat{\textbf{z}} + x\hat{\textbf{x}}) = \sum_{k_x} c_{k_x} T_{k_x}(z_0, z)e^{i k_x x}
\]

II.G k-Space Mesh

The range of values of \( k_x \) used for the above expansions is critical to the accuracy of the solutions. A choice must be made for both the maximum value of transverse wave vector, \( k_{x\text{max}} \), and the spacing, \( dk_x \). These values cannot be solely based on the requirements of accurately decomposing the source, \( c_{k_x} \). When transforming back to the spatial basis, the product \( c_{k_x} T_{k_x} \) appears in the inverse transform. The k-space mesh must be adequate to represent all features of this product. The transfer function \( T_{k_x}(z_0, z) \) can have resonances in \( k_x \) which are quite strong and narrow. These resonances can necessitate values of \( k_{x\text{max}} \) and \( dk_x \) that are larger and smaller respectively than would have been anticipated from just the source spectrum, \( c_{k_x} \). In fact for lossless media the resonance peaks are arbitrarily sharp. Mesh refinement will never lead to image convergence. When it was desired to isolate the effects of deviation of the real part of the material properties from minus one, a small imaginary part was also added. This imaginary part was chosen to be two orders of magnitude smaller than the deviation of the real part. This small imaginary part is always smaller than what is realizable in a real material.

Another consideration for the k-space mesh is the periodicity. The wave vector component spacing, \( dk_x \), determines the periodicity of the field patterns in the x direction. The spatial period, \( a \), is given by

\[
a = \frac{2\pi}{dk_x}
\]

One may wish to require the local fields in the region of interest to be isolated. For example, a slit source decomposed on a discrete mesh is actually an array of
slits separated by distance a. If we want to simulate the results for a single slit, we want the fields from neighboring slits to be negligible in the region of interest. To model the fields in the region within $\lambda$ of a slit narrow slit, it is more than sufficient if the neighboring slits are ten $\lambda$ away. Then $dk_x = \frac{2\pi}{10\lambda}$. Typically this is not as restrictive a requirement as representing resonance peaks in the transfer function.

The approach used for determining the k-space mesh in this paper was to check for convergence of the spatial fields. The value of $k_{x \text{max}} (dk_x)$ was sequentially doubled (halved) until the change in the spatial field pattern fell below some small threshold. In some cases this led to meshes in excess of 100K components.

II.H Image Resolution: Numerical Results

In this paper we explore eight different deviations from the ideal, $\mu = \epsilon = -1$, material properties. Each of the two material properties may deviate from minus one in four directions in the complex plain, figure II.4. Each of these deviations gives somewhat different behavior. The behavior due to each of these deviations was modeled using a thin slab of thickness $d = \lambda/10$, and an image to object plane distance of $2d$. The deviations modeled were logarithmically spaced since the focusing properties scale as the log of the deviation (Eq. II.11.) Also, the deviations modeled for $\epsilon$ were much larger than those for $\mu$. This was done to obtain a similar range of image resolutions for both parameters. As seen from Eq. II.10, resolution is much more sensitive to $\mu$ than $\epsilon$ for s-polarization.

II.H.1 transfer function

Real Deviations

For non-matched material properties the transfer function displays sharp peaks corresponding to resonances of the slab. This is seen in the first and third panes of figures II.5 and II.6. These resonances, whose positions are given by
Figure II.4: Values of $\mu$ and $\epsilon$ simulated are shown in the complex plane. For each value of $\mu$ and $\epsilon$, the transfer function and a real space image have been calculated. Positive real deviations lead to symmetric modes and negative real deviation lead to antisymmetric modes. $\mu$ deviations are shown in figure II.5 and II.7 $\epsilon$ deviations in figures II.6 and II.8.
Figure II.5: The transfer function of the slab. In this and subsequent figures the parameter that deviates from -1 is indicated in the upper right corner. The direction of the deviation in the complex plane is indicated by an arrow. Lighter shading of the line indicates larger deviation. The dashed line is the largest deviation that can still resolve $\lambda/10$ features.
Figure II.6: The solid circles mark the points for which z field dependence is shown. Circles in the top pane correspond to figure II.11 and circles in third pane correspond to figure II.12.
Figure II.7: Intensity of object and image focused by slab. The object is two square peaks of unit intensity, width $\lambda/100$ and separation $\lambda/10$. 
Figure II.8: Intensity of object and image focused by slab. The object is two square peaks of unit intensity, width $\lambda/100$ and separation $\lambda/10$. 
Figure II.9: Resonance merge. The dashed line in the upper two panes is for the value of $\mu$ that merges the two resonances. The bottom pane shows the center, $(x = 0)$, intensity versus $\mu$. This intensity peaks at the merge, $\mu = -0.45$. 
Figure II.10: Resonance merge. The dashed line in the upper two panes is for the value of \( \varepsilon \) that merges the two resonances. The bottom pane shows the center, \((x = 0)\), intensity versus \( \varepsilon \). This intensity peaks at the merge, \( \varepsilon = 3.45 \).
Figure II.11: Z dependence of the electric field excited by a unit plane wave of the specified transverse wave vector, $k_x$. The slab is delineated by the vertical grey lines. The solid line is for $\mu = -1$, $\epsilon = 1$, and the dashed line is for $\mu = \epsilon = -1$. The phase is indicated by shading as shown in the legend.
Figure II.12: Z dependence of the electric field excited by a unit plane wave of the specified transverse wave vector, $k_x$. The slab is delineated by the vertical grey lines. The solid line is for $\mu = -1$, $\epsilon = -3$, and the dashed line is for $\mu = \epsilon = -1$. The phase is indicated by shading as shown in the legend.
equation II.5, are modes of the system that can exist in the absence of excitation. As the deviations are made smaller the resonance shifts to larger values of $k_x$ until, at $\mu = \epsilon = -1$, the resonance has effectively shifted to infinity. In this case, the tail of the resonance gives the required gain within the slab to exactly compensate for the decay of the evanescent waves outside the slab yielding a transfer function of unity. For finite deviations from $\mu = \epsilon = -1$, the position of the resonance determines the maximum resolution of the slab. Beyond the resonance, the transfer function decays exponentially

$$|T_{k_x}| \sim e^{-2k_xd}$$

The resonance condition, equation II.5, derived by setting the denominator of the transfer function to zero, predicts two resonances for real deviations of the material parameters. One is near $k_x = k_0$, which is the boundary between propagating and evanescent waves. The other, at a larger $k_x$ value, marks the upper end of the pass band of the transfer function. For negative material property deviations (third pane of the figures), the lower resonance is symmetric and the upper resonance is antisymmetric. However, for the lower, symmetric mode, the zero in the denominator is cancelled by a zero in the numerator and no resonance is observed in the transfer function.

For positive deviations of the material properties this does not occur and both resonance peaks are evident in the transfer function (top pane of the figures). Both of these modes are symmetric. Increasing the deviation causes the location of the upper resonance to shift down and the lower resonance to shift up. There is a special value of the material properties for which the two resonances merge. For deviations of $\mu$ this value is $\mu = -0.45$ as seen in figure II.9. For deviations of $\epsilon$, the insensitive parameter, this special value occurs at a much larger deviation, $\epsilon = 3.45$, figure II.10. When the resonances are coincident there exists a doubly degenerate mode with strong gain over a wide band.
Imaginary Deviations

The deviations in the plus and minus imaginary direction yield materials with loss or gain respectively. For positive deviations, no resonances are observed (second pane of figures II.5 and II.6.) For negative deviations there is a bipolar peak near $k_0$, and the transfer function has values greater than one in the propagating range, $k_x/k_0 < 1$, (fourth pane in the figures.) This requires an active material with gain since propagating waves carry energy.

II.H.2 Spatial Images

The results of imaging a two “slit” source object are shown in figures II.7 and II.8. The slit spacing is $\lambda/10$. The individual slits have a width of $\lambda/100$, and unit intensity. The image intensity curve was generated using the transfer functions. In each graph the worst, (most deviant), transfer function that still resolved the object is shown dashed. This particular configuration has a focusing resolution of, $\sim \lambda/10$, the slit spacing. The corresponding curve in the transfer function plots is also dashed. The dashed curve in the transfer function has a band limiting resonant peak at $k_x/k_0 \sim 8$. This resolution is well beyond diffraction limited optics, which can at best resolve images of $\lambda/4$. A desired resolution of $\lambda/10$ permits only very small deviations of the sensitive parameter, $\mu$, on the order of one percent. The insensitive parameter may have rather large deviations extending even to positive real values. The criteria used for image resolution is as follows. The image must possess two peaks at locations consistent with the object, and the intensity of these peaks must be greater than any other peaks in the image, in particular they must be greater than the center peak. This criteria gives reasonable agreement with equation II.11.

The images formed by a slab, with properties near the resonance peak merge values, possess central peaks with large intensities. In fact the intensity of the peak increases with increasing property deviation up to the merge point. The third pane of figures II.9 and II.10 show the intensity of this peak as a function
of the material parameter. The case of epsilon deviation has a particularly large image intensity. At the peak the intensity is more than an order of magnitude larger than at any other value of the parameters.

II.H.3  Z Field Dependence

In figures II.11 and II.12 the z dependence of the electric field is shown with an excitation of a unit plane wave. For plane wave excitation, the magnitude of the electric field has no dependence on x as is clearly shown in equation II.1 and the y axis was chosen to be homogenous, therefore the z dependence is a complete description of the field. The material parameters chosen here are the largest $\epsilon$ deviations that still permitted $\lambda/10$ image resolution, $\mu = -1$ and $\epsilon = 1, -3$. For reference a perfect lens, $\mu = \epsilon = -1$, is shown also shown, (dashed). $k_x$ values were chosen to demonstrate the field behavior in the propagation range, evanescent pass band, resonant peaks and beyond the pass band. In figure II.11, there are two values of $k_x$ that are resonant shown in panes 2 and 4. The resonant fields are large, symmetric in $z$, and $\pi/2$ out of phase with the desired response. These field profiles are composed of two symmetrically coupled surface plasmon modes. The surface plasmons in pane 4 are more strongly coupled than in pane 2. In the pass band, pane 3, and the propagation range, pane 5, there is fair agreement with the perfect lens, both in magnitude and phase. Beyond the pass band, pane 1, the field is greater on the front surface of the slab than on the far surface, opposite to what is required for super lensing. One can see the characteristic $\pi$ phase shift of the response when going through a resonance, panes 1-3.

Similar results are seen in figure II.12 except that only one resonance is observed, and the mode is antisymmetric in $z$, composed of antisymmetrically coupled surface plasmons.
II.1 Aperture

\[
E_y(x, z_1) = A(x) \mathcal{F}^{-1} \{ e^{ik_2(z_1-z_0)} \mathcal{F} \{ E_y(x, z_0) \} \}
\]

\[
E_y(x, z_2) = \mathcal{F}^{-1} \{ T(z_1, z_2) \mathcal{F} \{ E_y(x, z_1) \} \}
\]

\[
E_y(x, z_3) = \mathcal{F}^{-1} \{ e^{ik_2(z_3-z_2)} \mathcal{F} \{ A(x) E_y(x, z_2) \} \}
\]

\[
A(x) = \begin{cases} 
1 & |x| < A_0/2 \\
0 & |x| \geq A_0/2 
\end{cases}
\]

II.1 Conclusion

The result of sub-wavelength focusing can be obtained using nothing but Maxwell’s equations in a slab of NIM with standard boundary conditions enforced.
outside the slab. Materials with negative dielectric constants, (metals and plasmas), are familiar, and materials with negative permeability have been recently demonstrated. Since Maxwell’s equations are not being called into question, these results are on firm ground. Sub wavelength focusing is possible. It is however, difficult. Even for the case of a thin slab with minimal working distance, as explored in this article, focusing resolution is very sensitive to at least one of the electromagnetic properties. In this case there is significant freedom in the other property including its loss characteristics and the choice of polarization gives control over which property is sensitive. The physical mechanism that limits the resolution for real deviations of the material properties, is the excitation of slab resonance modes. These modes mark the resolution cutoff of the transfer function.
II.K  Acknowledgement

This chapter, in full, will be submitted for publication, with authors D. Schurig, D. R. Smith, S. Schultz, S. A. Ramakrishna, J. B. Pendry.

II.L  Material Parameters Simulated

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Chapter III

Electromagnetic Wave Propagation in Media with Indefinite Permittivity and Permeability Tensors

III.A Abstract

We study the behavior of wave propagation in materials for which not all of the principle elements of the permeability and permittivity tensors have the same sign. These tensors are neither positive nor negative definite. We find that a wide variety of effects can be realized in such media, including negative refraction, near-field focusing and high impedance surface reflection. In particular a bilayer of these materials can transfer a field distribution from one side to the other, including near-fields, without requiring internal exponentially growing waves.

III.B Introduction

The range of available electromagnetic material properties has been broadened by recent developments in structured media, notably Photonic Band Gap ma-
Materials and metamaterials. These media have allowed the realization of solutions to Maxwell’s equations not available in naturally occurring materials, fueling the discovery of new physical phenomena and the development of devices. Photonic crystals, for example, have been utilized to modify the radiative density of states associated with nearby electromagnetic sources. Optical effects such as superradiance [23], enhanced or inhibited spontaneous emission [24], ultrarefraction [25] and even negative refraction [10, 11] have been predicted for various photonic lattice configurations.

The propagation characteristics of photonic lattices, while unique, are not conveniently expressed in a compact manner, and it is usually necessary to compute photonic band structure diagrams to deduce wave propagation behavior. By contrast, when the size and spacing of the constituent elements composing a medium are much smaller than the wavelength of interest, the composite can be treated as a continuous material—or metamaterial, with a permittivity tensor ($\varepsilon$) and a permeability tensor ($\mu$) [26]. This reduction of complexity simplifies the discussion of wave phenomena, without requiring undue attention to the detailed responding fields and currents characterizing the scattering elements. Typically, $\varepsilon$ and $\mu$ are positive definite, and the associated propagation behavior has been well-studied, including, more recently, chirality and bianisotropy [27, 28].

In 2000, it was shown experimentally that a metamaterial composed of periodically positioned scattering elements, all conductors, could be interpreted as having simultaneously a negative effective permittivity and a negative effective permeability [1]. Such a medium had been previously shown by Veselago to be consistent with Maxwell’s equations [3], but had never been demonstrated in a naturally occurring material or compound. A medium with simultaneously isotropic and negative $\varepsilon$ and $\mu$ supports propagating solutions whose phase and group velocities are antiparallel. Equivalently, such a material can be rigorously described as having a negative index of refraction[4]. An experimental observation of negative refraction was achieved using a metamaterial composed of wires and
split ring resonators deposited lithographically on circuit board material [2].

The prospect of negative refractive materials has generated considerable interest, as this simply stated material condition suggests the possibility of extraordinary wave propagation phenomena, including near-field focusing. Yet, negative refraction is not confined to materials with negative definite $\varepsilon$ and $\mu$, but can also be observed in materials with indefinite $\varepsilon$ and $\mu$ tensors. Lindell et al. [29] have shown that negative refraction can occur in uniaxially anisotropic media, and that such media are characterized by hyperbolic dispersion curves. We refer here to such materials as indefinite media for brevity. After a general discussion of the possible classes of indefinite media, we analyze the reflection and refraction properties for several configurations that may be relevant for unique applications. The components used to demonstrate the previous negative index metamaterials are appropriate building blocks for the classes of materials to be discussed here.

III.C Media

To simplify the following analysis, we assume a material whose anisotropic permittivity and permeability tensors are simultaneously diagonalizable, having the form

$$
\varepsilon = \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}
$$

$$
\mu = \begin{pmatrix}
\mu_x & 0 & 0 \\
0 & \mu_y & 0 \\
0 & 0 & \mu_z
\end{pmatrix}.
$$

(III.1)

We are interested in the manner in which a plane wave interacts with an anisotropic medium, in which not all of the diagonal components of the $\varepsilon$ and $\mu$ tensors have the same sign. To facilitate our analysis we consider an electromagnetic wave with the polarization directed along the $y$-axis, or

$$
E = \hat{y} e^{i (k_x x + k_z z - \omega t)}.
$$

(III.2)

From the electromagnetic wave equation, we find the following dispersion relation:

$$
k_z^2 = \varepsilon_y \mu_z \frac{\omega^2}{c^2} - \frac{\mu_x}{\mu_z} k_x^2,
$$

(III.3)
where we consider the +z-axis to be the forward reference direction. The wave vector has just one transverse component, $k_x$. Using only real values of $k_x$ we can expand any field distribution along the x direction using a Fourier series or transform, so we will restrict our discussion to real valued $k_x$, as is appropriate for the consideration of reflection/refraction phenomena.

The sign of $k_z^2$ determines the nature of the plane wave solutions. $k_z^2 > 0$ corresponds to real valued $k_z$ and propagating solutions. $k_z^2 < 0$ corresponds to imaginary $k_z$ and exponentially growing or decaying (evanescent) solutions. The value of $k_x$ for which $k_z^2 = 0$ is referred to as the cutoff, $k_c$. $k_c$ separates propagating from evanescent solutions. Based on the type of solutions of Eq. III.3, we identify four classes of media:

- **Always Evanescent**
  - $\varepsilon \mu_x < 0$
  - $\mu_x/\mu_z < 0$

- **Cutoff**
  - $\varepsilon \mu_x > 0$
  - $\mu_x/\mu_z > 0$

- **Anti-Cutoff**
  - $\varepsilon \mu_x < 0$
  - $\mu_x/\mu_z < 0$

- **Always Propagating**
  - $\varepsilon \mu_x > 0$
  - $\mu_x/\mu_z < 0$

At a given frequency, we can visualize the dispersion by plotting values of $k_z$ versus $k_x$ as shown in Fig. III.1. Waves in an *always evanescent* medium are evanescent for any direction of propagation. Waves in a *cutoff* medium are propagating for $k_x < k_c$, and evanescent for $k_x > k_c$. Examples of this medium class include free space, and any medium with $\varepsilon$ and $\mu$ tensors both positive or both negative definite. diagonal permittivity and permeability elements. The other two classes of media, *anti-cutoff* and *always propagating*, require indefinite material property tensors and have hyperbolic dispersion. Waves in *anti-cutoff* media are evanescent for $k_x < k_c$, and propagating for $k_x > k_c$. Waves in *always propagating* media propagate for all $k_x$[30].

We can examine the direction of energy flow for waves in *indefinite media* by calculating the group velocity, $v_g \equiv \nabla_k \omega(k)$. Since negative material property
Figure III.1: Iso-frequency curves for different classes of media. The cutoff transverse wave vector, $k_c$, is indicated for the cutoff and anti-cutoff media. Solid lines indicate real valued $k_z$. Dashed lines indicate imaginary values.
components are necessarily dispersive, we must use a causal, dispersive response function to represent them. For simplicity, we use the same, real-valued (lossless), response function, \(\xi(\omega)\), for all negative components of \(\mu\) or \(\varepsilon\). Positive components are assumed non-dispersive. Also for simplicity, we choose unit magnitude for all components. The slope of the response function is constrained by causality

\[
\xi(\omega_0) = -1 \quad \xi'(\omega_0) \omega_0 \geq 2 \tag{III.4}
\]

For the always propagating medium, then,

\[
\varepsilon = \mu = \begin{pmatrix}
\xi(\omega) & 0 & 0 \\
0 & \xi(\omega) & 0 \\
0 & 0 & 1
\end{pmatrix} \tag{III.5}
\]

The group velocity of a wave packet centered on a wave vector \(\mathbf{k} = k_x\mathbf{\hat{x}} + k_z\mathbf{\hat{z}}\) is then

\[
v_g = cf(k_x) \frac{k_x\mathbf{\hat{x}} - k_z\mathbf{\hat{z}}}{\omega_0/c} \tag{III.6}
\]

where \(f(k_x)\) is a dimensionless, positive, real scalar. Note that the group and phase velocity form an obtuse angle of nearly \(180^\circ\) for small \(k_x\), and nearly \(90^\circ\) for large \(k_x\). The direction of the group velocity is given for several other materials in Fig. III.2.

### III.D Refraction and Reflection

The unusual dispersive properties of indefinite media are manifest in reflection and refraction phenomena. We use \(\mathbf{k}\) to indicate the plane wave vector incident from the free space side of the interface, and \(\mathbf{q}\) to indicate the transmitted wave vector, inside the indefinite medium. The surface is assumed normal to the \(z\)-axis resulting in translational symmetry in the \(x\) and \(y\) directions and thus \(k_x = q_x\).

The nature of refraction into an indefinite medium is illustrated in Fig. III.3 for an always propagating material. The wave vector and group velocity of
Figure III.2: Each medium class has two sub-types, positive and negative. The sub-type indicates the direction of refraction, or equivalently, the sign of the $z$-component of the group velocity relative to the wave vector. Negation of both $\mu$ and $\varepsilon$ switches the sub-type. (For ease of illustration, the tensors in this table and those used for all specific calculations and plots in this article have elements of unit magnitude.)

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<td>$+k_x\hat{x} - k_z\hat{z}$</td>
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<td>-</td>
</tr>
<tr>
<td>positive always propagating</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>$-k_x\hat{x} + k_z\hat{z}$</td>
<td>-</td>
<td>+</td>
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<tr>
<td>negative always propagating</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>$+k_x\hat{x} - k_z\hat{z}$</td>
<td>+</td>
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</tr>
</tbody>
</table>
Figure III.3: Refraction from free space (blue) into negative, (green) and positive, (red) always propagating media. Only the real parts of the dispersion curves are shown. The free space curve is black and the always propagating medium is gray. The white, construction lines indicate the conservation of transverse wave vector, etc. The lengths of the group velocity vectors are not shown to scale.
the incident wave are parallel and the wave vector lies on the free space dispersion curve. The direction for the refracted wave is uniquely determined by three constraints: 1. The transverse component of the wave vector is conserved, \( k_x = q_x \). 2. The \( z \)-component of the group velocity must maintain its sign (for energy conservation.) 3. The group and phase velocity form an angle given by the group velocity expression, Eq. (III.6). We see that the \textit{negative always propagating} material exhibits negative refraction of the wave vector and positive refraction of the group velocity. The \textit{positive always propagating} material exhibits just the opposite behavior. Refraction into other classes of materials can be similarly characterized, and the refraction directions are shown in Fig. III.2.

The reflection properties of \textit{indefinite media} display interesting behavior, illustrated here for \textit{positive anti-cutoff} material. The reflection coefficient is given by

\[ \rho = \frac{\mu_x k_z - q_z}{\mu_x k_z + q_z} \]  

For unit magnitude \textit{anti-cutoff} material we have, from Eq. (III.3),

\[ q_z^2 = -\frac{\omega^2}{c^2} + k_x^2 = -k_z^2 \]  

Thus \( q_z = \pm ik_z \). The correct sign, +, is determined from a construct similar to Fig. III.3. We then have

\[ \rho = \frac{1 - i}{1 + i} = -i \]  

The magnitude of the reflection coefficient is unity, with a phase of -90° for propagating modes of all incident angles. An electric dipole antenna placed \( \lambda/8 \) away from the surface would thus be enhanced by interaction with the mirror surface. Customized reflecting surfaces are of practical interest, as they enhance the efficiency of nearby antennas, while at the same time providing shielding [31, 32]. In this case, the uniformity of the reflection coefficient over all angles of incidence may be advantageous. Furthermore, surface modes, which require evanescent solutions on both sides of the interface, are not supported on this interface. Fig. III.1 shows
no overlap of evanescent solutions (dashed line) between cutoff and anti-cutoff media (with the same cutoff wave vector.)

III.E Layered Media: Focusing

A motivating factor in the recent metamaterials effort has been the prospect of near-field focusing, as introduced by Pendry, who showed theoretically that a planar slab with isotropic $\varepsilon = \mu = -1$, and thickness, $2d$, could act as a lens with resolution well beyond the diffraction limit. A field source positioned a distance $d$ from the first interface will form an image a distance $d$ away from the other interface. Not only are the propagating components of the source refocused, but the decay of the evanescent components is reversed through the material, the net result being that the image exactly reproduces the source fields, even if the source field spatial distribution is much less than the wavelength. It is difficult, however, to realize significant sub-wavelength resolution in practice, as the required exponential growth of the large $kx$ field components across the negative index lens leads to extremely large field ratios[7, 33]. Sensitivity to material loss and other factors can significantly limit the sub-wavelength resolution. A proposed scheme to reduce this sensitivity is to section the planar lens into numerous separated subsections[34]. In the limit of a large number of very thin layers, the transverse and longitudinal effective material parameters exhibit extreme anisotropy, suggesting that anisotropic materials may be inherently advantageous for near-field focusing.

While it is not possible to focus near-fields with a single layer of an indefinite medium, it is possible to compensate for the dispersive effects of a negative always propagating layer with a positive always propagating layer. Fig. III.3 indicates that for the same incident plane wave, the $z$-component of the transmitted wave vector, $q_z$, is equal and opposite for these two materials. Combining equal lengths of these materials results in a composite indefinite medium with unit trans-
Figure III.4: Magnitude of the transfer function vs. transverse wave vector. The device is a bi-layer of positive and negative *always propagating* material with layers of equal thickness, $L_1/\lambda = 0.1, 0.2, 0.5, 1, 2$. The thinnest layer is the darkest curve. A realistic loss of $0.01i$ has been added to each diagonal component of $\varepsilon$ and $\mu$.

This bilayer accomplishes near-field focusing by converting incident evanescent waves to propagating waves within the bilayer, and back to evanescent waves on the opposite side. Thus, this bilayer can reproduce sub-wavelength features without the need for internal exponentially growing fields.

We can quantify the effects of non-ideal material properties, such as loss, by computing the transfer function, $T$, of the general, two-layer problem.

$$T = 8 \begin{bmatrix}
e^{i(\phi+\psi)} (1 - Z_0) (1 + Z_1) (1 - Z_2) + \\
e^{i(\phi-\psi)} (1 - Z_0) (1 - Z_1) (1 + Z_2) + \\
e^{i(-\phi+\psi)} (1 + Z_0) (1 - Z_1) (1 - Z_2) + \\
e^{i(-\phi-\psi)} (1 + Z_0) (1 + Z_1) (1 + Z_2)
\end{bmatrix}^{-1}$$

(III.10)
where the impedances are defined as

\[ Z_0 = \frac{q_{z1}}{\mu_{x1} k_z}, \quad Z_1 = \frac{\mu_{x1} q_{z2}}{\mu_{x2} q_{z1}}, \quad Z_2 = \frac{k_z}{\mu_{x2} q_{z2}}. \] (III.11)

and the individual layer phase advance angles are defined as

\[ \phi \equiv q_{z1} L_1 \quad \psi \equiv q_{z2} L_2. \]

\( L_1 \) is the thickness of the first layer and \( L_2 \) is the thickness of the second layer. For ideal \emph{positive always propagating} and \emph{negative always propagating} layers of equal thickness, the transfer function is unity for all incident plane waves. The transfer function with some realistic loss added is shown in Fig III.4.

A proposed implementation is shown in Fig. III.5. The top and bottom structures will focus S-polarized and P-polarized waves respectively. Combining the two structures results in a bilayer that focuses both polarizations and is \( x-y \) isotropic. The materials are formed from split ring resonators and wires with numerically and experimentally confirmed effective material properties [2]. Each split ring resonator orientation implements negative permeability along a single axis, as does each wire orientation for negative permittivity.

\section*{III.F Conclusion}

We have begun to explore the properties of media with indefinite \( \varepsilon \) and \( \mu \) tensors. \textit{Indefinite media} are governed by hyperbolic dispersion relations, previously found only in much more exotic situations, such as relativistic moving media [28]. Consideration of layered structures has led to useful and interesting reflection and refraction behavior, including a new mechanism for sub-diffraction focusing. We note that neither the analysis nor the fabrication of these media is complicated, and thus anticipate other researchers will quickly assimilate the principles and design structures with unique and technologically relevant properties.
Figure III.5: Split ring resonators are shown in black. Wires are shown in gray. Upper device: layer one has split ring resonators oriented in the $x$ direction and wires in the $y$ direction, and layer two has split ring resonators in the $z$ direction. Lower device: layer one has split ring resonators oriented in the $y$ direction and wires in the $x$ direction, and layer two has wires in the $z$ direction.
III.G  Acknowledgement

We thank Claudio Parazzoli (Phantom Works, Boeing) for motivating this work. This work was supported by DARPA through grants from ONR (Contract No. N00014-00-1-0632) and AFOSR (Contract No. 78535A/416250/440000) and a grant from AFOSR (Contract Number F49620-01-1-0440).

This chapter, in full, has been submitted for publication to Physical Review Letters, with authors D. R. Smith and D. Schurig.
Chapter IV

Sub-Diffraction Focusing Using Bilayers of Indefinite Media

IV.A Abstract

We analyze a new type of near field focusing device that consists of a bilayer of media with indefinite electromagnetic material property tensors. The dispersion of the media supports propagating waves for any transverse wave vector which results in a weaker dependence of spatial bandwidth on media lossiness.

IV.B Introduction

Pendry’s theoretical demonstration that a layer of negative index media could focus near as well as far fields and operate as a “perfect” lens has generated a lot of interest [6, 35, 36, 37, 38, 33, 7, 5, 39]. Here we discuss near field focusing in a broader context, and employing bilayers of a new class of media that are anisotropic [40]. The devices proposed here belong to a general class of bilayers that have compensating layers; the phase advance (or decay) across one layer is equal and opposite to the phase advance (or decay) across the other layer. Pendry’s “perfect” lens is a special case of a compensating bilayer. In this case the negative index media compensates for free space, and thus the device has the nice feature
that it possesses free space working distance. To be compensating, one layer must have normal components of the wave vector and group velocity of the same sign and the other layer must have normal components of opposite sign. This ensures that energy moving across the bilayer has opposite evolution in one layer relative to the other.

The device to be discussed in this letter is a compensating bilayer that employs a particular class of media we refer to as always propagating [40]. The most unusual property of this medium is that it supports propagating waves for all transverse wave vectors. Even waves that are evanescent in free space, when incident on this medium, are converted to propagating waves. There are two sub-classes of always propagating media that together can satisfy the requirements for a compensating bilayer. The main advantage of using this medium is less sensitivity to material lossiness. Bilayers that do not use an always propagating medium, such as the Pendry lens, must support large growing field solutions that are very sensitive to material loss [33, 37, 7].

Always propagating media are anisotropic, and in fact have both positive and negative principle components in the electromagnetic property tensors, $\varepsilon$ and $\mu$. Because these tensors are neither positive nor negative definite, we refer to this medium as an indefinite medium [40].

IV.C Dispersion

To simplify the following analysis, we assume a material whose anisotropic permittivity and permeability tensors are simultaneously diagonalizable, having the form

$$
\varepsilon = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \quad 
\mu = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix}.
$$
Consider an electromagnetic wave with the polarization directed along the $y$-axis, 

$$E = \hat{y} e^{i(k_xx + k_z z - \omega t)}.$$ 

From the electromagnetic wave equation, we find the following dispersion relation:

$$k_z^2 = \varepsilon y \mu_x \frac{\omega^2}{c^2} - \frac{\mu_x k_x^2}{\mu_z},$$

where we consider the $+z$-axis to be the forward reference direction. The wave vector has just one transverse component, $k_x$. Using only real values of $k_x$ we can expand any field distribution along the $x$ direction using a Fourier series or transform, so we will restrict our discussion to real valued $k_x$.

The sign of $k_z^2$ determines the nature of the plane wave solutions. Positive $k_z^2$ corresponds to real valued $k_z$ and propagating solutions. Negative $k_z^2$ corresponds to imaginary $k_z$ and exponentially growing and decaying (evanescent) solutions. For the devices discussed in this paper we will consider materials that are *always propagating*, that is $k_z$ real for all values of $k_x$. These materials satisfy

$$\varepsilon y \mu_x > 0 \quad \text{and} \quad \frac{\mu_x}{\mu_z} < 0$$

This requires material property tensors that have principle values that are not all of the same sign. The dispersion is hyperbolic [29] with asymptotes of slope, $m$, defined at large $k_x$ by

$$m \equiv \left| \frac{\partial k_z}{\partial k_x} \right| = \left| \frac{k_z}{k_x} \right| = \left| \sqrt{\frac{\mu_x}{\mu_z}} \right| \quad (IV.1)$$

As will be seen below, this slope is directly observable in spatial electric field intensity patterns. The constant frequency dispersion is shown in Fig. IV.1. Note that for all values of $k_x$ there are real values of $k_z$. Since this curve is a constant frequency contour the group velocity, $v_g = \nabla k \omega (k)$, must be normal to the curve. From the figure one can see that at large $k_x$, $v_g$ will also be normal to $k$. There are two types of *always propagating* material that correspond to the two possible directions of the group velocity. The simplest tensor forms for these two types are given in Eq. (IV.6).
Figure IV.1: Hyperbolic dispersion found in *always propagating* media (black). The asymptotic behavior is also shown (white.) Two wave vectors are shown that have the same transverse component, $k_x$. Both possible group velocity directions are shown for each wave vector. The normal ($z$) component of $k$ and $v_g$ can be either of the same sign or opposite sign.
Figure IV.2: From top to bottom: 1. the indices used to refer to material properties, 2. the conventions for the coefficients of each component of the general solution, 3. the sign structure of the material property tensors, 4. typical $z$-dependence of the electric field for an evanescent incident plane wave, 5. $z$-coordinate of the interfaces.

**IV.D General Bilayer Solution**

First we solve the general problem of an S-polarized wave incident on an anisotropic bilayer with unequal layer thickness. The property tensors are completely general except they are assumed simultaneously diagonalizable. The
The general solution for the electric field with unknown coefficients is given by

\[ E_y = e^{i(k_x x - \omega t)} \begin{cases} 
  e^{ik_z z} + \rho e^{-ik_z z} & z < 0 \\
  Ae^{ip_z z} + Be^{-ip_z z} & 0 < z < z_1 \\
  Ce^{iq_z z} + De^{-iq_z z} & z_1 < z < z_2 \\
  \tau e^{ik_z z} & z_2 < z
\end{cases} \]

The \( z \)-components of the wave vectors are

\[ k_z = \pm \sqrt{k_0^2 - k_x^2} \]  
\[ p_z = \pm \sqrt{\varepsilon_1 \mu_1 k_0^2 - \frac{\mu_1^2 k_x^2}{\mu_1}} \]  
\[ q_z = \pm \sqrt{\varepsilon_2 \mu_2 k_0^2 - \frac{\mu_2^2 k_x^2}{\mu_2}} \]

where \( k_0 = \omega^2/c^2 \) and the material parameters are relative to free space. The conventions for the material property subscripts and solution coefficients are shown in Fig. IV.2. The general solution includes exponentials with both signs of \( p_z \) and \( q_z \) so a choice of sign for these square roots is unnecessary. We choose the branch of the square root for \( k_z \) such that \( k_z \) is either real and positive or imaginary and positive. This ensures that the incident wave propagates toward the bilayer or decays in the direction toward the bilayer (away from the source). The transverse component (\( y \)) of the electric field is conserved across each of the three interfaces, at \( z = 0, z = z_1 \) and \( z = z_2 \), yielding three equations. Similarly the transverse component (\( x \)) of the magnetic field yields three more equations. The transverse magnetic field component is given by

\[ H_x = -\frac{1}{\omega} e^{i(k_x x - \omega t)} \begin{cases} 
  k_z (e^{ik_z z} - \rho e^{-ik_z z}) & z < 0 \\
  \frac{p_z}{\mu_1} (Ae^{ip_z z} - Be^{-ip_z z}) & 0 < z < z_1 \\
  \frac{q_z}{\mu_2} (Ce^{iq_z z} - De^{-iq_z z}) & z_1 < z < z_2 \\
  k_z \tau e^{ik_z z} & z_2 < z
\end{cases} \]

The six boundary matching equations are solved for the six unknown coefficients: \( \rho, A, B, C, D, \tau \). The transfer function from the front surface to the back surface
of the bilayer is given by the ratio of the transmitted electric field to the incident electric field

$$T = \frac{E_y(z_2)}{E_{yi}(0)} = \tau e^{ik_2z_2}$$

Inserting the value of $\tau$ from the solution of the system of six equations yields

$$T = 8 \begin{bmatrix} e^{i(\phi + \psi)}(1 - Z_0)(1 + Z_1)(1 - Z_2) + & e^{-i(\phi - \psi)}(1 + Z_0)(1 - Z_1)(1 - Z_2) + \\ e^{i(\phi - \psi)}(1 - Z_0)(1 - Z_1)(1 + Z_2) + & e^{-i(\phi + \psi)}(1 + Z_0)(1 + Z_1)(1 + Z_2) \end{bmatrix}^{-1}$$

(IV.3)

where the impedances are defined as

$$Z_0 \equiv \frac{p_z}{\mu_{1z}k_z} \quad Z_1 \equiv \frac{\mu_{1z}q_z}{\mu_{2z}p_z} \quad Z_2 = \frac{\mu_{2z}k_z}{q_z}.$$  (IV.4)

and the individual layer phase advance angles are defined as

$$\phi \equiv p_z z_1 \quad \psi \equiv q_z (z_2 - z_1).$$  (IV.5)

To obtain a unit transfer function, $T = 1$, it is sufficient that

$$\phi \pm \psi = 0 \quad \text{and} \quad 1 \mp Z_1 = 0.$$  

These two conditions are equivalent to layer compensation as discussed above, and the additional requirement of an impedance match between the layers.

### IV.E Simplest Case

We examine the archtypical focusing bilayer. In this case (and all others discussed in this paper), we will choose the $\varepsilon$ and $\mu$ tensors equal to each other and thus ensure that the focusing properties are independent of polarization. We also choose these tensors to be $x$-$y$ isotropic so that the focusing properties are independent of the $x$-$y$ orientation of the layers. This is the highest degree of symmetry
allowed for *always propagating* media. If we further choose unit magnitude for all tensor components we have

\[
\begin{align*}
\varepsilon_1 = \mu_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\
\varepsilon_2 = \mu_2 &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{align*}
\] (IV.6)

In this case the layer thickness must be equal for focusing, \( z_1 = z_2 - z_1 = d \). Plugging these values into Eq. (IV.2), (IV.3), (IV.4) and (IV.5), we find the transfer function is unity for all incident plane waves, \( T = 1 \). The magnitude is preserved and the phase advance across the bilayer is zero.

The internal field coefficients \((A, B, C, D)\) are plotted in Fig. IV.3. Evanescent incident waves \((k_x/k_0 > 1)\) carry no energy, but on entering the bilayer are converted to propagating waves. Since propagating waves do carry energy the forward and backward coefficients must be equal; the standing wave ratio must be and is unity.

Propagating incident waves, however, do transfer energy across the bilayer. We see in the figure that for propagating incident waves, \((k_x/k_0 < 1)\), the first layer, forward coefficient, \(|A|\), is larger in magnitude than the backward coefficient, \(|B|\). These roles are reversed in the second layer, \(|D| > |C|\). What we have referred to as “forward” really means positive \(z\)-component of the wave vector. This does not indicate the direction of energy flow which is given by the group velocity. The \(z\)-component of the group velocity must be positive in both layers to conserve energy across the interfaces. We can display the electric field quite simply in the limit \( k_x \gg k_0 \).

\[
E_y = e^{i(k_x x - \omega t)} \begin{cases} 
 e^{-k_x z} & z < 0 \\
 \sqrt{2} \cos [k_x z + \pi/4] & 0 < z < d \\
 \sqrt{2} \cos [k_x (2d - z) + \pi/4] & d < z < 2d \\
 e^{-k_x (z-2d)} & 2d < z 
\end{cases}
\]

we see that the internal field is indeed a standing wave, and is symmetric about the center of the bilayer. This field pattern is shown in Fig. IV.2.
Figure IV.3: The magnitude of coefficients of the internal field components. For values of $k_x > k_0$ (incident plane wave is evanescent) the forward and backward components have equal magnitude. The magnitude approaches $1/\sqrt{2}$ when $k_x \gg k_0$. 
IV.F Asymmetric Bilayers and Field Plots

We can relax the above symmetry to obtain some different behavior. In the previous discussion the property tensor elements were all unit magnitude leading to dispersion slope of one, as seen from Eq. (IV.1). We can introduce a different slope, $m$, as follows

\[
\varepsilon_1 = \mu_1 = \begin{pmatrix}
m_1 & 0 & 0 \\
0 & m_1 & 0 \\
0 & 0 & -1/m_1
\end{pmatrix}
\]
\[
\varepsilon_2 = \mu_2 = \begin{pmatrix}
-m_2 & 0 & 0 \\
0 & -m_2 & 0 \\
0 & 0 & 1/m_2
\end{pmatrix}
\]

Allowing the slope to differ in each layer we can still maintain a unit transfer function, $T = 1$, if we adjust the thickness of the layers appropriately.

\[
\frac{z_2 - z_1}{z_1} = \frac{m_1}{m_2}
\]

We have still maintained polarization independence and $x$-$y$ isotropy. The internal field for a bilayer with different slopes in each layer is shown in Fig. IV.4. The incident field is a localized source composed of many $k_x$ components. This source is equivalent to two narrow slits back illuminated by a uniform propagating plane wave. The plane wave components interfere to form a field intensity pattern that is localized in four beams, two for each slit. The beams diverge in the first layer and converge in the second layer to reproduce the incident field pattern on the far side. The plane waves that constructively interfere to form each beam have phase fronts parallel to the beam, (i.e. the wave vector is perpendicular to the beam.) The narrow slits yield a source which is dominated by large $k_x$ components. These components lie well out on the asymptotes of the hyperbolic dispersion, so all of the wave vectors point in just four directions, the four indicated in the plot. These
correspond to the positive and negative $k_x$ components in the source expansion and the forward and backward components of the solution ($A, B$ or $C, D$).

**IV.G Lossy Media: Approximate Analytical Transfer Function**

Lossy media is characterized by an imaginary component of the property tensors. We will use a single parameter for the imaginary component of all the principle elements of both $\varepsilon$ and $\mu$.

$$
\varepsilon_1 = \mu_1 = \begin{pmatrix}
  m + i\gamma & 0 & 0 \\
  0 & m + i\gamma & 0 \\
  0 & 0 & -1/(m + i\gamma)
\end{pmatrix}
$$

$$
\varepsilon_2 = \mu_2 = \begin{pmatrix}
  -m + i\gamma & 0 & 0 \\
  0 & -m + i\gamma & 0 \\
  0 & 0 & 1/(m + i\gamma)
\end{pmatrix}
$$

For this analysis we will use layers of equal slope and thickness, $z_1 = z_2 - z_1 = d$. Fig. IV.5 shows the effect of loss on the transfer function. Increasing loss limits the $k_x$ bandwidth. We can obtain a simple analytical approximation for the transfer function to see directly the trade-offs between bandwidth, slope, thickness and loss. We will assume that the loss parameter is small compared to the real part of the principle components of the material property tensors, $\gamma \ll 1/m, m$. We will also assume that $k_x \gg k_0$, limiting the range of validity of our approximate transfer function to large $k_x$. However, it will turn out that the approximate transfer function will give a realistic value of one for small $k_x$. In any case, we usually wish to find the upper limit of the pass band which will occur at large $k_x$ in most useful circumstances. If we evaluate the transfer function, Eq. (IV.3), to zeroth order in $\gamma$ for the impedance factors we obtain

$$
T = \sec (\phi - \psi).
$$
Figure IV.4: Internal electric field intensity plot for a localized two slit source. The four vector pairs in each layer indicate the only four orientations of internal plane waves. Plane waves from positive $k_z$ components of the source are indicated in white and negative in gray. The wave vectors are labeled with the solution component to which they correspond. $k_A$ and $k_C$ have positive $z$-components and $k_B$ and $k_D$ have negative $z$-components. The unmarked vector in each pair indicates the direction of the group velocity for that plane wave direction. $m_1 = 1$ and $m_2 = 1/2.$
Figure IV.5: The transfer function from the front surface to the back surface of the bilayer. $z_1 = z_2 - z_1 = \lambda$, $m = 1/3$ and $\varepsilon'' = \mu'' = .001, .002, .005, .01, .02, .05, .1$, from light to dark. For comparison, a single layer, isotropic near field lens is shown dashed. The single layer has thickness, $\lambda$, and $\varepsilon = \mu = -1 + 0.001i$.
If we then expand $\phi$ and $\psi$ to first order, we obtain

$$T = \text{sech} \left[ k_x d (1 + m^2) \gamma \right]$$

This approximate transfer function has excellent agreement with the full expression, Eq. (IV.3), in the appropriate limits. In particular it accurately gives the behavior at high $k_x$ values where the transfer function rolls off. The dependence of bandwidth, $\Delta k_x$, on thickness, $d$, loss, $\gamma$, and slope, $m$, is given by

$$\Delta k_x d (1 + m^2) \gamma \sim 1$$  \hspace{1cm} (IV.7)

Not surprisingly, larger bandwidth is obtained with thinner layers and smaller loss. Less intuitive is the role of the slope, $m$; smaller slope gives larger bandwidth. In Fig. IV.4 we observe that a smaller slope corresponds to beams that lie on a more direct path through the layer. For fixed thickness, $d$, this means a shorter path. One might suggest that $m = 0$, with beams going straight through the layer, would be the most desirable. However, our analysis is only valid for $m \gg \gamma$. In addition, since $1/m$ also appears as a principle component in the property tensors, there is a practical limit to how small $m$ can be. Arbitrarily large $\varepsilon$ and $\mu$ components are not realizable.

We note that Ramakrishnan and Pendry have proposed another device that also has reduced sensitivity to loss compared to the single layer “perfect” lens [34]. This device uses an infinite number of infinitesimally thin layers of isotropic, negative index medium. We have found that this configuration gives the same inverse relation between loss and bandwidth as Eq. (IV.7). This dependence is markedly different from that of a single layer which has exponential dependence.

$$\gamma \sim e^{-\Delta k_x d}$$

**IV.H Conclusion**

We have analyzed a new type of near field focusing device that consists of a bilayer of *always propagating* media. This unusual media possesses indefinite
material property tensors. We have found the slope of the dispersion relation to be a useful adjustable parameter that is directly observable in field intensity plots. Internal standing waves are employed to transfer externally evanescent waves across the device, and the dependence of spatial transfer function bandwidth on material lossiness is much more favorable than in single layer, isotropic devices.

IV.I Acknowledgement

This chapter, in full, will be submitted for publication, with authors D. Schurig and D. R. Smith.
Chapter V

Electromagnetic Spatial Filtering Using Media with Negative Properties

V.A Abstract

We show how bilayers of media with negative electromagnetic property tensor elements can be used to construct low, high and band pass spatial filters. These filters possess sharp adjustable roll offs and can operate in the near and far field regimes to select specific spatial variation components or beam angles.

V.B Introduction

There has been a great deal of interest in the “perfect” lens[6, 35, 36, 37, 38, 33, 7, 5, 39]. Researchers have found that the parameter range for which the “perfect” lens can focus near fields is quite limited[33, 37, 7]. In particular, it has been found that the lens must be thin and the losses small to have a spatial transfer function that operates significantly into the near field (evanescent) range. Even with small losses a thick lens will have a transfer function that rolls off quite steeply at the near/far field boundary. However, in the far field (propagating)
range the “perfect” lens is fairly robust, and a wide range of lens thickness and material loss can be tolerated.

This article discusses the exploitation of this effect which can result in a flat response for wave vectors in the propagating range and up to the evanescent boundary followed by a strongly attenuated response. This is a desirable characteristic of a spatial low pass filter. The “perfect” lens by itself would be of limited usefulness because the filter cutoff is fixed at a specific wave vector, but it is possible to tune this filter cutoff by employing compensating bilayers.

Compensating bilayers have the property that the phase advance (or decay) across one layer is equal and opposite to the phase advance (or decay) across the other layer. The “perfect” lens is a special case of a compensating bilayer. In this case the negative index media compensates for free space, and thus the device has the nice feature that it allows for free space working distance. To be compensating, one layer must have normal components of the wave vector and group velocity of the same sign and the other layer must have normal components of opposite sign. This ensures that energy moving across the bilayer has opposite evolution in one layer relative to the other.

Compensating bilayers can be composed of media with cutoffs that are different from free space. The cutoff can be tuned to a value below the free space cutoff, so the filter edge is in the propagating range. Then a bilayer can operate as an angular selector. The cutoff can also be tuned to a value above the free space cutoff to make a near field spatial filter.

Bilayers composed of isotropic media like the “perfect” lens can only function as low pass filters, however another type of media, which we refer to as anti-cutoff media, can be used to construct compensating bilayers that function as high pass filters. Anti-cutoff media require anisotropic electromagnetic property tensors that do not have principle components all of the same sign. These tensors are neither positive nor negative definite, so we refer to these media as indefinite media [40].
Band pass filters can also be realized as a four layer device composed of two adjacent compensating bilayers. The high and low cutoffs can be adjusted independently, and the two bilayers do not interfere with each other in the pass band, where the reflection coefficient is small.

V.C Dispersion

To simplify the following analysis, we assume a material whose anisotropic permittivity and permeability tensors are simultaneously diagonalizable, having the form

\[
\varepsilon = \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}, \quad \mu = \begin{pmatrix}
\mu_x & 0 & 0 \\
0 & \mu_y & 0 \\
0 & 0 & \mu_z
\end{pmatrix}.
\]

Consider an electromagnetic wave with the polarization directed along the \(y\)-axis,

\[
\mathbf{E} = \hat{y}e^{i(k_xx + k_z z - \omega t)}.
\]

From the electromagnetic wave equation, we find the following dispersion relation:

\[
k_z^2 = \varepsilon_y \mu_x k_0^2 \frac{\mu_x}{\mu_z} k_x^2,
\]

where \(k_0 = \omega/c\) and the material parameters are relative to free space. We consider the \(+z\)-axis to be the forward reference direction. The wave vector has just one transverse component, \(k_x\). Using only real values of \(k_x\) we can expand any field distribution along the \(x\) direction using a Fourier series or transform, so we will restrict our discussion to real valued \(k_x\).

We will assume there is at most a small imaginary part to the material property tensor components. In classifying the possible media behaviors we will neglect this imaginary part and treat the components as real. Then \(k_z^2\) is real and the sign of \(k_z^2\) determines the nature of the plane wave solutions. Positive \(k_z^2\) corresponds to real valued \(k_z\) and propagating solutions. Negative \(k_z^2\) corresponds to imaginary \(k_z\) and exponentially growing and decaying (evanescent) solutions.
Figure V.1: Dispersion curves for *cutoff* and *anti-cutoff* media for several different values of the cutoff, $k_c$. Solid lines indicate real valued $k_z$, and dashed lines indicate imaginary values. A wave vector, $\mathbf{k}$, is shown, and both possible group velocity directions associated with that wave vector.
When $\varepsilon_y\mu_z > 0$ there is a value of $k_x$ for which $k_z^2 = 0$. This is referred to as the cutoff, $k_c$, and is given by

$$\frac{k_c}{k_0} = \sqrt{\varepsilon_y\mu_z}. $$

$k_c$ separates propagating from evanescent solutions. For the devices discussed in this paper we will consider the following types of media

- **cutoff** $\varepsilon_y\mu_x > 0$ and $\mu_x/\mu_z > 0$
- **anti-cutoff** $\varepsilon_y\mu_x < 0$ and $\mu_x/\mu_z < 0$

Waves in a *cutoff* medium are propagating for $k_x < k_c$, and evanescent for $k_x > k_c$. Examples of this medium class include free space, and any medium with $\varepsilon$ and $\mu$ tensors both positive or both negative definite. *Anti-cutoff* media require indefinite material property tensors and have hyperbolic dispersion [29]. Waves in *anti-cutoff* media are evanescent for $k_x < k_c$, and propagating for $k_x > k_c$. Fig. V.1 shows both classes of dispersion, for several different values of the cutoff, $k_c$.

There are two sub-types in each dispersion class. These two types are distinguished by the two possible group velocity directions that are normal to the dispersion curve. Since this curve is a constant frequency contour the group velocity, $v_g = \nabla k\omega(k)$, must be normal to the curve. One can switch types by negating both the $\mu$ and $\varepsilon$ tensors simultaneously.

**V.D Transfer Matrix**

First we solve the general problem of an $S$-polarized wave incident on an anisotropic multi-layer structure. We only require a bilayer solution for low and high pass filters, but band pass filters require four layers, so we will outline the general multi-layer procedure. In a single layer, the general solution for the
electric field with unknown coefficients is given by

\[ E_y = e^{i(k_xx - \omega t)} \begin{cases} 
    A e^{ik_z z} + B e^{-ik_z z} & z < 0 \\
    C e^{ip_z z} + D e^{-ip_z z} & 0 < z < d \\
    E e^{ik_z (z-d)} + F e^{-ik_z (z-d)} & d < z 
\end{cases} \]

The z-components of the wave vectors are

\[ k_z = \pm \sqrt{k_0^2 - k_x^2} \]
\[ p_z = \pm \sqrt{\varepsilon_y \mu_x k_0^2 - \frac{\mu_x \mu_z}{\varepsilon_z} k_x^2} \]

The general solution includes exponentials with both signs of \( p_z \) so a choice of sign for this square root is irrelevant. We choose the branch of the square root for \( k_z \) such that \( k_z \) is either positive real or positive imaginary. This ensures that the incident wave (the one with coefficient \( A \)) propagates toward the multi-layer or decays in the direction toward the multi-layer (away from the source). The transverse component (\( y \)) of the electric field is conserved across both interfaces, at \( z = 0 \), and \( z = d \), yielding two equations. Similarly the transverse component (\( x \)) of the magnetic field yields two more equations. The magnetic field is given by

\[ H_x = -\frac{e^{i(k_xx - \omega t)}}{\omega} \begin{cases} 
    k_z (A e^{ik_z z} - B e^{-ik_z z}) & z < 0 \\
    \frac{p_z}{\mu_z} (C e^{ip_z z} - D e^{-ip_z z}) & 0 < z < d \\
    k_z (E e^{ik_z (z-d)} - F e^{-ik_z (z-d)}) & d < z 
\end{cases} \]

These four boundary matching equations are solved for the first four of the unknown coefficients: \( A, B, C, D \). Then the two front face (\( z = 0 \)) coefficients, \( A \) and \( B \), can be expressed as a linear combination of the two rear face (\( z = d \)) coefficients, \( E \) and \( F \), in the form of a transfer matrix.

\[
\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \alpha - \beta \zeta_+ & -\beta \zeta_- \\ \beta \zeta_- & \alpha + \beta \zeta_+ \end{pmatrix} \begin{pmatrix} E \\ F \end{pmatrix}
\]
where

\[
\alpha \equiv \cos (p_z d) \\
\beta \equiv i \sin (p_z d) \\
\zeta_\pm \equiv \frac{1}{2} \left( \frac{p_z}{\mu_x k_z} \pm \frac{\mu_x k_z}{p_z} \right)
\]

We can then create a multi-layer structure by multiplying the transfer matrices of the individual layers to form a combined transfer matrix. Each matrix transforms the back surface coefficients to the front surface coefficients which, in turn, become the back surface coefficients of the next layer.

\[
M = M_0 M_1 \ldots M_i \ldots M_N
\]

We then impose our boundary condition and normalization. The incident wave has unit magnitude, and there is no wave returning from infinity to the back face.

\[
\begin{pmatrix}
1 \\
\rho
\end{pmatrix} = M
\begin{pmatrix}
\tau \\
0
\end{pmatrix}
\]

Solving, we find the reflection and transmission coefficient of the overall multi-layer structure.

\[
\tau = \frac{1}{M_{11}} \\
\rho = \frac{M_{21}}{M_{11}}
\]

Since the transmitted wave coefficients are referred to the back face, the transmission coefficient is identical to the transfer function.

V.E Spatial Filters

We will use the following two basis matrices to represent all media used for filter devices in this article.

\[
\sigma_0 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \\
\sigma_1 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}
\]
Figure V.2: Reflection and transmission coefficients versus incident angle for low, high and band pass filters. For the high and low pass filters, the values for $\theta_c \equiv \sin^{-1}(k_c/k_0)$ are $\pi/12$, $\pi/6$, $\pi/4$, $\pi/3$ and $5\pi/12$ going from dark to light lines. The band pass filter has cutoffs, $\pi/6$ and $\pi/3$, with transmission shown with a dark line and reflection shown with a light one. $\gamma = 0.01$ and $k_0d = 4\pi$ for all filters except $\theta_c \equiv \pi/12$ where $k_0d = 6\pi$. 
Figure V.3: Reflection and transmission coefficients versus transverse wave vector, $k_x/k_0$, for low, high and band pass filters. For the high and low pass filters, the values for $k_c/k_0$ are 1, 2, 5, 10 and 20 going from dark to light lines. The band pass filter has cutoffs, 5 and 10, with transmission shown with a dark line and reflection shown with a light one. $\gamma = 0.01$ and $k_c d = 4\pi$ for all filters; the filters with higher cutoffs are thinner.
Figure V.4: Reflection and transmission of Gaussian beams incident on a band pass filter. The band pass filter has $\theta_c = 24^\circ$ and $58^\circ$, $k_c d = 4\pi$, and $\gamma = 0.01$. The beams have width $10\lambda_0$ and incident angles $9^\circ$, $34^\circ$, and $69^\circ$. The scale on the transmission side is compressed to compensate for the attenuation of the beam.
Figure V.5: Low, high and band pass filtering of a square wave object. The square wave has a fundamental period of $\lambda_0$. The low pass filter has cutoff, $k_c/k_0 = 5$. The band pass filter has cutoffs, $k_c/k_0 = 5$ and $20$, and the high pass filter has cutoff, $k_c/k_0 = 20$. $\gamma = 0.01$ and $k_cd = 4\pi$ for all three filters.
The first is just the identity matrix and the second is an indefinite matrix. Note that they are both $x$-$y$ isotropic. Though the solutions are computed above for a specific orientation - where the plane of incidence is the $x$-$z$ plane - the devices discussed here will work equally well for a $y$-$z$ plane of incidence, or any angle in between.

Low pass filtering only requires isotropic media. We write the material properties explicitly in terms of the cutoff, $k_c$.

\[
\begin{align*}
\varepsilon_1 &= \mu_1 = \left( \frac{k_c}{k_0} + i\gamma \right) \sigma_0 \\
\varepsilon_2 &= \mu_2 = \left( -\frac{k_c}{k_0} + i\gamma \right) \sigma_0
\end{align*}
\]

$\gamma \ll 1$ is the parameter that introduces material loss. The cutoff, $k_c$, determines the upper limit of the pass band. This bilayer device has one layer of positive and one layer of negative media. Note that $\varepsilon = \mu$ for both layers, so this device will be polarization independent. We have included a small imaginary part, representing material loss. Adjusting this loss parameter and the layer thickness controls the filter roll off characteristics.

High pass filtering requires indefinite material property tensors.

\[
\begin{align*}
\varepsilon_1 &= \mu_2 = \frac{k_c}{k_0} \sigma_1 + i\gamma \sigma_0 \\
\varepsilon_2 &= \mu_1 = -\frac{k_c}{k_0} \sigma_1 + i\gamma \sigma_0
\end{align*}
\]

Here the cutoff, $k_c$, determines the lower limit of the pass band. Note that $\varepsilon \approx -\mu$ for both layers, so this device will not be polarization independent. Swapping $\varepsilon \leftrightarrow \mu$ will effectively change the layer order. However, the layer order only affects some subtle properties of the internal field and does not affect the overall transmission or reflection properties, so this device will be externally polarization independent.
Band pass filters are composed of four layers. A bilayer for the low pass filtering and another bilayer for the high pass filtering. Since the second bilayer reflects waves back into the first, one cannot rigorously compute the band pass coefficients by multiplying two bilayer solutions together. A four layer transfer matrix solution is required.

The transmission and reflection coefficient is shown for low, high and band pass filters with various cutoffs in Figs. V.2 and V.3. Fig. V.2 shows filters with cutoffs in the free space, propagating range, \( k_c < k_0 \). The independent variable is given as an angle,

\[ \theta = \sin^{-1} \frac{k_x}{k_0}, \]

since in this range the incident plane waves propagate in real directions. For incident propagating waves, the reflection and transmission coefficients must, and do obey

\[ |\rho|^2 + |\tau|^2 < 1 \]

to conserve energy. Fig. V.3 shows filters with cutoffs in the free space, evanescent range, \( k_c > k_0 \). The independent variable is given as, \( k_x/k_0 \). These filters can be used to process near fields. In this range, transmission and reflection coefficients need not be bound to values less than one since evanescent waves do not carry energy. The upper \( k_x \) roll off seen in the figure is undesirable for a high pass filter. This roll off, discussed in detail elsewhere [41], is determined by the loss, \( \gamma \), and the thickness, \( d \).

Fig. V.4 shows the field intensity pattern for several Gaussian beams incident on a band pass filter with cutoffs in the propagating range. This multi-layer structure transmits beams that are in a mid angle range and reflects beams that are incident at small and large angles. This device is unusual in two ways. Standard materials cannot reflect normally incident beams and transmit higher angle ones. Also, though an upper critical angle is not unusual, it can only occur when a beam is incident from a higher index media to a lower index media, and not for a beam incident from free space, as it is in this case. The action of the
compensating layers also permits a greater transmittance with less distortion than is possible with any single layer of normal materials. While it is possible to implement an angular filter with a channeled, “mini-blinds” structure, such a device could not be both translationally and rotationally invariant like the homogenous implementation described here.

Fig. V.5 shows the effect of three different filters on an electromagnetic spatial variation that has significant near field content. The spatial variation is analogous to a diffraction grating back illuminated with a plane wave. The results of low, high and band pass filtering are shown. The high pass filter shows the edge detection properties of high pass filters. In this case, the edge information is predominately in the near field components that would be lost in normal optical components.

V.F Conclusion

We have shown how media with negative electromagnetic property tensor elements can be used to construct high, low and band pass spatial filters with cutoffs tunable both above and below the near/far field boundary. The far field filters are particularly flexible and can be used either in transmission or reflection. We believe that these filters could find many uses in image processing and communications applications.

V.G Acknowledgement

This chapter, in full, will be submitted for publication, with authors D. Schurig and D. R. Smith.
Bibliography

[30] These unusual classes of dispersion are not unknown in the context of electrons moving in a periodic potential. When an electronic band structure has a saddle point the effective mass tensor is indefinite and analogous behavior is found.