According to Wikipedia ([https://en.wikipedia.org/wiki/Dipole_antenna](https://en.wikipedia.org/wiki/Dipole_antenna)) the ideal half-wave dipole has a directivity of 2.15 dBi, which is very close to the directivity found here, 2.104 dBi. The discrepancy is likely within the numerical error introduced by the course mesh. (The finite radius of the wire simulated and the corresponding length adjustment may also contribute.)
The symmetry planes chosen are:

- x-normal (YZ) - magnetic ($H_t = 0$)
- y-normal (ZX) - electric ($E_t = 0$)
- z-normal (XY) - none

The objects have mirror symmetry in the x-normal and y-normal planes, but not in the z-normal plane, since we have two different objects along the z-direction.

The excitation mode we would like to have for our origin-centered, y-oriented dipole antenna, has local electric field lines connecting the symmetric charges in the two dipole halves. Thus the electric field is normal to the $y = 0$ plane, and a y-normal electric symmetry plane may be used.

The corresponding magnetic field circulates around the current flowing along the y-oriented dipole wires. Any plane containing the y-axis could be set as a magnetic symmetry plane, including the x-normal plane.

See the electric dipole fields below.
4. The ideal directivity of a uniform aperture is given by

\[ D_u = \frac{4\pi}{\lambda^2} A_p \]

where \( \lambda \) is the operating free-space wavelength and \( A_p \) is the physical aperture area. In our case the physical aperture area, or cross-sectional area is

\[ A_p = \pi r^2 = \pi \left( \frac{10\lambda}{2} \right)^2 = 25\pi\lambda^2 \]

so that the ideal directivity is

\[ D_u = \frac{4\pi}{\lambda^2} 25\pi\lambda^2 = 100\pi^2 = 987 = 30 \text{ dBi} \]

In our simulation we have found a directivity of 24.7 dBi. There are several possible explanations for this discrepancy of 5 dB.

* The feed is omnidirectional so half the power does not enter the lens to be focused to the desired direction. (This accounts for about 3dB of the 5dB discrepancy.)
* The lens only functions as an ideal converter of spherical-waves to planar-waves in the geometric limit, i.e when the size of the lens is extremely large compared to wavelength. For our modest electromagnetically sized lens the aperture fields are not exactly uniform and plane-wave like, as can be seen in the near-field plot.
* The feed (our half-wave dipole) is an imperfect source of spherical waves, so a perfect conversion to a plane-wave like aperture field would not be expected even from an ideal lens.