

Trigonometry for the Loop Antenna

$$\hat{\mathbf{r}} = \hat{\mathbf{x}} \sin \theta \cos \phi + \hat{\mathbf{y}} \sin \theta \sin \phi + \hat{\mathbf{z}} \cos \theta$$

$$\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos \theta \cos \phi + \hat{\mathbf{y}} \cos \theta \sin \phi - \hat{\mathbf{z}} \sin \theta$$

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi$$

$$\theta' = \frac{\pi}{2}$$

$$\hat{\mathbf{r}}' = \hat{\mathbf{x}} \cos \phi' + \hat{\mathbf{y}} \sin \phi'$$

$$\hat{\boldsymbol{\theta}}' = -\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}}' = -\hat{\mathbf{x}} \sin \phi' + \hat{\mathbf{y}} \cos \phi'$$

$$\psi \equiv \phi' - \phi$$

$$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}' = (\hat{\mathbf{x}} \sin \theta \cos \phi + \hat{\mathbf{y}} \sin \theta \sin \phi + \hat{\mathbf{z}} \cos \theta) \cdot (\hat{\mathbf{x}} \cos \phi' + \hat{\mathbf{y}} \sin \phi') = \sin \theta (\cos \phi \cos \phi' + \sin \phi \sin \phi') = \sin \theta \cos \psi$$

$$\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}}' = (\hat{\mathbf{x}} \sin \theta \cos \phi + \hat{\mathbf{y}} \sin \theta \sin \phi + \hat{\mathbf{z}} \cos \theta) \cdot (-\hat{\mathbf{x}} \sin \phi' + \hat{\mathbf{y}} \cos \phi') = \sin \theta (-\cos \phi \sin \phi' + \sin \phi \cos \phi') = -\sin \theta \sin \psi$$

$$\hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}}' = (\hat{\mathbf{x}} \cos \theta \cos \phi + \hat{\mathbf{y}} \cos \theta \sin \phi - \hat{\mathbf{z}} \sin \theta) \cdot (-\hat{\mathbf{x}} \sin \phi' + \hat{\mathbf{y}} \cos \phi') = \cos \theta (-\cos \phi \sin \phi' + \sin \phi \cos \phi') = -\cos \theta \sin \psi$$

$$\hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}}' = (-\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi) \cdot (-\hat{\mathbf{x}} \sin \phi' + \hat{\mathbf{y}} \cos \phi') = \sin \phi \sin \phi' + \cos \phi \cos \phi' = \cos \psi$$

$$\hat{\boldsymbol{\phi}}' = \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}}') + \hat{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}}') + \hat{\boldsymbol{\phi}}(\hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}}') = -\hat{\mathbf{r}} \sin \theta \sin \psi - \hat{\boldsymbol{\theta}} \cos \theta \sin \psi + \hat{\boldsymbol{\phi}} \cos \psi$$